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ATTITUDE DYNAMICS OF A SPACECRAFT WITH A SOLAR STABILIZER

Vladislav Sidorenko*

A spacecraft can be oriented toward the Sun by means of solar radiation pressure. To provide a restoring torque a spacecraft should be designed in such a way that, under the conditions of solar orientation, the center of the radiation pressure is placed behind the center of mass from the point of view of an observer on the side of the Sun. In particular one can suppose that the spacecraft is equipped with a solar stabilizer of conic or spherical shape. Our aim is to study the dynamics of the spacecraft with the solar stabilizer after its separation from the booster. Usually an active attitude control system is used to slow down initial fast rotation. But there are several ways to control the rotational motion by means of solar rudders – movable plates mounted on the spacecraft in the form of a windmill or propeller. We discuss the outcome if the simplest strategy is chosen: a simultaneous deviation of solar rudders through a given angle to produce a propellor torque.

BASIC ASSUMPTIONS

We start with the following assumptions regarding the design of the spacecraft:

- the spacecraft is equipped with an inflated spherical solar stabilizer attached on a long bar (Figure 1);
- the solar rudders consist of N perfectly reflecting plates which are uniformly distributed at a distance R from the symmetry axis of the spacecraft in a plane at a distance l from the center of mass O;
- the ellipsoid of inertia of the spacecraft is an ellipsoid of revolution. The symmetry axis of the ellipsoid of inertia coincides with the geometric symmetry axis of the spacecraft with solar stabilizer.

The spacecraft is assumed to be in a circular heliocentric orbit with a mean motion ω_0.

MOTION EQUATIONS

First we introduce two right-hand orthogonal coordinate systems with their origin at the spacecraft center of mass O.

* Keldysh Institute of Applied Mathematics,
Miusskaya Sq. 4, 125047 Moscow, RUSSIA.

The coordinate system $O\xi\eta\zeta$ is fixed rigidly in the body of the spacecraft; the axis $O\zeta$ is aligned with the axis of spacecraft geometrical symmetry.

The system $OXYZ$ is the orbital coordinate system. The axis OZ is directed toward the Sun, and the axis OY is along the tangent to the orbit in the direction of motion of the spacecraft.

The position of the fixed coordinate system with respect to the orbital coordinate system can be defined by means of Euler angles ψ, ϑ, φ (Figure 2). Of particular interest is the angle ϑ (the nutation angle) corresponding to the angle between the direction toward the sun and axis of symmetry.

The unit vectors of the coordinate systems $OXYZ$ and $O\xi\eta\zeta$ will be denoted as $\mathbf{e}_X, \mathbf{e}_Y, \mathbf{e}_Z$ and $\mathbf{e}_\xi, \mathbf{e}_\eta, \mathbf{e}_\zeta$ respectively.

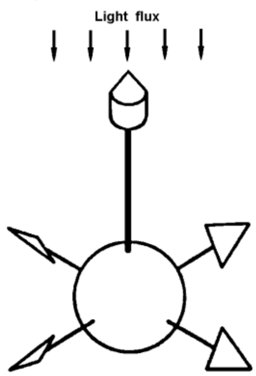

Figure 1. The spacecraft with a solar stabilizer.

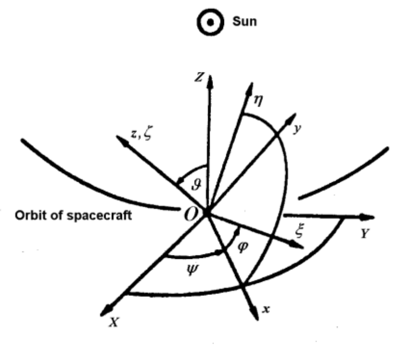

Figure 2. The coordinate systems used

The dynamical motion equations are

$$J\frac{d\boldsymbol{\omega}}{dt} + \boldsymbol{\omega} \times J\boldsymbol{\omega} = \mathbf{M}^s + \mathbf{M}^p. \tag{1}$$

Here $J = diag(A, A, C)$ is the inertia tensor of the spacecraft, $\mathbf{M}^s$ is the restoring solar torque, and $\mathbf{M}^p$ is the propellor torque from solar rudders.

The restoring solar torque is given by

$$\mathbf{M}^s = m(\vartheta)SLP\frac{\mathbf{e}_\zeta \times \mathbf{e}_Z}{|\mathbf{e}_\zeta \times \mathbf{e}_Z|},$$

where S and L are a characteristic area and a characteristic length of the spacecraft, and P is the specific radiation pressure. The dimensionless function $m(\vartheta)$ is determined by the geometry of the spacecraft . In particular, for the spacecraft with a spherical stabilizer we would have $m(\vartheta) \approx \sin\vartheta$.

The restoring solar torque is a potential torque and can be specified by a force function

$$U_s = SLP\int_0^\vartheta m(\vartheta')d\vartheta'.$$

The expression for the propellor torque $\mathbf{M}^P$ is

$$\mathbf{M}^P = 2PS_* \sum_{\sigma=1}^{N} (\mathbf{e}_Z, \mathbf{n}_\sigma) \,|\, (\mathbf{e}_Z, \mathbf{n}_\sigma) \,|\, \mathbf{n}_\sigma \times \mathbf{r}_\sigma \,. \qquad (2)$$

Here $\mathbf{n}_\sigma$ is the normal to the rudder of index σ, $\mathbf{r}_\sigma$ is the radius vector of this rudder in the fixed coordinate system, S_* is the area of a solar rudder. The magnitude and direction of the propellor torque depend on the orientation of the spacecraft and on the orientation of the rudder plates with respect to the plane $O\xi\eta$. The desired orientation is achieved by rotating the solar rudders through an angle χ with respect to their nominal position, in which the rudder plates are parallel to the plane $O\xi\eta$. The choice of the sign of χ is determined by the direction in which the rudders rotate as observed from behind these rudders, from the center of mass of the spacecraft. For the nominal position of the rudders ($\chi = 0^\circ$) and also when they are rotated through an angle $\chi = 90^\circ$, no propellor torque arises.

To close the dynamical equations (1) we use the kinematic relations

$$\omega_\xi = a_{X\xi}\left(\omega_0 + \cos\psi \frac{d\vartheta}{dt}\right) + a_{Y\xi}\sin\psi \frac{d\vartheta}{dt} + a_{Z\xi}\frac{d\psi}{dt},$$

$$\omega_\eta = a_{X\eta}\left(\omega_0 + \cos\psi \frac{d\vartheta}{dt}\right) + a_{Y\eta}\sin\psi \frac{d\vartheta}{dt} + a_{Z\eta}\frac{d\psi}{dt},$$

$$\omega_\zeta = a_{X\zeta}\omega_0 + a_{Z\zeta}\frac{d\psi}{dt} + \frac{d\varphi}{dt}.$$

Here the coefficients $a_{X\xi}, a_{X\eta}, \ldots$ denote the scalar products of the unit vectors $\mathbf{e}_X, \mathbf{e}_Y, \mathbf{e}_Z$ and $\mathbf{e}_\xi, \mathbf{e}_\eta, \mathbf{e}_\zeta$:

$$a_{X\xi} = (\mathbf{e}_X, \mathbf{e}_\xi),\; a_{X\eta} = (\mathbf{e}_X, \mathbf{e}_\eta), \ldots$$

Next we introduce the dimensionless variables and parameters

$$\Omega_x = \left(\frac{A}{SLP}\right)^{1/2} (\omega_\xi \cos\varphi - \omega_\eta \sin\varphi), \quad \Omega_y = \left(\frac{A}{SLP}\right)^{1/2} (\omega_\xi \sin\varphi + \omega_\eta \cos\varphi),$$

$$\Omega_z = \left(\frac{A}{SLP}\right)^{1/2} \omega_\varsigma, \quad \tau = \left(\frac{SLP}{A}\right)^{1/2} t, \quad \lambda = \frac{C}{A}, \rho = \frac{R}{L}, \delta = \frac{l}{L}, \varepsilon_p = \frac{S_*}{S}, \varepsilon_0 = \left(\frac{A}{SLP}\right)^{1/2} \omega_0 .$$

Here $\Omega_x, \Omega_y, \Omega_z$ denote the projections of the angular velocity vector onto the axes of the semi-fixed coordinate system $Oxyz$ (Figure 2). The parameter ε_p characterize the effect of the solar rudders on the motion of the spacecraft. We assume below that ε_0 and ε_p are the small parameters of the same order of magnitude:

$$\varepsilon_0 = \kappa_0 \varepsilon, \; \varepsilon_p = \kappa_p \varepsilon, \; 0 < \varepsilon << 1.$$

Now the equations of motion accept the form

$$\frac{d\Omega_x}{d\tau} = -(\lambda\Omega_z - \Omega_y ctg\,\vartheta)\Omega_y - m(\vartheta) + \varepsilon\kappa_p m_x^p\,,$$

$$\frac{d\Omega_y}{d\tau} = (\lambda\Omega_z - \Omega_y ctg\,\vartheta)\Omega_x + \varepsilon\kappa_p m_y^p\,,\quad \lambda\frac{d\Omega_z}{d\tau} = \varepsilon\kappa_p m_z^p\,,$$

$$\frac{d\psi}{d\tau} = (\Omega_y - \varepsilon\kappa_0 a_{Xy})/\sin\vartheta\,,\quad \frac{d\vartheta}{d\tau} = \Omega_x - \varepsilon\kappa_0 a_{Xx}\,, \tag{3}$$

$$\frac{d\varphi}{d\tau} = \Omega_z - \varepsilon\kappa_0 a_{Xz} - ctg\,\vartheta(\Omega_y - \varepsilon\kappa_0 a_{Xy})\,.$$

Here

$$m_x^p = 2\sum_{\sigma=1}^{N} f_\sigma\,|\,f_\sigma\,|[\delta\sin\chi\cos(\varphi+\Phi_\sigma) - \rho\cos\chi\sin(\varphi+\Phi_\sigma)],$$

$$m_y^p = 2\sum_{\sigma=1}^{N} f_\sigma\,|\,f_\sigma\,|[\delta\sin\chi\sin(\varphi+\Phi_\sigma) + \rho\cos\chi\cos(\varphi+\Phi_\sigma)],$$

$$m_z^p = 2\rho\sin\chi\sum_{\sigma=1}^{N} f_\sigma\,|\,f_\sigma\,|,\quad \Phi_\sigma = 2\pi\sigma/N,$$

$$f_\sigma = (\mathbf{e}_Z, \mathbf{n}_\sigma) = \cos\chi\cos\vartheta - \sin\chi\sin\vartheta\cos(\varphi+\Phi_\sigma)\,.$$

SPECIAL EVOLUTIONARY VARIABLES[1]

Let us consider first some properties of unperturbed motion. In the case $\varepsilon = 0$ Eqs. (3) become

$$\frac{d\Omega_x}{d\tau} = -(\lambda\Omega_z - \Omega_y ctg\,\vartheta)\Omega_y - m(\vartheta)\,,$$

$$\frac{d\Omega_y}{d\tau} = (\lambda\Omega_z - \Omega_y ctg\,\vartheta)\Omega_x\,,\quad \lambda\frac{d\Omega_z}{d\tau} = 0\,, \tag{4}$$

$$\frac{d\psi}{d\tau} = \Omega_y/\sin\vartheta,\quad \frac{d\vartheta}{d\tau} = \Omega_x\,,\quad \frac{d\varphi}{d\tau} = \Omega_z - \Omega_y ctg\,\vartheta\,.$$

System (4) has a family of the partial solutions

$$\Omega_x \equiv 0,\quad \Omega_y \equiv \Omega_{y0},\quad \Omega_z \equiv \Omega_{z0},\quad \vartheta \equiv \theta,$$

$$\psi = W\tau + \psi_0,\quad \varphi = \omega_{\varphi 0}\tau + \varphi_0, \tag{5}$$

where the constants ψ_0 and φ_0 are arbitrary and the constants $\Omega_{y0}, \Omega_{z0}, \omega_{\varphi 0}, W,$ and θ are connected by the relations

$$\Omega_{y0} = W\sin\theta,\quad \lambda\Omega_{z0} = W\cos\theta - \frac{m(\theta)}{W\sin\theta},$$

$$\omega_{\varphi 0} = -\frac{1}{\lambda}\left[(\lambda-1)W\cos\theta + \frac{m(\theta)}{W\sin\theta}\right].$$

Solutions (5) correspond to motions in which the symmetry axis of the spacecraft moves at a constant angular velocity W over a constant angular distance θ around the direction toward the Sun (the *OZ* axis), while the spacecraft itself rotates around the symmetry axis at a constant angular velocity $\omega_{\varphi 0}$. Such motions are called "regular precessions".

A sufficient condition for stability of regular precession is

$$\left(\frac{\partial^2 V}{\partial \vartheta^2}\right)_{\vartheta=\theta} > 0, \quad V(\vartheta, W, \theta) = \frac{1}{2}\left(\frac{v - u\cos\vartheta}{\sin\vartheta}\right)^2 - U_s, \tag{6}$$

where

$$u(W,\theta) = W\cos\theta - \frac{m(\theta)}{W\sin\theta}, \quad v(W,\theta) = W - \frac{m(\theta) ctg\,\theta}{W}.$$

The quantities u and v in Eqs. (6) are the projections of the angular momentum vector of the spacecraft onto the dynamic symmetry axis and onto the direction toward the Sun.

For the application of the perturbation technique we replace the variables $\Omega_x, \Omega_y, \Omega_z$, and ϑ to the special evolutionary variables W, θ, c, and υ. The variables W and θ specify the reference regular precession, while the variables c and υ characterize the amplitude and the phase of the nutation oscillations (the oscillations of the angle ϑ) in the motion which is close to the reference precession.

MOTION EQUATIONS IN THE SPECIAL EVOLUTIONARY VARIABLES

After some simple calculations the equations for $dW/d\tau$, $d\theta/d\tau$, $dc/d\tau$, and $d\upsilon/d\tau$ can be obtained. Without writing them out we note only that in general

$$dW/d\tau,\ d\theta/d\tau \sim \varepsilon,\ dc/d\tau \sim \varepsilon c,$$

$$d\upsilon/d\tau,\ d\psi/d\tau,\ d\varphi/d\tau \sim 1.$$

To study the long term changes in the motion properties the averaging method is applied[2]. In the first approximation of the averaging method we find (we use the same notation for average variables)

$$\frac{dW}{d\tau} = \varepsilon G_W(W,\theta,c), \quad \frac{d\theta}{d\tau} = \varepsilon G_\theta(W,\theta,c),$$
$$\frac{dc}{d\tau} = \varepsilon c G_c(W,\theta,c). \tag{7}$$

Here

$$G_{W(\theta)}(W,\theta,c) =$$

$$= (-)\kappa_p \left[\overline{m}_z^p \frac{\partial v}{\partial \theta(W)} - \left(\overline{m}_y^p \sin\theta + \overline{m}_z^p \cos\theta \frac{\partial u}{\partial \theta(W)}\right)\right] \Big/ D + O(c^2),$$

$$D = \frac{\partial(u,v)}{\partial(W,\theta)}, \quad \overline{m}_y^p = \frac{N}{2\pi}\int_0^{2\pi/N} m_y^p d\varphi, \quad \overline{m}_z^p = \frac{N}{2\pi}\int_0^{2\pi/N} m_z^p d\varphi.$$

The lengthy expression for the function $G_c(W,\theta,c)$ is omitted.

QUALITATIVE ANALYSIS OF THE MOTION ON THE BASIS OF THE AVERAGED EQUATIONS

In the phase space of the system (7) the condition $c=0$ defines an integral manifold. This manifolds consists of the solutions which become regular precession at $\varepsilon \to 0$. Next we note the absence of the terms which are linear in c in the right parts of the equations for $dW/d\tau$ and $d\theta/d\tau$. It indicates that the evolution of the variables W and θ depends relatively weekly on the amplitude of the nutational oscillations. So we can focus our attention on the behaviour of the solutions on the manifold $c=0$. Figure 3 provides the examples for two values of the rudder rotational angle χ. Taking into consideration the sign of the function $G_c(W,\theta,0)$ we can define on the manifold $c=0$ attracting ($G_c(W,\theta,0)<0$) and repelling ($G_c(W,\theta,0)>0$) regions. The change in the sign of χ leads to a change in the direction of the space trajectories and to a transformation of attracting regions into repelling regions and vice verse.

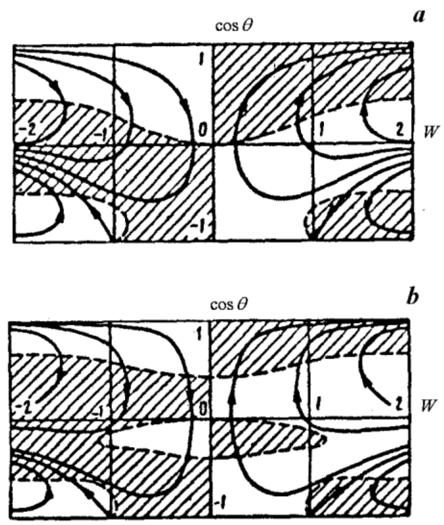

Fig. 3. Changes in the parameters of the reference regular precession due to the propellor torque: a - $\chi = 75^0$**, b -** $\chi = 15^0$**. The repelling regions are shaded.**

Let us assume that the angular momentum of the spacecraft $\mathbf{L}$ is large ($1 << |\mathbf{L}| << 1/\varepsilon$) and makes an angle θ_L ($\theta_L \neq 0, 180^0$) with the axis OZ. Such a situation can appear after the spacecraft separation from the booster stage. If the motion of the spacecraft is a regular precession then the precession parameters are $|W_*| \approx m(\theta_*)/(\sin\theta_* |\mathbf{L}|)$, $\theta_* \approx \theta_L$ or $\theta_* \approx 180^0 - \theta_L$. Since we have $|W_*| << 1$, this motion can be called a “slow” precession.

Depending on the sign of χ there are two variants of the slow precession evolution due to propellor torque. In the simplest variant the propeller torque puts the spacecraft into the regime of solar orientation and causes it to progressively “unwind “ from the symmetry axis. In the more complex variant of the evolution the spacecraft first goes into a regime of countersolar orientation. The angle ϑ then decreases asymptotically to 90^0 so the longitudinal axis of the spacecrafts assumes an orientation perpendicular to the direction to the Sun.

REFERENCES

[1]V.V. Sidorenko, "Capture and escape from resonance in the dynamics of the rigid body in viscous medium". *Journal of Nonlinear Science.* Vol. 4, No. 1, 1994, pp. 35-57.

[2]V.I. Arnold, V.V. Kozlov, and A.I. Neishtadt, *Mathematical aspects of the classical and celestial mechanics. Dynamical systems – 3.* Berlin: Springer, 1988.