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ON OPTIMIZATION OF EARTH COVERAGE CHARACTERISTICS FOR COMPOUND SATELLITE CONSTELLATIONS BASED ON ORBITS WITH SYNCHRONIZED NODAL REGRESSION

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The Earth coverage provided by the satellites' swath for the new type of satellite constellations – compound, multi-tiered, constellations with satellites' orbits of different altitude and inclination and synchronized nodal regression – is considered. The method for compound satellite constellation design is described and illustrated. Special attention is given to both continuous and periodic Earth coverage. It is shown that using compound satellite constellations for providing different types of Earth coverage makes it possible to sufficiently improve the Earth coverage, as compared to the traditional constellations based on common altitude and inclination for all the satellites of the constellation, and, as a consequence, to get new opportunities for the satellite constellation design for different types of prospective space systems (remote sensing, communications, etc.) regarding increasing the quality of solving their tasks or/and minimization of the number of the satellites required. At the same time, the condition of synchronized nodal regression of all the satellites in the compound constellations considered provides the delta-V (fuel) budget on-board the satellites, which is necessary for constellation station keeping, being on the same level as for traditional satellite constellations.

INTRODUCTION

For many years since the beginning of space era the satellite constellations, both for continuous and discontinuous (periodic) Earth observation missions, have been designed using orbits with common values of altitude and inclination for all the satellites in the constellation. The reason is quite understandable: if the values of two mentioned parameters are different for the satellites in the constellation, the orbital structure, in common case, becomes unstable due to different nodal regression and an extra delta-V (fuel) budget on-board the satellites is required for constellation station keeping. Due to this reason almost every known solution for design of satellite constellations for coverage of large Earth regions is based on the use of circular orbits with equal altitudes and inclinations. Indeed, all the known methods for the continuous and periodic coverage satellite constellations design – both pioneering approaches (References 1–4) to the problem, and

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all subsequent works in the area till the present time (for instance, References 5–11) – are based on circular orbits with equal values of altitude and inclination.

An a priori assumed condition of constant values of altitude and inclination for the satellites of the constellation is associated with inefficient Earth coverage by the satellite swath, characterized by unlike upper and lower latitude Earth coverage. It leads to the decrease of quality of the space observation missions for lower latitudes or/and to the increase of the number of satellites in the constellations. In this paper a method for the optimization of continuous and periodic Earth coverage basing on the extension of traditional satellite constellation optimization domain with constellations incorporating orbits with different altitudes and inclinations is presented. It is shown and illustrated that the method suggested provides a much more efficient Earth coverage and, as a consequence, leads to the improvement of continuous and periodic coverage characteristics or/and the reduction of the required number of satellites in the constellation.

DESCRIPTION OF THE METHOD

Every beginning satellite constellations researcher, while starting the study of the optimization problem of the Earth coverage, initially notes that for an arbitrary satellite constellation with equal altitudes and inclinations the coverage of lower and upper latitudes is quite uneven with regard to the density of observational stream: the coverage of lower latitudes is a great deal worse than that of the upper ones. At the same time, given that the flight dynamics for such satellites doesn't allow any changes, the researcher gradually reconciles with this thought and takes this fact for granted. In this paper we do not follow this way.

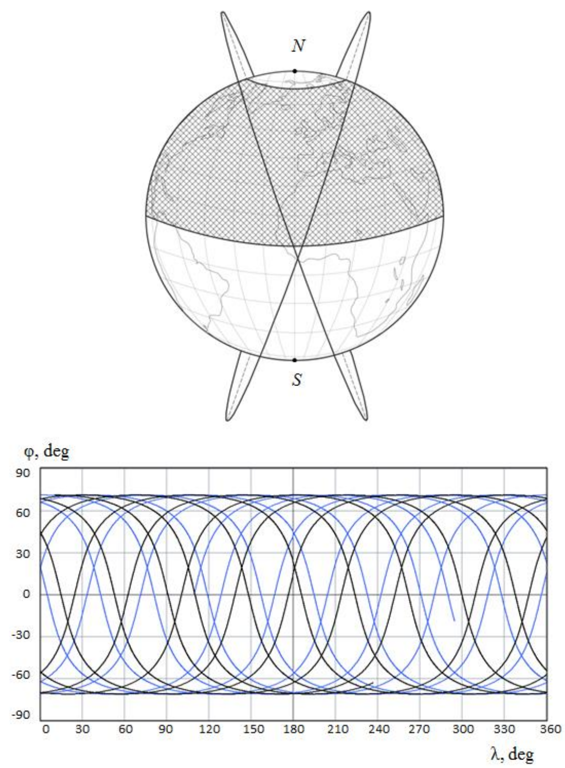

Figure 1. Traditional two-satellite constellation and a corresponding ground-track (H = 3000 km, i = 71°)

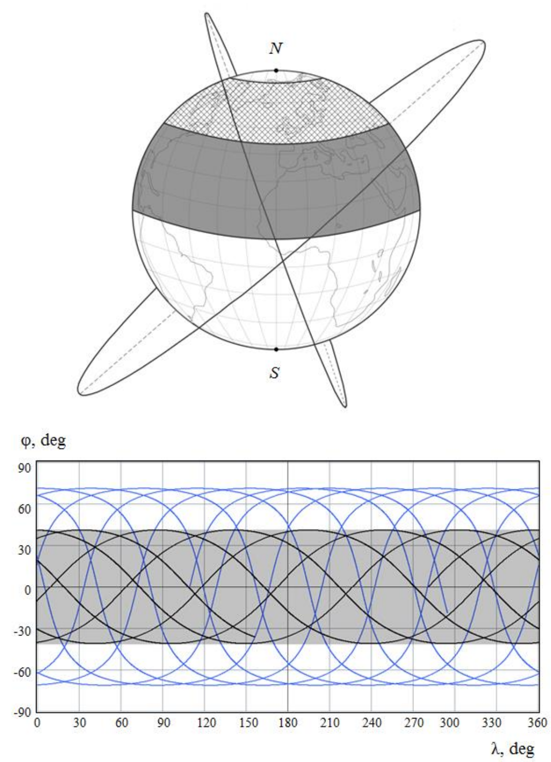

Figure 2. Compound two-satellite constellation incorporating two NR-synchronized orbits and a corresponding ground-track (H_1 = 3000 km, i_1 = 71°, H_2 = 5545 km; i = 41°)

Indeed, as one can see in Figure 1, the density of tracks' overlap and, correspondingly, density of the satellite swaths' overlap, will be redundant on the upper latitudes even in case of critical coverage of near-equatorial regions. Traditional prejudices aside, we intuitively realize that we would like to move a part of redundant overlaps to the lower latitudes. We also understand that when utilizing the orbits with equal inclinations, we cannot achieve this purpose because of the peculiarities of the satellites' motion along such orbits. At the same time, varying the inclinations of certain satellites in general case is followed by different nodal regression of the satellites of the constellation, and by rapid orbital structure degradation, and – in the case of its station-keeping – by enormous, practically unacceptable required delta-V on-board budget.

Solution which underlies the suggested method consists in using so-called multi-tiered constellations, which are characterized by an equal nodal regression rate for all their satellites[12]. Satellites in different tiers have different inclinations, and thus a more effective, more even, coverage of the Earth surface is provided. In Figure 2 one can see, how the Earth coverage is transformed even with addition of the second tier. We can see that, just as we wanted, redundant coverage of the upper latitudes "travelled" to the lower latitudes, providing a more even resulting coverage of the whole observed latitudinal belt.

In general, the Earth coverage could be provided by the compound, multi-tiered, satellite constellation, incorporating separate conventional systems (subsystems, tiers) with similar values of altitude and inclination for the satellites of each tier. These systems consist of $N = \sum_{j=1}^{m} n_j$ satellites, where m is the number of tiers in the system, n_j– the number of satellites on the j^{th} tier. The values of altitude H_j (radius r_j) and inclination i_j for each tier of the multitiered constellation are calculated implementing special method so that the nodal regression rate is kept equal for all the satellites in the constellation, as will be shown further. The term of "tiering" used to describe these systems implies different tiers including one or more satellites. Tiers may be arbitrarily distributed in the near-Earth space on circular orbits with various inclinations.

The considered approach to forming the tiers of the compound (multi-tiered) constellations is based on designing special combinations of altitude H_j and inclination i_j of the satellites on these tiers, thus providing equal ascending nodes regression rate for all the satellites in constellation.

Let us designate two or more orbits as *NR (Nodal Regression)-synchronized orbits*, if they differ in altitude and inclination values and have equal ascending nodes precession rate. Taking this into account, the reviewed method of the tiers formation results directly from the following property of multitiered satellite systems: arbitrarily chosen m satellites of the multitiered constellation, one per each of m tiers, possess *NR-synchronized orbits.*

Each tier of the system is characterized by the values of altitude (radius) and inclination, these two parameters determining the magnitudes of the ascending node $\Delta\Omega_j$ regression and the nodal period T_{Ω_j}:

$$\Delta\Omega_j(r_j, i_j), \;\; T_{\Omega_j}(r_j, i_j), \;\; j = \overline{1, m}. \tag{1}$$

Equal ascending nodes precession rate condition for satellites in different tiers is as follows:

$$\frac{\Delta\Omega_1}{T_{\Omega_1}} = \frac{\Delta\Omega_2}{T_{\Omega_2}} = \ldots = \frac{\Delta\Omega_m}{T_{\Omega_m}}, \tag{2}$$

where $\Delta\Omega_j$ and T_{Ω_j} are, respectively, ascending node regression per one revolution and nodal period for the satellite on the j^{th} tier. Their values can be calculated by the following Equations[13]:

$$T_{\Omega_1} = 2\pi \frac{r_j^{3/2}}{\sqrt{\mu}} \left[1 - \frac{3}{8} J_2 \left(\frac{R_E}{r_j} \right)^2 \left(7\cos^2 i_j - 1 \right) \right]; \tag{3}$$

$$\Delta\Omega_j = -3\pi J_2 \left(\frac{R_E}{r_j} \right)^2 \cos i_j . \tag{4}$$

The basic relation between parameters of the *NR-synchronized orbits* of the satellites composing a multitiered system could be given by the following Equation:

$$\varphi(r_1 + \Delta r_j, i_1 + \Delta i_j) = \frac{\Delta\Omega_j (r_1 + \Delta r_j, i_1 + \Delta i_j)}{T_{\Omega_j} (r_1 + \Delta r_j, i_1 + \Delta i_j)} = C_{cr}, \quad j = \overline{2, m} \tag{5}$$

$$C_{cr} = \frac{-3\pi J_2 \left(\frac{R_E}{r_1} \right)^2 \cos i_1}{2\pi \frac{r_1^{3/2}}{\sqrt{\mu}} \left[1 - \frac{3}{8} J_2 \left(\frac{R_E}{r_1} \right)^2 \left(7\cos^2 i_1 - 1 \right) \right]} \tag{6}$$

In this relation the parameters of the first tier are held fixed. Altitude and inclination for the orbits of the j^{th} tier are written in deviations from the corresponding parameters of the first tier. The relation is specified by a constant value C_{cr} which is calculated from the parameters of the first tier. It is evident from these equations, that one could NR-synchronize the tiers by means of varying the parameters Δr_j and Δi_j.

As it results from the aforesaid, the equations for obtaining the orbit parameters of the satellites comprising a multi-tiered constellation can be written in the form

$$\varphi(r_1 + \Delta r_j, i_1 + \Delta i_j) = C_{cr}, \tag{7}$$

where the constant value C_{cr} is defined by the parameters of the first-tier circular orbits.

Equation (5) is solved numerically for one of the unknown variables Δr_j, Δi_j. The restriction of equality was imposed on the remaining variables on account of the requirements to the constellation.

The way of solving of Equation (7) is graphically analyzed in Figure 3. The variation range of the orbit parameters is reviewed with respect to the first-tier-orbits. The figures represent various dependences which may be of some interest in terms of the methodology for synchronizing j^{th} tier with the first one. For the sake of simplicity the figures feature each j^{th} tier designated with an index of “2”.

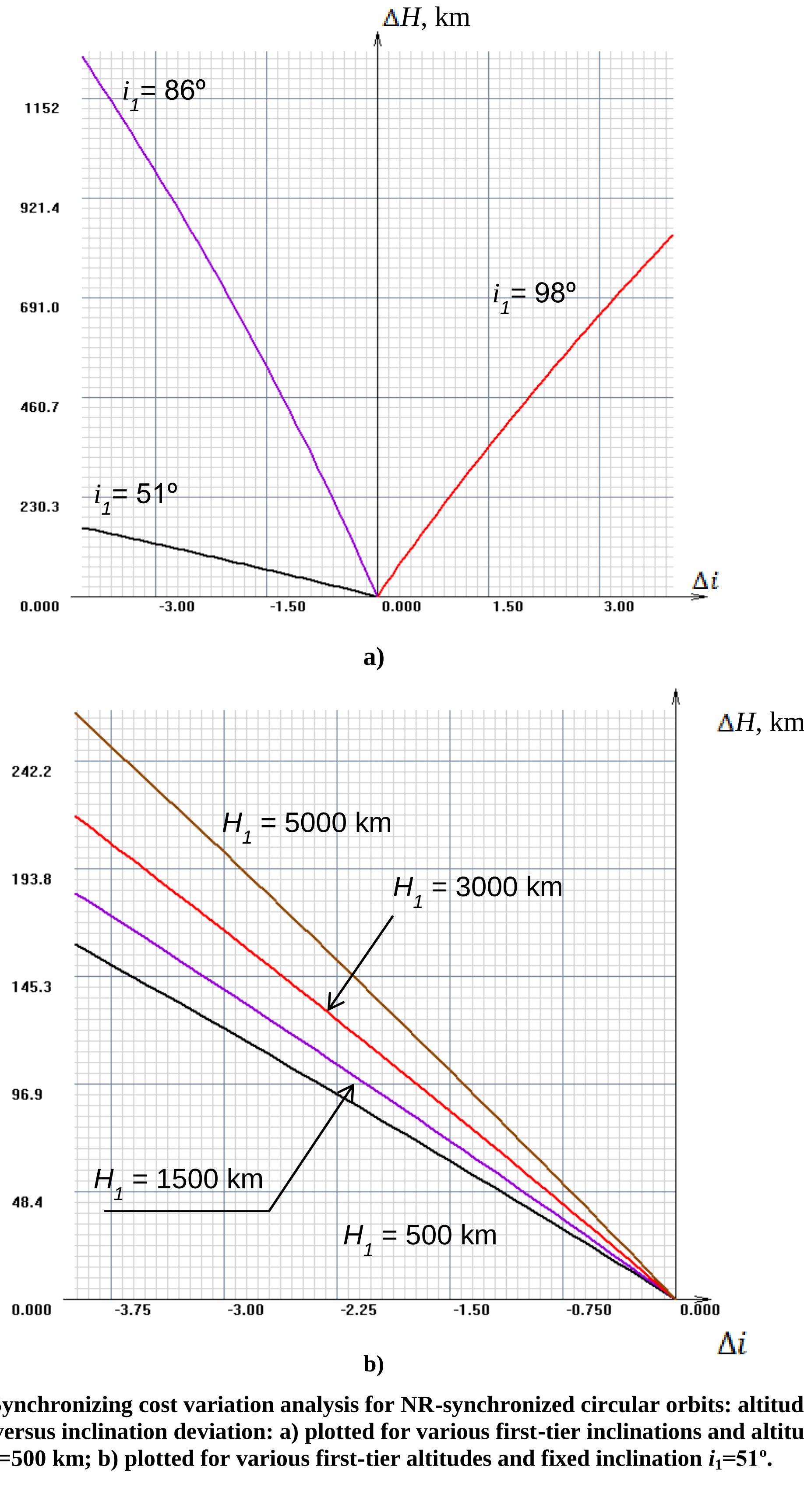

Figure 3. Synchronizing cost variation analysis for NR-synchronized circular orbits: altitude deviation versus inclination deviation: a) plotted for various first-tier inclinations and altitude H_1=500 km; b) plotted for various first-tier altitudes and fixed inclination i_1=51°.

For circular orbits considred the function was plotted, as follows:

$$\Delta r_j = f_r(\Delta i_j / r_1, i_1) \tag{8}$$

Figure 3 presents a graphical analysis of this dependence by plotting the orbit radius deviation versus the inclination deviation. Figure 3 reveals the variation of synchronizing cost required for altering the orbit altitude and for orbital plane change. Figure 3a presents the trend for various first-tier inclinations i_1 and fixed first-tier altitude H_1, while Figure 3b incorporates the dependences for fixed i_1 value with H_1 being varied.

It is shown in the Figure 3a, that with first-tier inclination drawing near 90° (polar inclination) higher synchronizing cost is required for obtaining a second tier: for the same inclination deviation the resulting altitude (radius) deviation would be greater. This fact is due to the orbital precession growing weaker at such inclinations. The same tendency of synchronizing cost growth is true for the case presented on Figure 3b: the higher the first-tier altitude, the lesser the precession rate becomes, and the greater synchronizing cost are required for obtaining the second tier. It also should be noted, that in order to obtain a second synchronized tier at a higher altitude the inclination (in case of posigrade orbits) should be reduced, i.e. $\Delta i_j < 0$.

Taking the aforesaid into account, the essence of the method suggested consists in selection of the satellite orbits in different tiers of the multi-tiered constellation for a given amount of satellites in each tier (with regard to additional optimization of the number of satellites in separate tiers, as well as in constellation as a whole) for the required observed Earth region and set parameters of the on-board equipment. As one can see from a quantitative description of the method conducted above, this method allows considerably extending the domain of optimized satellite constellation parameters by means of getting an opportunity of distributing the satellites in several tiers with various values of altitude and inclination. However the question now arises, to what extent this could contribute to the improvement of obtained optimal constellations' parameters in comparison with the best known solutions for the traditional case of a single-tier satellite distribution, i.e. distribution on the orbits of equal altitudes and inclinations.

To answer this question, let us now consider the implementation of the method described above for two basic types of the Earth surface coverage – continuous and periodic coverage. Orbital formation of separate subsystems of the entire compound constellation could be based (as it is supposed henceforth) on the use of the existing theoretical apparatus for the optimization of traditional satellite constellations on orbits with equal altitudes and inclinations. However, an optimal (rational) fusion of separate tiers into one compound constellation requires a development of an additional theoretical basis – separately for either of the two essentially different problems of continuous and periodic coverage.

With regard to the aforesaid, specially developed theoretical elements of multi-tiered satellite constellations optimization for continuous and periodic coverage are briefly described below. Numerical characteristics of the best obtained multi-tiered constellations and best known traditional single-tiered constellations are also given below in common comparison conditions for continuous and periodic Earth coverage.

IMPLEMENTATION FOR CONTINUOUS EARTH COVERAGE

Theoretical Approach

It appeared to be useful to implement an N-satellite ($N = 4, 6, \ldots$) Walker monostructure as a basic element for different tiers while designing a multi-tiered system. Such Walker system with a fixed altitude H and inclination i can be designated by the following formula:

$$\Omega_{ij} = \begin{cases} 0, i=1,\ j=1,2,\ldots,N/2, \\ \pi, i=2,\ j=1,2,\ldots,N/2, \end{cases} \quad u_{ij} = \begin{cases} 0, i=1,2;\ j=1, \\ \Delta\cdot(j-1), i=1,2;\ j=2,\ldots,N/2, \end{cases} \tag{9}$$

where $\Delta = 4\pi/N$; N — the number of satellites in the system.

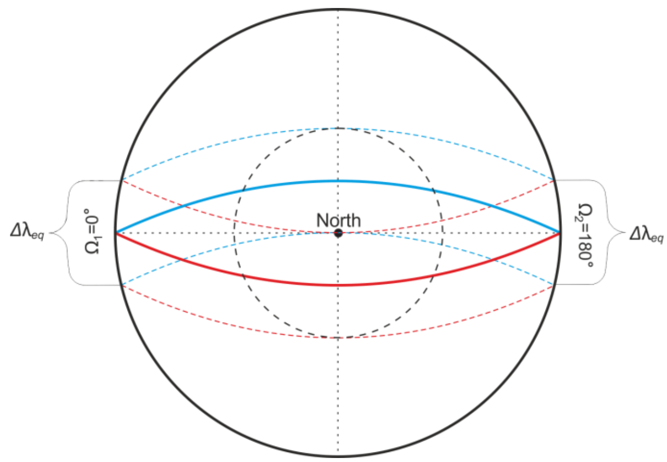

Figure 4. Typical Boat System (the polar coverage case is shown)

The aforementioned two-plane orbital structure with even number of satellites is presented in Figure 4. We will denote it basing on its peculiar shape as a "Boat System".

Let us consider the process of designing the two-tiered satellite system out of several Boat Systems. We assume that the first tier of the compound system is used for polar cap coverage and consists of one N_1–satellite Boat System with lesser altitude H_1 and greater inclination i_1 than that in the second tier of the compound constellation. Such first tier orbital structure ensures the coverage of the polar cap (the constraint $i_1 + \theta_1 > \pi/2$ is met) and some partial coverage for the lower latitudes covering the arc $\Delta\lambda_{eq}$ on the equator:

$$\Delta\lambda_{eq} = 2\arcsin\left(\sin\left(\arccos\left(\cos\theta_1 \Big/ \cos\frac{2\pi}{N_1}\right)\right)\Big/\sin i_1\right) \tag{10}$$

where N_1 – the number of satellites in the first tier of the Boat System; geocentric angle θ_1 of coverage of the first tier satellite is defined by equation: $\theta_1 = \arcsin\left(\frac{R_E + H}{R_E}\sin\beta_1\right) - \beta_1$; β_1 – span angle of satellite on-board equipment.

The main idea of designing double-tiered satellite system out of basic Boat Systems consists in adding to the first tier the two Boat Systems on the second tier basing on greater altitudes H_2 and lower inclinations i_2 for fulfilling the missing coverage of lower latitudes as well in providing special relative phasing of satellites within these two Boat Systems.

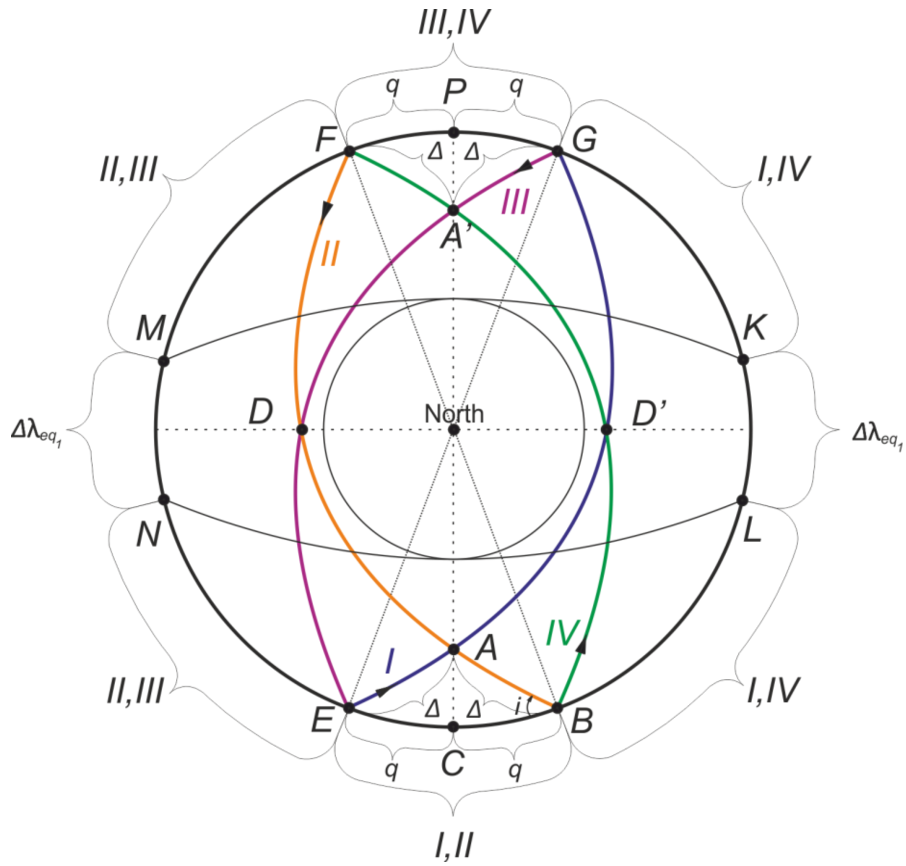

Figure 5. Satellite phasing derivation

Let us number the four orbital planes which form such constellation out of two Boat Systems according to the sequence of their ascending nodes *E*, *F*, *G*, *B*, as shown in Figure 5.

Due to the fact that continuous coverage of the lower latitudes by such constellation is lacking, it proved reasonable for provision of this coverage to synchronize successive (i.e. at regular time intervals) passes of the *A* point by satellites from the I and II orbits, as well as the *A′* point by the satellites from III and IV orbits.

The conditions for such synchronization of the motion of satellites from orbits I, II, III and IV are represented in the following way:

$$\begin{cases} u_j^{(1)} = (j-1)\dfrac{2\pi}{n} \\ u_j^{(2)} = \pi + (j-1)\dfrac{2\pi}{n} - \dfrac{\pi}{n} - 2\Delta \end{cases}; \tag{11}$$

$$\begin{cases} u_j^{(3)} = p + (j-1)\dfrac{2\pi}{n} \\ u_j^{(4)} = p + \pi + (j-1)\dfrac{2\pi}{n} - \dfrac{\pi}{n} - 2\Delta \end{cases}, \quad (12)$$

where $u_j^{(k)}$ – an argument of latitude of the j^{th} satellite from k^{th} orbit; $k = 1,2,3,4;\quad j = 1,2,...,n$; $n = N/2$; N – a number of satellites in the Boat System ($N/2$ is a number of satellites in each of the orbit I, II, III, IV); Δ – an angular distance between the ascending node E (G) of the first (third) orbit and the point A (A′) of the intersection with the second (fourth) orbit, derived from the spherical triangle ACE by the formula $\Delta = arctg(tg\, q / \cos i_2)$; q – an angular distance EC (PG) from the ascending node of the satellites of the first (third) orbit to the PC line of symmetry of the resulting orbital structure formed by two Boat Systems (see Figure 5).

The procedure of optimization for such constellation of two Boat Systems consists in enumeration of the number of satellites N, relative phasing value p of the orbital structures (11) and (12) and the angular distance q. Such procedure is performed after altitudes H_1, H_2 and inclinations i_1, i_2 were selected for each tier.

Below the results of such optimization are presented in comparison with the known variants of traditional single-tiered constellations.

Numerical Results

As a reference, Table 1 lists the best known satellite constellations for the global one-fold continuous Earth coverage for a given number of satellites N. These variants result from the minimization of the required geocentric width θ of the satellite coverage area under the given constraints on the set of all possible inclinations and orbits equal for all the satellites of the constellation. At the same time, Table 1 lists the altitude of the orbits and the span angle of on-board equipment calculated for the elevation angle of 10°.

Table 1. Best traditional, single-tiered, variants of satellite constellations for global continuous Earth coverage obtained on the set of all possible inclinations.

№	1	2	3	4	5
N	20	22	24	26	28
θ, deg	36.48	34.01	32.81	31.11	30.16
H, km	2744	2355	2184	1959	1841
β, deg	43.54	45.99	47.19	48.89	49.84

Table 2 contains the double-tiered satellite constellations obtained under the conditions of each variant from Table 1. The altitude H listed in Table 1 for the traditional single-tiered constellation is assumed as altitude of the orbits for first tier H_1 of the developed double-tiered constellation. Inclination i_1 of the first-tier orbits, as well as altitude H_2 and inclination i_2 of the orbits of the second tier were computed according to the method described above.

Table 2. Double-tiered satellite constellations for global continuous Earth coverage under conditions of the constellations listed in the Table 1.

№	β, deg	N	Constellation (number of satellites in the j^{th} tier, altitude H, km, inclination i, deg, longitude of ascending node Ω, deg, argument of latitude u, deg)
1	43.54	18	N_1 = 10; H_1 = 2744.0 (i_1 = 75.10); Ω_1 = 0; u_{11} = 0; u_{12} = 72.00; u_{13} = 144.00; u_{14} = 216.00; u_{15} = 288.00; Ω_2 = 180.00; u_{21} = 0; u_{22} = 72.00; u_{23} = 144.00; u_{24} = 216.00; u_{25} = 288.00; N_2 = 8; H_2 = 5174.6 (i_2 = 54.00); Ω_1 = 37.97; u_{11} = 0; u_{12} = 180.00; Ω_2 = 322.03; u_{21} = 319.28; u_{22} = 139.28; Ω_3 = 217.97; u_{31} = 271.74; u_{32} = 91.74; Ω_4 = 142.03; u_{41} = 231.03; u_{42} = 51.03.
2	45.99	20	N_1 = 12; H_1 = 2355.0 (i_1 = 69.00); Ω_1 = 0; u_{11} = 0; u_{12} = 60.00; u_{13} = 120.00; u_{14} = 180.00; u_{15} = 240.00; u_{16} = 300.00; Ω_2 = 180.00; u_{21} = 0; u_{22} = 60.00; u_{23} = 120.00; u_{24} = 180.00; u_{25} = 240.00; u_{26} = 300.00; N_2 = 8; H_2 = 4199.9 (i_2 = 45.50); Ω_1 = 36.81; u_{11} = 0; u_{12} = 180.00; Ω_2 = 323.19; u_{21} = 325.35; u_{22} = 145.35; Ω_3 = 216.81; u_{31} = 263.39; u_{32} = 83.39; Ω_4 = 143.19; u_{41} = 228.75; u_{42} = 48.75.
3	47.19	20	N_1 = 12; H_1 = 2184.0 (i_1 = 70.75); Ω_1 = 0; u_{11} = 0; u_{12} = 60.00; u_{13} = 120.00; u_{14} = 180.00; u_{15} = 240.00; u_{16} = 300.00; Ω_2 = 180.00; u_{21} = 0; u_{22} = 60.00; u_{23} = 120.00; u_{24} = 180.00; u_{25} = 240.00; u_{26} = 300.00; N_2 = 8; H_2 = 4269.7 (i_2 = 45.00); Ω_1 = 35.03; u_{11} = 0; u_{12} = 180.00; Ω_2 = 324.97; u_{21} = 322.73; u_{22} = 142.73; Ω_3 = 215.02; u_{31} = 276.77; u_{32} = 96.77; Ω_4 = 144.97; u_{41} = 239.49; u_{42} = 59.49.
4	48.89	20	N_1 = 12; H_1 = 1959.0 (i_1 = 76.00); Ω_1 = 0; u_{11} = 0; u_{12} = 60.00; u_{13} = 120.00; u_{14} = 180.00; u_{15} = 240.00; u_{16} = 300.00; Ω_2 = 180.00; u_{21} = 0; u_{22} = 60.00; u_{23} = 120.00; u_{24} = 180.00; u_{25} = 240.00; u_{26} = 300.00; N_2 = 8; H_2 = 5058.3 (i_2 = 43.00); Ω_1 = 32.17; u_{11} = 0; u_{12} = 180.00; Ω_2 = 327.83; u_{21} = 310.10; u_{22} = 130.10; Ω_3 = 212.17; u_{31} = 256.99; u_{32} = 76.99; Ω_4 = 147.83; u_{41} = 207.09; u_{42} = 27.09.
5	49.84	24	N_1 = 12; H_1 = 1841.0 (i_1 = 80.00); Ω_1 = 0; u_{11} = 0; u_{12} = 60.00; u_{13} = 120.00; u_{14} = 180.00; u_{15} = 240.00; u_{16} = 300.00; Ω_2 = 180.00; u_{21} = 0; u_{22} = 60.00; u_{23} = 120.00; u_{24} = 180.00; u_{25} = 240.00; u_{26} = 300.00; N_2 = 12; H_2 = 5101.8 (i_2 = 56.00); Ω_1 = 30.68; u_{11} = 0; u_{12} = 120.00; u_{13} = 240.00; Ω_2 = 329.33; u_{21} = 336.70; u_{22} = 96.70; u_{23} = 216.70; Ω_3 = 210.68; u_{31} = 236.68; u_{32} = 356.68; u_{33} = 116.68; Ω_4 = 149.33; u_{41} = 213.38; u_{42} = 333.38; u_{43} = 93.38.

Span angles of the satellite on-board equipment for the double-tiered constellations listed in Table 2 were assumed in each case equal to the ones in Table 1.

By analyzing the constellations from Table 1 and Table 2 it may be stated that the use of double-tiered constellations allows considerably reducing the required number of satellites in the constellation for the considered one-fold continuous Earth coverage, as compared to the analogous traditional single-tiered constellations. Depending on the variant considered, the number of satellites could be reduced by 2–6.

IMPLEMENTATION FOR PERIODIC EARTH COVERAGE

Theoretical Approach

So-called Secure Route Constellations are used as tier-composing orbital formations for the synthesis of compound satellite constellation for periodic coverage[11]. Orbits of Secure Route Constellation are geosynchronous and their ground tracks repeat every m satellite revolutions and n effective astronomical days in accordance with the following condition of geosynchrony:

$$T_{tr} = m \cdot T_{nod} = n \cdot T_{ef}\,, \tag{14}$$

where T_{nod} is a satellite nodal period; T_{ef} – effective astronomical day; m, n – integer relatively prime numbers characterizing the number of revolutions and the number of efficient astronomical days within the repetition period of the satellite track T_{tr}. Together numbers m and n define a repetition factor of a geosynchronous orbit – m/n. The value $m/n = T_{ef}/T_{nod}$ is a rational number and stands for the repetition factor of the geosynchronous orbit. At the same time, a notion of a real repetition factor $k = T_{ef}/T_{nod}$ is considered, being applicable to both geosynchronous orbits and orbits with non-repeating tracks.

A phase structure of the satellite is defined according to the following considerations: the satellites of Secure Route Constellation move along the same route with a constant relative time-lag $\Delta t = \tau / N$, where τ is a revisit time for the single-satellite coverage of a given Earth region, N is a number of satellites in Secure Constellation. The use of such systems allows providing the Earth regions revisit time on par with the best variants of periodic coverage constellations, obtained in classes of Walker systems and “Streets of Coverage” constellations[11]. Given that the phase structure of Secure Constellations is calculated in a relatively simple way, and due to efficient performance of such constellations, they could be efficiently used for the mass calculations.

Satellite orbits in the second tier (their altitude H_2 and inclination i_2) are so selected, that two conditions are satisfied simultaneously: geosynchrony condition for the satellite motion in each tier and equal nodal regression condition for the satellite orbits from different tiers. The algorithm used to this end incorporates the aforementioned concept of the repetition factor $k = T_{ef}/T_{nod}$ as a real number, which is defined by the following formula for the orbits in the second tier:

$$k_2 = \left\{ \omega_E \frac{(R_E + H_2)^{3/2}}{\sqrt{\mu}} \left[1 - \frac{3}{8} J_2 \left(\frac{R_E}{R_E + H_2} \right) \left(7\cos^2 i - 1 \right) \right] + \frac{3\pi}{2} J_2 \left(\frac{R_E}{R_E + H_2} \right)^2 \cos i \right\}^{-1} \tag{15}$$

where ω_E is an Earth rotation rate, μ – gravitational parameter; R_E – Earth radius, J_2 is a coefficient characterizing oblateness of the Earth ($J_2 = 0.0010827$).

The following scheme is introduced to calculate the repetition factor m_2/n_2 of geosynchronous orbits in the second tier by the given repetition factor m_1/n_1 and inclination i_1 for the first tier and a fixed inclination i_2 for the second tier orbits: 1) the altitude H_1 of geosynchronous orbit, corresponding to the given repetition factor m_1/n_1 and inclination i_1, is calculated[11]; 2) the altitude H_2 is calculated from the synchronization condition specified by Eq. (2); 3) real repetition factor k_2, corresponding to altitude H_2 and inclination i_2, is calculated from Eq. (15); 4) from the real repetition factor k_2 and inclination i_2 the integer numbers m_2 and n_2 are derived, so that for a minimally possible number n_2 altitude $H_2^*(m_2/n_2, i_2)$ is close to earlier calculated value H_2 at a given level of accuracy.

Phase structure (satellites' values of the longitude of ascending node and argument of latitude) for each tier of the constellation is defined by the way of distributing the satellites according to the concept of Secure Constellations, as it was described above. Optimization of the orbital structure in each tier is performed for one of p latitude belts (denoted as a *basic* belt for this tier), which result from the fragmentation of the entire observation belt (where p is a number of tiers in the compound constellation). The matching of tiers and these basic latitudinal belts is established in decreasing order of the orbits' inclinations (or according to the sequence of latitudinal belts from pole to equator). However, only the tier with the smallest inclination provides the coverage of a single belt, namely of the belt closest to equator. Satellites of all the remaining tiers of the constellation are involved in the coverage of two or more latitude belts (despite the fact that the orbital structures in each tier are optimized only for the single corresponding basic belt coverage, as was mentioned above). For instance, in a case of a double-tiered constellation the first latitude belt (the one closer to the pole) is covered only by the satellites with a greater inclination, while the second latitude belt is covered by all the satellites of such double-tiered constellation.

To provide expedient coverage of the second and subsequent (if there are more than two tiers) latitude belts, the following optimization procedure of the satellite motion in various tiers is conducted.

On the first step, the optimal initial relative phase position (for the longitude of ascending node and argument of latitude) is obtained for previously developed orbital structures of the first and second tiers by the criterion of minimal maximum revisit time (MRT) of the second belt. Such optimization is conducted by the way of two-dimensional "rotation" of these two orbital structures, which implies a constant-step joint variation of longitude of ascending node and argument of latitude with a quality analysis of the resulting constellation and selection of the best values (it should be mentioned here, that a technical solution could be obtained to prevent the enumeration of these values within the entire range (0, 360°) of their definition).

The second stage of optimization is necessary in case there is a third tier in the constellation. It is on this stage that we perform the optimization of the position of the third tier with respect to the optimal orbital constellation obtained on the first step. In this particular case our goal is to minimize the revisit time of the third latitudinal belt. In the case of fourth tier (subsequent tiers), the third (subsequent) stage of optimization is conducted. The variant of orbital structure obtained on the last optimization stage is assumed as an ultimate variant of the entire orbital structure of the multi-tiered constellation.

Numerical Results

Table 3 lists the variants of traditional single-tiered satellite constellations for periodic coverage of the latitude belt 0..70° at the specified span angles of on-board sensors, calculated according to Reference 11. Table 4 lists variants of double-tiered constellations, calculated in accordance with the suggested method under the conditions of corresponding variants from the Table 3.

Table 3. Best known traditional (single-tiered) 4-satellite constellations for periodic coverage of the latitude belt 0..70° for the specified span angles β of on-board equipment

№	β, deg	Constellation (inclination *i*, deg; altitude *H*, km, longitude of ascending node Ω, deg, argument of latitude *u*, deg)		MRT τ, hours
1	39.09	i = 81, H = 707.7	$\Omega_1 = 0$; $\Omega_2 = 180.00$; $\Omega_3 = 360.00$; $\Omega_4 = 180.00$ $u_1 = 0$; $u_2 = 270.00$; $u_3 = 180.00$; $u_4 = 90.00$	11.93
2	41.97	i = 72, H = 693.7	$\Omega_1 = 0$; $\Omega_2 = 180.00$; $\Omega_3 = 360.00$; $\Omega_4 = 180.00$ $u_1 = 0$; $u_2 = 270.00$; $u_3 = 180.00$; $u_4 = 90.00$	11.90
3	46.52	i = 72, H = 1234.0	$\Omega_1 = 0$; $\Omega_2 = 62.30$; $\Omega_3 = 124.60$; $\Omega_4 = 186.90$ $\mathrm{u}_1 = 0$; $\mathrm{u}_2 = 270.00$; $\mathrm{u}_3 = 180.00$; $\mathrm{u}_4 = 90.00$	4.12
4	51.80	i = 64, H = 863.2	$\Omega_1 = 0$; $\Omega_2 = 270.70$; $\Omega_3 = 181.40$; $\Omega_4 = 92.10$ $u_1 = 0$; $u_2 = 190.30$; $u_3 = 20.60$; u4 = 210.90	10.25
5	56.13	i = 72, H = 382.0	$\Omega_1 = 0$; $\Omega_2 = 180.00$; $\Omega_3 = 360.00$; $\Omega_4 = 180.00$ $u_1 = 0$; $u_2 = 90.00$; $u_3 = 180.00$; $u_4 = 270.00$	11.88
6	73.27	$i = 72$, H = 238.2	$\Omega_1 = 0$; $\Omega_2 = 90.00$; $\Omega_3 = 180.00$; $\Omega_4 = 270.00$ $u_1 = 0$; $u_2 = 0$; $u_3 = 0$; $u_4 = 0$	5.94

By analyzing the data from Tables 3–4 one can conclude that the use of double-tiered constellations in common comparison conditions considered here, allows reducing the required number of satellites by 1–2 satellites (2–3 satellites instead of four) as compared to the best known variants of traditional single-tiered constellations. At the same time, in some cases the coverage quality is for a given region, i.e. the MRT value is decreased.

Table 4. Double-tiered satellite constellations for periodic coverage of the latitude belt 0..70° under conditions of the satellite constellations listed in Table 3

№	β, deg	N	Constellation (inclination i, altitude H, longitude of ascending node Ω, argument of latitude u)		MRT τ, hours
1	39.09 (36.5)	$N = 2$	$N_1 = 1$; $i_1 = 81$, $H_1 = 707.7$ $N_2 = 1$; $i_2 = 49$, $H_2 = 4298.8$	$\Omega_{11} = 0$, $u_{11} = 0$ $\Omega_{21} = 180.00$, $u_{21} = 90.00$	11.03
2	41.97	$N = 3$	$N_1 = 1$; $i_1 = 72$, $H_1 = 693.7$ $N_2 = 2$; $i_2 = 51$, $H_2 = 2279.4$	$\Omega_{11} = 0$, $u_{11} = 0$ $\Omega_{21} = 180.00$, $u_{21} = 270.00$ $\Omega_{22} = 360.00$, $u_{22} = 180.00$	9.83
3	46.52 (42.90)	$N = 3$	$N_1 = 1$; $i_1 = 72$, $H_1 = 1234.0$ $N_2 = 2$; $i_2 = 51$, $H_2 = 2955.0$	$\Omega_{11} = 0$, $u_{11} = 0$ $\Omega_{21} = 62.31$, $u_{21} = 270.00$ $\Omega_{22} = 186.90$, $u_{22} = 90.00$	3.30
4	51.80	$N = 2$	$N_1 = 1$; $i_1 = 72$, $H_1 = 863.2$ $N_2 = 1$; $i_2 = 51$, $H_2 = 2486.0$	$\Omega_{11} = 0$, $u_{11} = 0$ $\Omega_{21} = 181.40$, $u_{21} = 20.60$	10.37
5	56.13 (45.0)	$N = 3$	$N1 = 1$; $i_1 = 72$, $H_1 = 382.0$ $N2 = 2$; $i_2 = 51$, $H_2 = 1902.4$	$\Omega_{11} = 0$, $u_{11} = 0$ $\Omega_{21} = 180.00$, $u_{21} = 90.00$ $\Omega_{22} = 360.00$, $u_{22} = 180.00$	10.65
6	73.27 (52.0)	$N = 3$	$N_1 = 1$; $i_1 = 72$, $H_1 = 238.2$ $N_2 = 2$; $i_2 = 51$, $H_2 = 1725.0$	$\Omega_{11} = 0$, $u_{11} = 0$; $\Omega_{21} = 90.00$, $u_{21} = 0$ $\Omega_{22} = 180.00$, $u_{22} = 0$	5.69

CONCLUSIONS

1. A new satellite constellation design method is suggested. The method allows considerably extending the optimization domain of the orbital structure parameters due to the distribution of satellites in various tiers of the constellation, each tier being characterized by its own pair "altitude H – inclination i". The given parameters H and i are so synchronized for the separate tiers that equal nodal regression is provided for all satellites, and thus the required delta-V for the station-keeping of such constellations is on par with that of the traditional single-tiered constellations.

2. Theoretical results for the compound satellite constellation design are present in this paper, separately for the cases of periodic and continuous coverage.

3. The comparison of the obtained variants of double-tiered satellite constellations for global continuous one-fold coverage with best known traditional single-tiered constellations (which have equal altitudes and inclinations coinciding with those of one of the tiers of the compound constellation), revealed that the use of suggested method allows reducing the required number of satellites by 2–6 in common comparison conditions considered.

4. The comparison of the obtained variants of double-tiered satellite constellations for periodic coverage with best known traditional single-tiered constellations (which have equal altitudes and

inclinations coinciding with those of one of the tiers of the compound constellation), revealed that the use of such multi-tiered constellations allows decreasing the required number of satellites up to 2 times (2–3 satellites instead of four). At the same time in some cases we achieve simultaneously the improvement of coverage characteristics decreasing MRT.

REFERENCES

[1] Luders R.D. Satellite Networks for Continuous Zonal Coverage // American Rocket Society. J. Feb. 1961. V. 31. P. 179-184.

[2] Ullock, M.H., Shoen A.H., Optimum Polar Satellite Networks for Continuous Earth Coverage // AIAA Journal, Vol. 1, 1963, pp. 69-72.

[3] Walker J.G. Circular Orbits Patterns Providing Continuous Whole Earth Coverage // Royal Aircraft Establishment. Technical Report. No. 70211. Nov. 1970.

[4] Mozhaev G.V. The Problem of Earth Continuous Coverage and Kinematically Regular Constellations // Cosmic Research. J. of USSR Academy of Sciences.1972. V. 10. No. 6. P. 833-840; 1973. V. 11. No. 1. P. 59-69.

[5] Lang, T.J., Hanson, J.M. Orbital Constellations which Minimize Revisit Time // AAS Paper 83-402. AAS/AIAA Astrodynamics Conference. Lake Placid, New York. 1983.

[6] Rider L. Optimized Polar Orbit Constellation for Redundant Earth Coverage // J. of the Astronautical Sciences. 1985. V. 33. No. 2. P. 147-161.

[7] Adams W.S., Rider L. Circular Polar Constellations Providing Continuous Single or Multiple Coverage Above a Specified Latitude // J. of the Astronautical Sciences. 1987. V. 35. No. 2. P. 155-192.

[8] Razoumny Yu. N. Method of Parameters Optimization of the One-Route Earth Artificial Satellite Systems for the Earth Periodical Coverage // Earth Research from Space. J. of USSR Academy of Sciences. No. 5. 1991. P. 39-46.

[9] Lang, T.J. Walker Constellations to Minimize Revisit Time in Low Earth Orbit // AAS Paper 03-178, AAS/AIAA Space Flight Mechanics Meeting. Ponce, Puerto Rico, 2003.

[10] Lang, T.J. Streets of Coverage Constellations to Minimize Revisit Time in Low Earth Orbit // AAS Paper, № 05-154, 15th AAS/AIAA Space Flight Mechanics Meeting, Copper Mountain, Colorado, Jan. 2005.

[11] Razoumny Yu. N. New Research Methodology for Earth Periodic Coverage and Regularities in Parametric Localization of Optimal Low-Earth-Orbit Satellite Constellations // AAS Paper 13-917. AAS/AIAA Astrodynamics Specialist Conference. Hilton Head, South Carolina. 2013.

[12] Razoumny Yu. N., Baranov A. A., Razoumny V. Yu. et al. Concept of Multitiered Satellite Constellation Design and Orbital Stability Keeping // IWSCFF-2013-04-03, Proceedings of the 7th International Workshop on Satellite Constellation and Formation Flying, Lisbon, Portugal, 2013.

[13] Kalil F., Martikan F. Derivation of Nodal Period of an Earth Satellite and Comparison of Several First-order Secular Oblateness Results. // AIAA Journal, September, 1963, pp. 2041–2046.

[14] Mozhaev G.V. The Problems of Optimization of Satellite Systems Motion: State of Research and Prospects // Proceedings of the 5th International Workshop on Satellite Constellation and Formation Flying, Yevpatoria, Ukraine, 2008.