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ACTIVE MAGNETIC ATTITUDE CONTROL SYSTEM PROVIDING THREE-AXIS INERTIAL ATTITUDE

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Active magnetic attitude control system providing arbitrary inertial attitude of a satellite is considered. Control and gravitational torques are taken into account. Planar motion is considered for the periodical solutions. Stability is assessed using Floquet theory, optimal algorithm parameters are chosen. Overall gravity effect is assessed.

INTRODUCTION

A three-axis active magnetic attitude control system and related algorithms are of great interest and importance if one considers small satellites. Being the low-cost, reliable and small, magnetorquers are especially attractive for these satellites. However, magnetic control is limited due to the underactuation problem. Control torque along the geomagnetic field induction vector cannot be implemented. As a result, it is impossible to implement an arbitrary in terms of the direction torque at each moment. Therefore it seems impossible to achieve necessary three-axis attitude using numerically simple locally optimal algorithms. One approach is to complement magnetorquers with other actuators. These actuators should be also compact, lightweight, use no fuel. Among these novel actuators fluid rings can be noted.[1] These actuators however are not tested properly and have greater mass and storage requirements. Another possible solution is to move to optimal control or find it from the boundary problem.[2] However onboard computer of a small satellite may be incapable of solving such a problem.

This paper follows our previous work [3] where the asymptotical stability of the necessary attitude with some values of the control gains is proven. There was also present a simple way to choose the control parameters. It was assumed the satellite is affected by the control torque only. We can outline only one paper with a comprehensive analytical approach to the problem.[4] It is shown that the three-axis magnetic attitude is achievable, though only small vicinity of the necessary attitude is taken into account, the satellite is considered to be a spherically-symmetrical one, and the whole analysis can be hardly interpreted in order to implement on a spacecraft. Similar assumption on a spherically-symmetrical satellite was made in [5]. An asymptotical stability of the

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necessary attitude is shown using the Lyapunov function approach, however the analysis shows that the assumption on the satellite inertia tensor is key to its overall success. The analysis is not valid for a three-axial satellite and therefore is of limited technical importance. Probably the most important paper on the three-axis magnetic control apart from [4] is [6]. Performance of the three-axis magnetic attitude control system of the Gurwin-Techsat small satellite is present. Only the necessary attitude maintenance is shown, however the work shows the possibility to overcome the underactuation issue.

Here the planar motion of the satellite subjected to the magnetic control torque and the disturbing gravitational one is considered. Poincare method is used to assess the periodic motions of the satellite. The same approach is used in the three-dimensional motion, however only generating stationary point is considered. Floquet theory is used to assess the stability of the system of equations.

1. PROBLEM STATEMENT

The choice of the geomagnetic field model is one of the most crucial points for the success of the work. Two different models will be used for analytical and numerical analysis. Well-known IGRF is used for numerical analysis. We use the averaged geomagnetic field model (simplified direct dipole model) for analytical analysis. To introduce this model we need to notify a reference system $O_aY_1Y_2Y_3$ where O_a is the Earth's center, the O_aY_3 axis is directed along with the Earth's axis, O_aY_1 lies in the Earth's equatorial plane and is directed to the ascending node of the satellite's orbit, the O_aY_2 axis is directed so the system is right-handed. If the magnetic induction vector source point is translated to O_a then the cone is tangent to the O_aY_3 axis, its axis lies in the $O_aY_2Y_3$ plane. The cone half-angle is given by $\operatorname{tg}\Theta = \dfrac{3\sin 2i}{2\left(1-3\sin^2 i+\sqrt{1+3\sin^2 i}\right)}$.[7]

Introduced reference frame is inconvenient for the motion representation since both the control torque (due to the geomagnetic induction vector expression) and the gravitational torque (due to the radius-vector expression) are overburdened. The most convenient inertial reference frame regarding the gravitational torque representation is the orbital one $O_aX_1X_2X_3$. O_aX_3 axis is directed along the normal to the orbital plane, O_aX_1 is directed to the ascending node of the satellite orbit, O_aX_2 is directed so that the whole reference frame is right-handed. Clearly the frames $O_aX_1X_2X_3$ and $O_aZ_1Z_2Z_3$ are related by the rotation by the angle $\delta = \Theta - i$ along the O_aX_1 axis. Angle δ does not exceed 10° so it may be considered a small parameter in some cases.

We will also use the bound reference frame $Ox_1x_2x_3$, its axes coincide with the principal axes of inertia of the satellite. Mutual attitude of $O_aX_1X_2X_3$ and $Ox_1x_2x_3$ reference frames is given by the direct cosines matrix $\mathbf{A}$. We use Euler equations and related variables to represent the satellite motion. The state vector comprises of the variables $\omega_1,\omega_2,\omega_3,\alpha,\beta,\gamma$. Here ω_i are the absolute angular velocity components in the bound frame $Ox_1x_2x_3$ (i=1,2,3), Euler angles α,β,γ give the satellite and therefore the bound frame $Ox_1x_2x_3$ attitude with respect to $O_aX_1X_2X_3$. Euler angles rotation sequence is chosen in such a way that the direction cosines matrix $\mathbf{A}$ has the form

$$\mathbf{A}=\begin{pmatrix} \cos\alpha\cos\beta & \sin\beta & -\sin\alpha\cos\beta \\ -\cos\alpha\sin\beta\cos\gamma+\sin\alpha\sin\gamma & \cos\beta\cos\gamma & \sin\alpha\sin\beta\cos\gamma+\cos\alpha\sin\gamma \\ \sin\alpha\cos\gamma+\cos\alpha\sin\beta\sin\gamma & -\cos\beta\sin\gamma & -\sin\alpha\sin\beta\sin\gamma+\cos\alpha\cos\gamma \end{pmatrix}. \qquad (1)$$

Motion equations for the satellite with arbitrary tensor of inertia $\mathbf{J}_x = diag(A,B,C)$ are

$$\begin{aligned} &A\frac{d\omega_1}{dt}=(B-C)\omega_2\omega_3+M_{1x}, && \frac{d\alpha}{dt}=\frac{1}{\cos\beta}\left(-\omega_3\sin\gamma+\omega_2\cos\gamma\right),\\ &B\frac{d\omega_2}{dt}=-(A-C)\omega_1\omega_3+M_{2x}, && \frac{d\beta}{dt}=\omega_2\sin\gamma+\omega_3\cos\gamma, \\ &C\frac{d\omega_3}{dt}=-\left(B-A\right)\omega_1\omega_2+M_{3x}, && \frac{d\gamma}{dt}=\omega_1-\mathrm{tg}\,\beta\left(\omega_2\cos\gamma-\omega_3\sin\gamma\right), \end{aligned} \qquad (2)$$

where M_{1x}, M_{2x}, M_{3x} are the torque (both control and disturbing ones) components in the bound frame $Ox_1x_2x_3$.

The control dipole moment

$$\mathbf{m}=-k_\omega\mathbf{B}\times\boldsymbol{\omega}-k_a\mathbf{B}\times\mathbf{S} \qquad (3)$$

is often constructed using different reasoning. It is inspired by the PD-controller construction.[5]

2. PLANAR MOTION ANALYSIS

Eq. (2) are quite tough for the three-dimensional motion analysis. However these equations allow the planar motion problem statement. These equations can be analyzed analytically. In order for the planar motion to exist the satellite should the axisymmetrical one ($A=C$). The orbit should be the polar one. With these assumptions taken into account Eq. (2) allow the planar motion where $\alpha=\gamma=0$, $\omega_1=\omega_2=0$. The planar motion of the satellite is described by the equations

$$\ddot\beta+\omega^2\sin\beta+\varepsilon\dot\beta=\lambda\sin\left(2u-2\beta\right) \qquad (4)$$

where the derivative by the argument of latitude is denoted by a dot. We introduce dimensionless angular velocity according to relation $\omega_3=\Omega\omega_0$, argument of latitude instead of time and parameters

$$\lambda=\frac{3\left(B-A\right)}{2A}\omega_0^2,\ \omega^2=\frac{2k_aB_0^2}{A\omega_0^2},\ \varepsilon=\frac{k_\omega B_0^2}{A\omega_0}.$$

Consider the control torque to be the prevailing one. The gravitational torque is considered small so $\lambda << 0$. The unperturbed motion equation is therefore

$$\ddot\beta+\omega^2\sin\beta+\varepsilon\dot\beta=0. \qquad (5)$$

The simplest case of the generating solution is the stationary point $\beta=0$, $\dot\beta=0$ or $\beta=\pi$, $\dot\beta=0$. The first stationary point is the stable one. Consider this point as the generating solution and use the Poincare method [8] to find periodic solutions arising due to the small gravitational torque influence. The solution can be found as a series of functions of different order of the small parameter λ. Grouping terms of the order of small parameter we obtain differential equations for the first order deviation and their solution

$$\beta_1 = -\frac{1}{\left(\omega^2 - 2\varepsilon - 4\right)\left(\omega^2 + 2\varepsilon - 4\right)}\left(\left(4 - \omega^2\right)\sin 2u + 2\varepsilon\cos 2u\right). \tag{6}$$

In case of the unstable stationary point $\beta = \pi$, $\dot{\beta} = 0$ the first-order approximation solution

$$\beta_1 = \frac{1}{4\varepsilon^2 + \left(\omega^2 + 4\right)^2}\left(\left(4 + \omega^2\right)\sin 2u + 2\varepsilon\cos 2u\right). \tag{7}$$

Note that the resonance cannot occur in (6) and (7). This is due to the practically interesting satellite parameters – positional and damping control gains values. These can lead to the solution-related parameters values $\varepsilon < 0.3$ and $\omega^2 < 0.3$.

Now we move to the motion arising from the oscillating solution. This is performed in two steps. First, $\varepsilon = 0$ case is considered. It is shown in [3] that the positional part of the magnetic control has no effect on the angular velocity damping. Gravitational torque also has no effect on the angular velocity being the conservative one. This will eventually lead to the slow rotation and minute effect of the damping control torque part. So the damping effect on the generating oscillations is assessed. Second, full Eq. (5) is considered. Step one result is used as generating solution. Existence of periodical solutions on small time intervals is proven and approximate analytical expressions for these solutions are found. The following analysis is divided into subsections according to this general plan. Neglecting the damping control part in (5) leads to the generating solution

$$\beta = 2\arcsin\left(k\,\mathrm{sn}\,\varphi\right), \tag{8}$$

$$\dot{\beta} = 2\omega k\,\mathrm{cn}\,\varphi, \tag{9}$$

where $\varphi = \omega u + \varphi_0$, $\mathrm{sn}\,\varphi$ and $\mathrm{cn}\,\varphi$ are elliptic sine and cosine, k is these functions modulus. It is equal to the root of the energy integral of Eq. (5) in case $\varepsilon = 0$. Solution (8) is periodic and the period is

$$T = \frac{4\mathrm{K}\left(k\right)}{\omega}.$$

Relations (8)-(9) give the two-parametric solution of (5) in case $\varepsilon = 0$. We do not take into account the rotational solution of (5) focusing on the oscillations only. Clearly adding the damping component in solution (5) leads to the decrease of the energy integral and solution (8)-(9) changes. We assess this change utilizing the van-der-Pole method. In order to do so we consider the parameters of the solution (8)-(9) k and φ_0 as variable ones and get the derivative of (8)-(9). Comparison with the equation of motion leads to the phase and modulus change formulae.

$$\dot{\varphi} = \omega + \varepsilon\left(\frac{\mathrm{cn}\left(\varphi,k\right)\mathrm{sn}\left(\varphi,k\right)}{\mathrm{dn}\left(\varphi,k\right)} + \frac{1}{1-k^2}\mathrm{E}\left(\arcsin\mathrm{sn}\left(\varphi,k\right),k\right)\mathrm{cn}^2\left(\varphi,k\right)\right),$$
$$\dot{k} = -\varepsilon k\,\mathrm{cn}^2\left(\varphi,k\right).$$

Modulus change equation averaged over the generating solution period is

$$\dot{k} = -\frac{\varepsilon\omega}{4k\,\mathrm{K}(k)}\left[\mathrm{E}\left(\mathrm{am}\frac{4\mathrm{K}(k)}{\omega},k\right) + \left(k^2-1\right)\frac{4\mathrm{K}(k)}{\omega}\right]. \quad (10)$$

Eq. (10) may be severely simplified if one considers only a discrete set of frequencies $\omega = \frac{2}{n}$, where n is a natural number. This means that instead of the continuous set of the positional control gain values only a discrete set is considered. However practically interesting positional control gain k_a values lead to $\omega < 0.5$. Therefore if $n \geq 4$ we have practically important set of values of ω. Further modulus evolution assessment requires the full elliptic integrals decomposition into series by the order of k. The number of terms defines the modulus evolutionary solution accuracy. If only the second term is taken into account we get the exponential modulus change

$$k = k_0 \exp\left(-\frac{\varepsilon}{2}u\right).$$

We only take into account the second term and do not take into account the phase change. In this case the generating solution has the form

$$\begin{aligned} \beta &= 2\arcsin\left(k_0 \exp\left(-\frac{\varepsilon}{2}u\right)\mathrm{sn}\,\varphi\right), \\ \dot{\beta} &= 2\omega k_0 \exp\left(-\frac{\varepsilon}{2}u\right)\mathrm{cn}\,\varphi. \end{aligned} \quad (11)$$

Solution (11) can be used as the generating one for the first-order approximation of Eq. (4). The equations arising a tough for the straightforward solving. The existence of periodic solutions can be proven however. Eq. (4) allow periodicity leading to the modulus value k satisfying the equation

$$\mathrm{K}(k) = \frac{\pi\omega m}{4n}. \quad (12)$$

The full elliptic integral is a monotonous function rising from $\pi/2$ to the infinity. So Eq. (12) allows only one solution k^*. Choosing m and n (and therefore elliptic integral values) different modulus values can be obtained. Practically valid control parameters lead to $\omega < 0.5$. Taking into account $\mathrm{K}(k) \geq \pi/2$ this leads to $m/n \geq 4$. If we choose for example $n = 1$ then changing m leads to different modulus values. Greater n values allow denser interval $[0,1]$ decomposition. One of parameters of two-parametric solution of Eq. (4) is considered found. To find the second one we construct the phase balance equation [8] for $n = 1$

$$\int_0^{\pi m} \sin\left[2u - 2\beta_0(k,\varphi_0)\right]\dot{\beta}_0(k,\varphi_0)\,du = 0. \quad (13)$$

This equation allows the second parameter – phase φ_0 – to be determined. Numerical analysis of Eq. (13) provides the phase value for each modulus. Therefore the periodic solution in presence of gravitational torque exists.

Periodic solutions of Eq. (4) can be found numerically according to the relation

$$\beta(u+\pi)=\beta(u)=\beta^*.$$

Periodic solution arising from $\beta(0)=0$ or $\beta(0)=0$ are already found in Eq. (6) however it is valid for the zeroed initial velocity. This analytical solution can be used to characterize periodic solutions amplitude (maximum β deviation from the time interval $u\in[0,\pi]$) despite the fact that the velocity values are different. Figure 1 introduces the periodic solution amplitude for both the analytical and numerical approaches. Periodic solutions in case $\beta^*\neq 0$ should be found numerically.

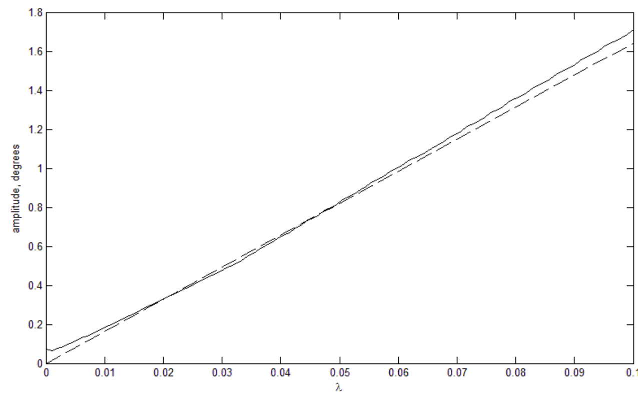

Figure 1. Numerical and analytical amplitudes

Periodic solutions amplitude depends on β^* linearly just as in case of the stable generating point (6). However for each fixed gravitational torque value λ amplitude dependence on the initial condition β^* is strongly nonlinear. The net dependence of the amplitude on both β^* and λ can be represented as

$$\beta_{\max}=k_1(\beta^*)\lambda+k_0(\beta^*).$$

Parameters $k_1(\beta^*)$ and $k_0(\beta^*)$ for discrete argument values in range $\beta^*\in[0,\pi]$ were found using the least-squares method and numerically obtained amplitude values. Further dependence on the gravitational torque can be approximated using the polynomials

$$k_1(\beta^*)=\sum_{j=0}^{5}a_j\beta^{*j},\ k_0(\beta^*)=\sum_{j=0}^{3}b_j\beta^{*j}.$$

The least squares method leads to

$$\begin{aligned}&k_1(\beta^*)=0.01115\beta^{*9}-0.21552\beta^{*8}+1.7586\beta^{*7}-\\&-7.85184\beta^{*6}+20.71312\beta^{*5}-32.12662\beta^{*4}+\\&+26.42569\beta^{*3}-7.42894\beta^{*2}-1.72622\beta^*+0.27445,\\&k_0(\beta^*)=0.03867\beta^{*3}-0.32723\beta^{*2}+1.65024\beta^*-0.01085\end{aligned}$$

with mean-square deviation not more than 0.017. These relations allow quite accurate periodic solutions amplitude approximation for $\beta^*\neq 0$.

3. THREE-DIMENSIONAL MOTION

Three-dimensional motion of the satellite can be studied in the vicinity of stable generating stationary point using Poincare method. Small parameter characterizing the gravitational torque influence should be introduced to equations of motion. We assume that all three principal inertia moments are close, for example $B = A + \lambda A, C = A + \lambda\mu A,$ where $\lambda = o(1)$, $\mu = O(1)$. In this case dimensionless gravitational torque $\mathbf{M}_g = 3\omega_0^2 \overline{\mathbf{M}}_g$ can be introduced. We also introduce the derivative with respect to the argument of latitude instead of time and dimensionless angular velocity. Introducing notation $\mathbf{x} = (\boldsymbol{\Omega}, \alpha, \beta, \gamma)$ we can rewrite the equations of motion in a form

$$\frac{d\mathbf{x}}{du} = \mathbf{f}(\mathbf{x}) + \lambda \mathbf{g}(\mathbf{x}).$$

The solution can be found in a form $\mathbf{x} = \mathbf{x}_0 + \varepsilon \mathbf{x}_1 + O(\lambda^2)$, where $\mathbf{x}_0$ is a generating solution (stationary point in the case considered), $\mathbf{x}_1 = (w_1, w_2, w_3, \alpha_1, \beta_1, \gamma_1)$. This solution can be substituted to the equations of motion,

$$\frac{d\mathbf{x}_0}{du} + \lambda \frac{d\mathbf{x}_1}{du} = \mathbf{f}(\mathbf{x}_0) + \lambda\left(\mathbf{F}(\mathbf{x}_0)\mathbf{x}_1 + \mathbf{g}(\mathbf{x}_0)\right) + O(\lambda^2).$$

Terms of the same order of λ can be grouped together thus leading to the first order approximation equation

$$\begin{aligned}
\dot{w}_1 &= \eta\chi\left[-\left(B_2^2 + B_3^2\right)w_1 + B_1B_2w_2 + B_1B_3w_3\right] + \\
&\quad +2\eta\left[B_1B_2\alpha_1 + B_1B_3\beta_1 - \left(B_2^2 + B_3^2\right)\gamma_1\right], \\
\dot{w}_2 &= \eta\chi\left[B_1B_2w_1 - \left(B_1^2 + B_3^2\right)w_2 + B_2B_3w_3\right] + \\
&\quad +2\eta\left[-\left(B_1^2 + B_3^2\right)\alpha_1 + B_2B_3\beta_1 + B_1B_2\gamma_1\right], \\
\dot{w}_3 &= \eta\chi\left[B_1B_3w_1 + B_2B_3w_2 - \left(B_1^2 + B_2^2\right)w_3\right] + \\
&\quad +2\eta\left[B_2B_3\alpha_1 - \left(B_1^2 + B_2^2\right)\beta_1 + B_1B_3\gamma_1\right] + \frac{3}{2}\sin 2u, \\
\dot{\alpha}_1 &= w_2, \dot{\beta}_1 = w_3, \dot{\gamma}_1 = w_1,
\end{aligned} \tag{14}$$

where $\eta = \frac{k_a B_0^2}{A\omega_0^2}$, $\chi = \frac{k'_\omega}{k_a}$, $k'_\omega = \omega_0 k_\omega$ (damping control gain is altered so it has the same dimension as the positional control gain). Forced solution of (14) can be found as

$$\begin{aligned}
\alpha_1 &= a_1 \sin 2u + a_2 \cos 2u, \\
\beta_1 &= b_1 \sin 2u + b_2 \cos 2u, \\
\gamma_1 &= c_1 \sin 2u + c_2 \cos 2u.
\end{aligned} \tag{15}$$

Substituting (15) into (14) allows the coefficients in (15) be obtained from the equation

$$\begin{pmatrix} B_1B_2 & -\chi B_1B_2 & B_1B_3 & -\chi B_1B_3 & 2-\left(B_2^2+B_3^2\right) & \chi\left(B_2^2+B_3^2\right) \\ \chi B_1B_2 & B_1B_2 & \chi B_1B_3 & B_1B_3 & -\chi\left(B_2^2+B_3^2\right) & 2-\left(B_2^2+B_3^2\right) \\ 2-\left(B_1^2+B_3^2\right) & \chi\left(B_1^2+B_3^2\right) & B_2B_3 & -\chi B_2B_3 & B_1B_2 & -\chi B_1B_2 \\ -\chi\left(B_1^2+B_3^2\right) & 2-\left(B_1^2+B_3^2\right) & \chi B_2B_3 & B_2B_3 & \chi B_1B_2 & B_1B_2 \\ B_2B_3 & -\chi B_2B_3 & 2-\left(B_1^2+B_2^2\right) & \chi\left(B_1^2+B_2^2\right) & B_1B_3 & -\chi B_1B_3 \\ \chi B_2B_3 & B_2B_3 & -\chi\left(B_1^2+B_2^2\right) & 2-\left(B_1^2+B_2^2\right) & \chi B_1B_3 & B_1B_3 \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \\ b_1 \\ b_2 \\ c_1 \\ c_2 \end{pmatrix} = \frac{3}{2\eta}\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}.$$

After this linear system of equations is solved the first order approximation can be assessed.

The stability analysis of the equations of motion in the wide range of control parameters k_a, k_ω is conducted using the Floquet theory [9]. Eq. (2) are linearized in the vicinity of necessary attitude (zero angles and angular velocity) however this position is not an equilibrium one

$$\begin{aligned}
\dot{\Omega}_1 &= \eta\chi\left[-\left(B_2^2+B_3^2\right)\Omega_1 + B_1B_2\Omega_2 + B_1B_3\Omega_3\right] + \\
&\quad +2\eta\left[B_1B_2\alpha + B_1B_3\beta - \left(B_2^2+B_3^2\right)\gamma\right] + 3\frac{C-B}{A}\left(\alpha\cos u\sin u - \gamma\sin^2 u\right), \\
\dot{\Omega}_2 &= \eta\chi\frac{A}{B}\left[B_1B_2\Omega_1 - \left(B_1^2+B_3^2\right)\Omega_2 + B_2B_3\Omega_3\right] + \\
&\quad +2\eta\frac{A}{B}\left[-\left(B_1^2+B_3^2\right)\alpha + B_2B_3\beta + B_1B_2\gamma\right] + 3\frac{A-C}{B}\left(\alpha\cos^2 u - \gamma\cos u\sin u\right), \\
\dot{\Omega}_3 &= \eta\chi\frac{A}{C}\left[B_1B_3\Omega_1 + B_2B_3\Omega_2 - \left(B_1^2+B_2^2\right)\Omega_3\right] + \\
&\quad +2\eta\frac{A}{C}\left[B_2B_3\alpha - \left(B_1^2+B_2^2\right)\beta + B_1B_3\gamma\right] + 3\frac{B-A}{C}\left(\cos u\sin u + \beta\sin^2 u - \beta\cos^2 u\right), \\
\dot{\alpha} &= \Omega_2,\ \dot{\beta} = \Omega_3,\ \dot{\gamma} = \Omega_1.
\end{aligned} \tag{16}$$

Homogenous (16) part can be analyzed with the Floquet approach. Instead of initial damping control gain k_ω altered one $k'_\omega = k_\omega/\omega_0$ is used again. This control gain corresponds to the dimensionless angular velocity in control torque (3). Positional deviation $\mathbf{S}$ is dimensionless also. Hence altered control gains allow the comparison of damping and positional control parts basing on the control gains values. Though both dimensional, their relation is dimensionless. Characteristic exponents of equations (16) are present in Figures 2-9.

Note that geomagnetic field components in (16) may be written in inertial reference frame using different models. IGRF is further used in numerical analysis. Comparison of averaged geomagnetic field model and IGRF leads to one important conclusion. Stability areas are generally greater in case IGRF is used. This is due to the non-uniform induction vector rotation which leads to slightly broadened area of directions for the control torque.

Fig. 2 and 3 bring characteristic exponents for the «Chibis-M» satellite, inertia tensor $\mathbf{J}_x = diag(1.0255,\ 1.5393,\ 1.8172)$ kg·m^2 [10]. Orbital inclination is 30 and 60 degrees respectively for these and further figures pairs, orbital attitude is 600 km. Characteristic exponents depend on the values of control gains.

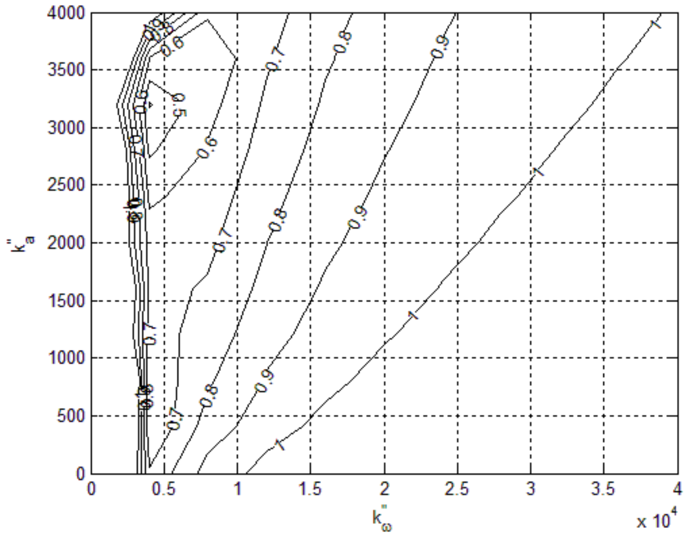

Figure 2. Characteristic exponents. Orbit inclination 30°.

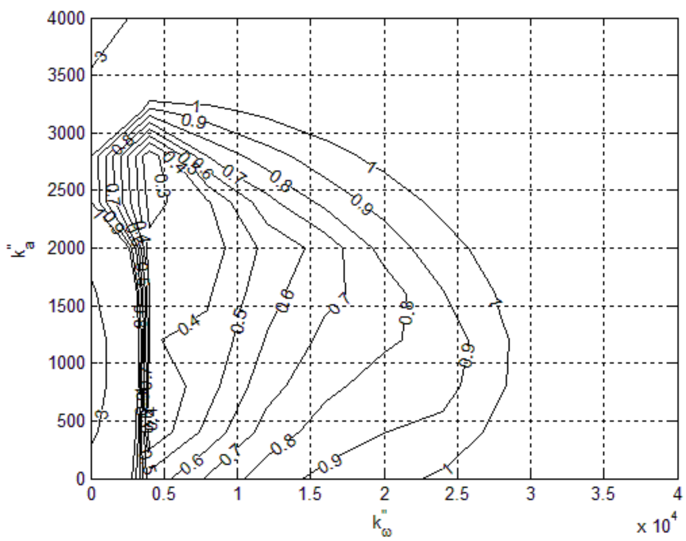

Figure 3. Characteristic exponents. Orbit inclination 60°.

Figures 2 and 3 prove that the high-inclined orbit leads to the smaller stability area but greater are with small characteristic exponents. This is due to the gravitational torque that acts as a restoring one for some orbital and satellite parameters. In fact gravitational torque can implement some part of the desired PD-controller inspired torque that is inaccessible for the magnetic control system. This effect varies with different satellite parameters and orbital inclination. Gravitational torque can be a disturbing one if its component along the geomagnetic induction vector is antiparallel to that component of the ideal PD-controller. Nevertheless gravitational torque can be advantageous even in this case. It doesn't allow the satellite to stick for long in the attitude leading to small control (ideal PD-controller control is directed almost along the geomagnetic induction vector). Figures 4 and 5 bring characteristic exponents for the same system but without the gravitational torque.

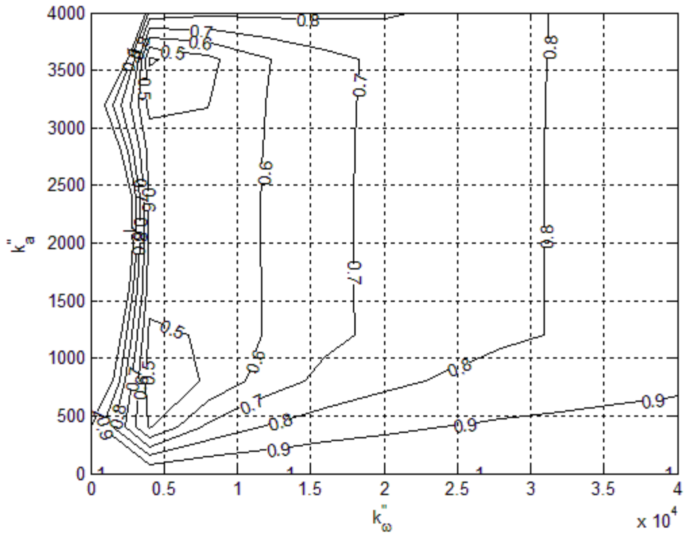

Figure 4. Characteristic exponents. Orbit inclination 30°. Without gravitational torque

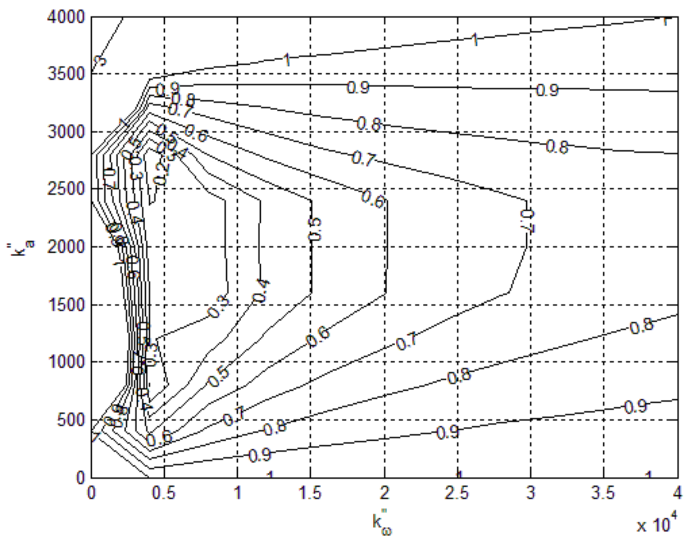

Figure 5. Characteristic exponents. Orbit inclination 60°. Without gravitational torque

Pairs of Figures 2-3 and 4-5 comparison reveals gravitational torque influence. Stability area decreases if gravity is taken into account.

Figures 6 and 7 bring characteristic exponents for the satellite with inertia tensor $\mathbf{J}_x = diag(1,\ 1.1,\ 1.3)\ \text{kg}\cdot\text{m}^2$.

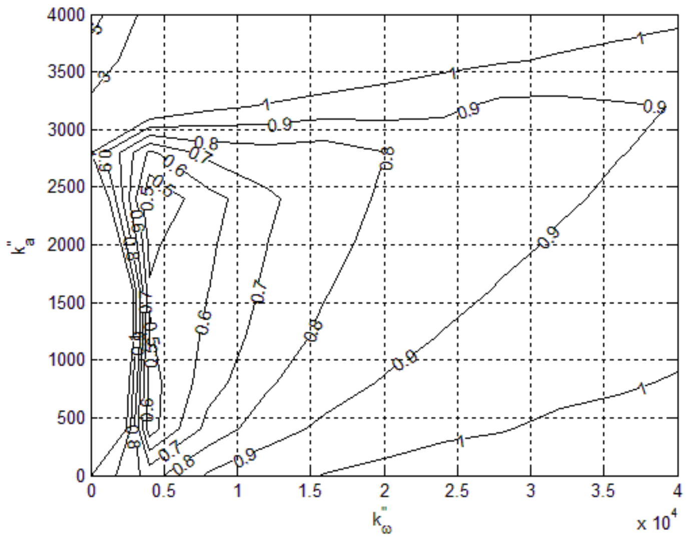

Figure 6. Characteristic exponents. Orbit inclination 30°.

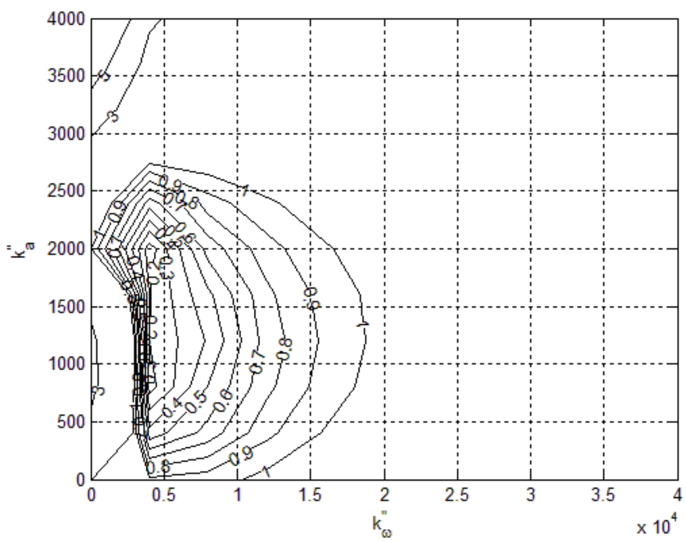

Figure 7. Characteristic exponents. Orbit inclination 60°.

Gravitational torque has reduced influence with the inertia tensor being close to the spherically-symmetrical one. Gravitational torque has negligible effect on the satellite with small size and mass. Nevertheless the stability area is still greater for the low-inclined orbit. Figures 8 and 9 bring characteristic exponents for the satellite with inertia tensor $\mathbf{J}_x = diag(0.17,\ 0.15,\ 0.2)$ kg·m^2.

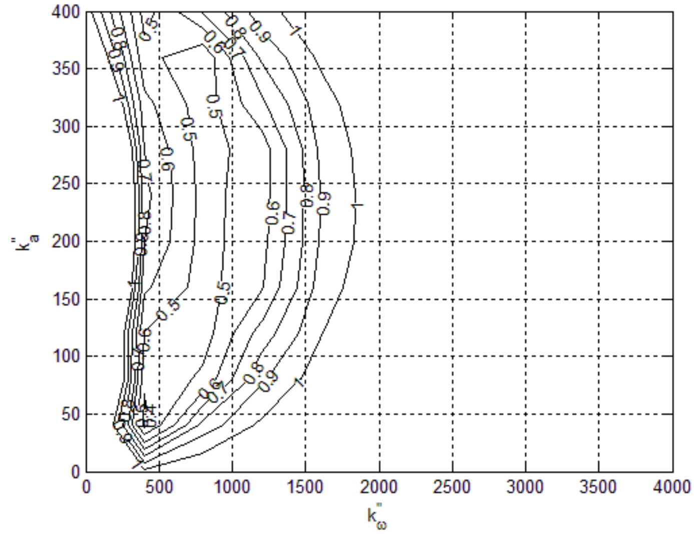

Figure 8. Characteristic exponents. Orbit inclination 30°

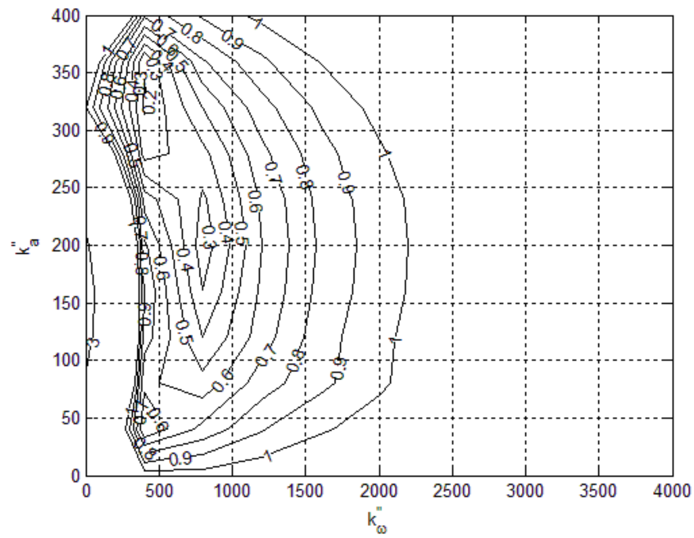

Figure 9. Characteristic exponents. Orbit inclination 60°

Control gains are smaller for this case since the angular moment is small compared to above considered cases [3]. Increased stability area for the highly-inclined orbit is due to the geomagnetic induction vector greater variability.

CONCLUSION

Magnetic attitude control system providing a three-axis inertial attitude in presence of gravitational torque is considered. Planar motion periodic solutions are assessed. Simple approximate formulas are found ensuring good accessible attitude accuracy and its easy evaluation.

Floquet theory is used to assess the stability depending on the control gains. Stability area in the field of control gains are found for a set of typical satellite inertia tensors and orbit inclinations. These areas are compared with those obtained in absence of gravitational torque. This torque influence is shown to be controversial though easily predictable using present techniques. Gravity generally acts as a disturbing one leading to the decreased stability area. This stability area shrinking depends on the satellite and orbit geometry. Overall three-axis magnetic attitude is shown to be achievable in presence of gravitational torque.

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