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ATTITUDE STABILIZATION OF A CHARGED SPACECRAFT SUBJECT TO LORENTZ FORCE

Yehia A. Abdel-Aziz,[*] and Muhammad Shoaib[†]

In this paper, the possibility of the use of Lorentz force, which acts on charged spacecraft, is investigated as a means of attitude control. We assume that the spacecraft is moving in the Earth's magnetic field in an elliptical orbit under the effects of the gravitational and Lorentz torques. We derived the equation of the attitude motion of a charged spacecraft in pitch direction. The effect of the orbital elements on the attitude motion is investigated with respect to the magnitude of the Lorentz torque. The oscillation of angular velocity in pitch direction due to Lorentz force is given for various values of charge to mass ratio. The stability of the attitude orientation is analyzed; and regions of stability are provided for various values of charge to mass ratio. Finally, an analytical method is introduced to study the behavior of all the equilibrium positions. The numerical results confirm that the charge to mass ratio can be used as a semi-passive control for Lorentz-Augmented spacecraft.

INTRODUCTION

The problem of charged spacecraft subject to Lorentz force has recently received renewed attention in the Literature (see, References 1-2). Most recent results concerns the use of Lorentz force for orbital perturbation and controlling the relative motion (see, References 3-8). However, for the problem of attitude stabilization of a charged spacecraft a few important results have also been derived (see, References 9-12).

The present work considers the attitude orientation of an electrostatically charged spacecraft. We have taken into account the effects of gravitational and Lorentz Torques to study the attitude stablization of spacecraft moving in low Earth orbit (LEO). We are using the same model for Lorentz torque as in reference 12 to analyize the effects of orbital elements on the magnitude of Lorentz torque, and invistigate the oscillation in angular velocity in pitch direction for various values of charge to mass ratio. Both numerical and analytical techniques are used to identify stable and unstable regions for equlibrium positions for various values of charge to mass ratios.

[*] Associate Professor, National Research Institute of Astronomy and Geophysics (NRIAG), Elmarsed Street 11721, Helwan, Cairo Egypt. Email: yehia@nriag.sci.eg

[†] Assistant Professor, Department of Mathematics, University of Ha'il, PO BOX 2440, Ha'il, Saudi Arabia, email: safridi@gmail.com

SPACECRAFT MODEL AND TORQUE DUE TO LORENTZ FORCE

A rigid spacecraft is considered whose center of mass moves in the Newtonian central gravitational field of the earth in an elliptic orbit. We suppose that the spacecraft is equipped with an electrostatically charged protective shield, having an intrinsic magnetic moment. The rotational motion of the spacecraft about its center of mass is analyzed, considering the influence of gravity gradient torque T_G and the torque T_L due to Lorentz forces. The torque T_L results from the interaction of the geomagnetic field with the charged screen of the electrostatic shield.

The rotational motion of the satellite relative to its center of mass is investigated in the orbital coordinate system $C_{x_o y_o z_0}$ with C_{x_o} tangent to the orbit in the direction of motion, C_{y_o} lies along the normal to the orbital plane, and C_{z_o} lies along the radius vector r of the point O_E relative to the center of the Earth. The investigation is carried out assuming the rotation of the orbital coordinate system relative to the inertial system with the angular velocity Ω. As an inertial coordinate system, the system O_{XYZ} is taken, whose axis $OZ(k)$ is directed along the axis of the Earth's rotation, the axis $OX(i)$ is directed toward the ascending node of the orbit, and the plane coincides with the equatorial plane. Also, we assume that the satellite's principal axes of inertia $C_{x_b y_b z_b}$ are rigidly fixed to a satellite (i_b, j_b, k_b). The satellite's attitude may be described in several ways, in this paper the attitude will be described by the angle of yaw ψ the angle of pitch θ, and the angle of roll φ, between the axes $C_{x_b y_b z_b}$ and O_{XYZ}. The three angles are obtained by rotating satellite axes from an attitude coinciding with the reference axes to describe attitude in the following way:

- The angle of precession ψ is taken in plane orthogonal to Z -axis.
- θ is the notation angle between the axes Z and z_0.
- ϕ is angle of self -rotation around the Z -axis

According to[13], we can write the relationship between the reference frames $C_{x_b y_b z_b}$ and $C_{x_o y_o z_0}$ as given by the matrix A which is the matrix of direction cosines $\alpha_i, \beta_i, \gamma_i$, $(i = 1,2,3)$.

$$A = \begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{pmatrix}, \quad (1)$$

where

$$
\begin{aligned}
&\alpha_1 = \cos\psi\cos\phi - \sin\psi\sin\phi\cos\theta,\\
&\alpha_2 = -\cos\psi\sin\phi - \cos\theta\sin\psi\cos\phi,\\
&\alpha_3 = \sin\theta\sin\psi,\\
&\beta_1 = \sin\psi\cos\varphi + \cos\theta\cos\psi\sin\varphi,\\
&\beta_2 = -\sin\psi\sin\phi + \cos\theta\cos\psi\cos\phi,\\
&\beta_3 = -\sin\theta\cos\psi),\\
&\gamma_1 = \sin\theta\sin\phi,\\
&\gamma_2 = \sin\theta\cos\phi,\\
&\gamma_3 = \cos\theta,
\end{aligned} \tag{2}
$$

and $$\vec{\alpha} = \alpha_1 i_b + \alpha_2 j_b + \alpha_3 k_b,\ \vec{\beta} = \beta_1 i_b + \beta_2 j_b + \beta_3 k_b,\ \vec{\gamma} = \gamma_1 i_b + \gamma_2 j_b + \gamma_3 k_b, \tag{3}$$

As stated above, it is assumed that the spacecraft is equipped with elctrostatic cahrge, therefore we can write the torque due to Lorentz force as follows[12].

$$\vec{T}_L = (T_{Lx}, T_{Ly}, T_{Lz}) = q\vec{\rho}_0 \times A^T(\vec{V}_{rel} \times \vec{B}_o), \tag{4}$$

or $$\vec{T}_L = (T_{Lx}, T_{Ly}, T_{Lz}) = \rho_0 \times A^T\left(R_L, T_L, N_L\right)^T, \tag{5}$$

$$\vec{\rho}_0 = x_0 i_b + y_0 j_b + z_0 k_b = q^{-1}\int_S \sigma\rho dS. \tag{6}$$

$\vec{\rho}_0$ is the radius vector of the charged center of the spacecraft relative to its center of mass and A^T is the transpose of the matrix A of the direction cosines $\vec{\alpha}$, $\vec{\beta}$, $\vec{\gamma}$, $\left(R_L, T_L, N_L\right)$ are the components of Lorentz force into the radial, transverse, and normal components respectively yields,

$$R_L = \frac{q}{m}\frac{B_0}{r^2}\left[\omega_e[1-\sin^2 i\sin^2(\omega+f)] - \sqrt{\mu/p^3}\cos i\left(1+e\cos f\right)^2\right], \tag{7}$$

$$T_L = \frac{q}{m}\frac{B_0}{\sqrt{\mu p}}\left[\begin{matrix}\frac{\dot{r}}{r}\sqrt{\mu/p^3}\cos i\left(1+e\cos f\right)^2 + 2\omega_e\sqrt{\mu/p^3}\times \\ \sin^2 i\sin(\omega+f)\cos(\omega+f)\left(1+e\cos f\right)^2\end{matrix}\right],$$

(8)

$$N_L = \frac{q}{m}\frac{B_0}{\sqrt{\mu p}}\left[\begin{array}{c} 2\left[\omega_e[1-\sin^2 i\sin^2(\omega+f)]-\sqrt{\mu/p^3}\cos i\,(1+e\cos f)^2\right]\times \\ \sqrt{\mu/p^3}\cos i\,(1+e\cos f)^2+\frac{\dot{r}}{r}\sqrt{\mu/p^3}\,\frac{\cos i}{\sqrt{1-\sin^2 i\sin^2(\omega+f)}}\times \\ (1+e\cos f)^2-2\frac{\mu}{p^3}\frac{\sin^3 i\cos^2(\omega+f)\sin(\omega+f)}{1-\sin^2 i\cos^2(\omega+f)}(1+e\cos f)^4 \end{array}\right]. \quad (9)$$

where, B_0 is the strength of the magnetic field in Wb.m., is the charge-to-mass ration of the spacecraft, μ is the Earth's gravitational parameter a ,e, i , ω and f a are semi-major axis, eccentricity, inclination of the orbit on the equator, argument of the perigee, and the true anomaly of the spacecraft orbit respectively.

SPACECRAFT ATTITUDE MOTION EQUATIONS

The attitude motion of the spacecraft is expressed by Euler's equations[13]. Therfore, the equation of the attitude dynamics of a rigid spacecraft due to gravity gradient and Lorentz torques is expressed as

$$\dot{\vec{\omega}} I + \vec{\omega}\times\vec{\omega} I = \vec{T}_G + \vec{T}_L, \quad (10)$$

where $T_G = 3\Omega^2 \vec{\gamma}\times\vec{\gamma} I$ is the well known formula of the gravity gradient torque, I = diag (A,B,C) is the inertia matrix of the spacecraft, Ω is the orbital angular velocity, $\vec{\omega}$ is the angular velocity vector of the spacecraft. The angular velocity of the spacecraft in the inertial reference frame is $\vec{\omega} = (p,q,r)$, where

$$\begin{aligned} p &= \dot{\psi}\sin\theta\sin\phi + \dot{\theta}\cos\phi \\ q &= \dot{\psi}\sin\theta\cos\phi - \dot{\theta}\sin\phi\ , \\ r &= \dot{\psi}\cos\theta + \dot{\phi}. \end{aligned} \quad (11)$$

The system of Eq. (10) admits Jacobi´s integral[14]

$$\frac{1}{2}\vec{\omega}\cdot\vec{\omega} I\text{-}V_0 = \text{h} \quad , \quad (12)$$

Where V_0 is the potential of the problem and takes the following expression[15]

$$V_0 = \frac{3}{2}\Omega^2(\gamma \cdot \gamma \mathrm{I}) + \rho_0 \cdot \left(\mathrm{R_L}, \mathrm{T_L}, \mathrm{N_L}\right)^{\mathrm{T}} \left(\beta_1, \beta_2, \beta_3\right)$$
$$= \frac{3}{2}\Omega^2(\gamma \cdot \gamma \mathrm{I}) + x_0 \mathrm{R_L}\beta_1 + y_0 \mathrm{T_L}\beta_2 + \mathrm{z}_0 \mathrm{N_L}\beta_3,$$

(13)

where, the first part is potential due to the gravitational potential and the rest of the part due to Lorentz force.

ATTITUDE MOTION IN THE PITCH DIRECTION

Assume the attitude motion of the charged spacecraft in the pitch direction, i.e. $\psi = \phi = 0, \theta \neq 0.$ Applying this condition in Euler equation of the attitude motion of the spacecraft in Eq (10), we derive the second order differential equation of the motion.

$$A\frac{d^2\theta}{dt^2} = (C - B)(3\Omega^2 - 1)\sin\theta\cos\theta + (z_0\ N_L - y_0 T_L)\sin\theta + (y_0 N_L - z_0 T_L)\cos\theta. \qquad (14)$$

Let $y_0 = k\ z_0,$

(15)

whereis arbitrary number. Then equation (12) takes the following form.

$$A\frac{d^2\theta}{dt^2} = (C - B)(3\Omega^2 - 1)\sin\theta\cos\theta + z_0(N_L - kT_L)\sin\theta + z_0(kN_L - T_L)\cos\theta.$$

(16)

In right hand side of this equation, the first term represents the gravity gradient torque, the second and third terms represent the Lorentz torque.

Using equation (12, 13), we can write the Jacobi´s integral as follows

$$h = \frac{1}{2}(\frac{d\theta}{dt})^2 - \frac{3}{2}\Omega^2(B\sin^2\theta + C\cos^2\theta) - z_0(kN_L\cos\theta - T_L\sin\theta)$$

(17)

The quantity *h* corresponds to the energy in the rotating refrence frame, which we can use to calculate the minimum and maximum energy required for stable equilibrium positions, and the energy needed to move from unstable position to stable position.

EFFECT OF ORBITAL ELEMENTS ON TORQUE

In this section, we study the effects of orbital elements on the magnitude of Torque due to Lorentz force.

Using Equation (5), and $z_0 = 1,\ k = 1.1,\ e = 0.001,\ i = 15^{o}, and\ \ f = 60^{o}.$ We can calculate the components of the Lorentz torque, and the magnitude of the torque as function of semimajor axis and charge to mass ratio .

, (18)

, (19)

. (20)

Using the values $z_0 = 1,\ k = 1.1,\ a = 6900km,\ i = 15^{o}, and\ \ f = 60^{o}$, we get the components of Lorentz torque as a function of eccentricity and charge to mass ration (*asthe following:*

, (21)

(22)

(23)

Using Equations (18-20), we can calculate the magnitude of the Lorentz torque as a function of semi-major axis and charge to mass ratio. Figure (1-left) shows the magnitude of Lorentz torque for semi-major (a) between 6500 km to 12000 km with three different values of *, ,* and *. The figure shows that* the magnitude of Lorentz torque is decreasing with the altitude of the satellite increases and the magnitude of the torque is affected by charge to mass ratio. Similarly, from equations (21-23) we can calculate the magnitude of the torque as a function of eccentricity and charge to mass ratio; shown in Figure (1-rigth). This figure shows the magnitude is nearly constant up to and when , the magnitude of torque increases with the increasing value of eccentricity.

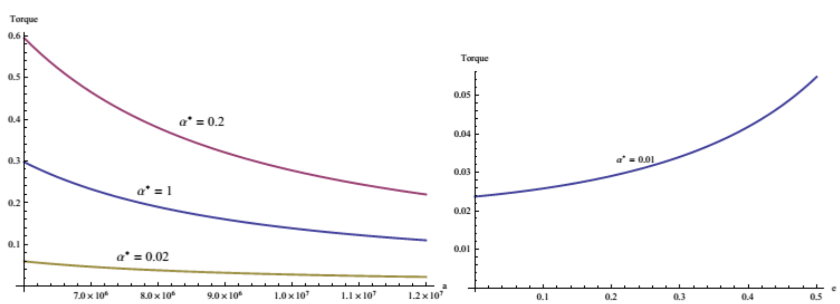

Figure 1. Magnitude of the Lorentz torque as function of (Left) semi-major axis and (right) eccentricity

Using the values $z_0 = 1,\ k = 1.1,\ e = 0.001,\ a = 6900km, and\ \ f = 60^{o}$, we get the components of Lorentz torque as a function of inclination and charge to mass ratio (*asthe following:*

(24)

(25)

(26)

Similarly using the values $z_0 = 1,\ k = 1.1, a = 6900km, i = 15^{o} and\ a = 6900km$, we get the components of Lorentz torque as a function of eccentricity and charge to mass ratio (*asthe following:*

, (27)

, (28)

(29)

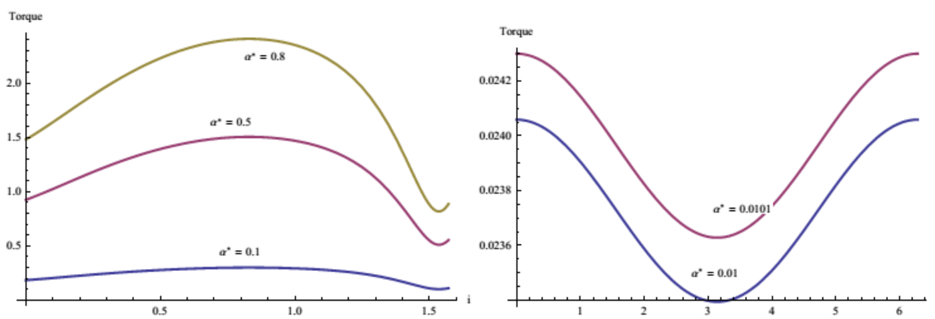

Figure 2. Magnitude of the Lorentz torque as function of (Left) inclination and (right) true anomaly

Using Equations (24-26), we can calculate the magnitude of the Lorentz torque as a function of inclination and charge to mass ratio. Figure (2-left) shows the magnitude of Lorentz torque when . The figure shows that the magnitude of Lorentz torque is increasing with inclination increasing till and after that the torque is decreasing with increasing value of inclination. Similarly, from equations (27-29) we can calculate the magnitude of the torque as a function of true anomaly and charge to mass ratio. Figure (2-rigth) shows the magnitude of torque is decreasing with the increasing value of true anomaly up to and when, the magnitude of the torque starts increasing.

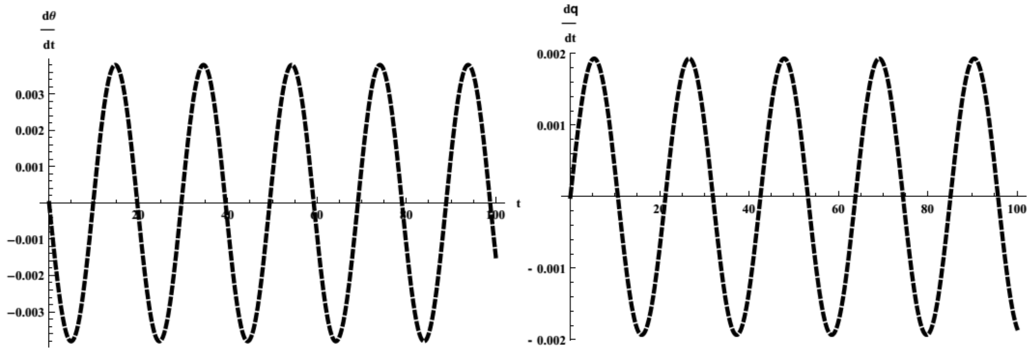

Figure 3. Oscillation in $\frac{d\theta}{dt}$ **due to Lorentz torque when (left)** , **B < C and (right)** .

Figures (3-4) shows the oscillation in angular velocity due to Lorentz torque. It is clear from Figure (3-left) that the oscillation in angular velocity is between -0.004 and 0.004 when , *and* $B > C$ but when *(Figure* 3,right) and $B > C$ the oscillation is between -0.002 and 0.002. Similarly (Figure 4-left) the oscillation in angular velocity is between -0.3 and 0.3 when *and* $B > C$. In the case of figure (4-right) where , *and* $B > C$, the oscillation is between -0.3 and 0.3. It is obvious that the difference in behavior in the oscillation in Figure (3, left) is due to charge to mass ratio. In addition, the effects of the components of moment of inertia of the charged spacecraft are clear when we compare figure (3) and figure (4).

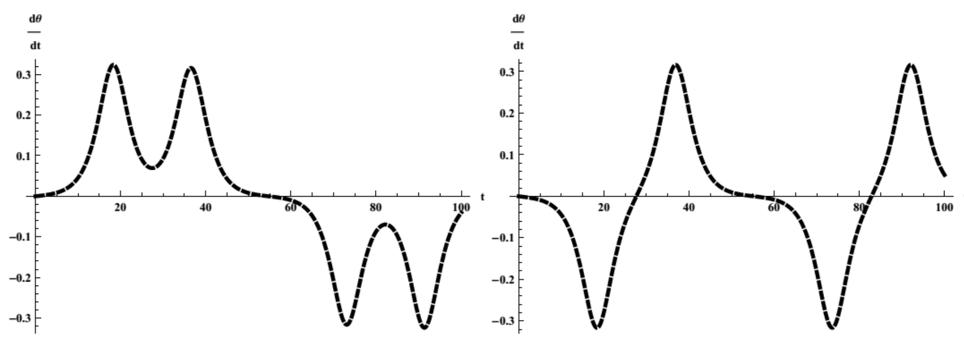

Figure 4. Oscillation in $\frac{d\theta}{dt}$ **due to Lorentz torque when (left)** $B > C$ **and (right)** ,

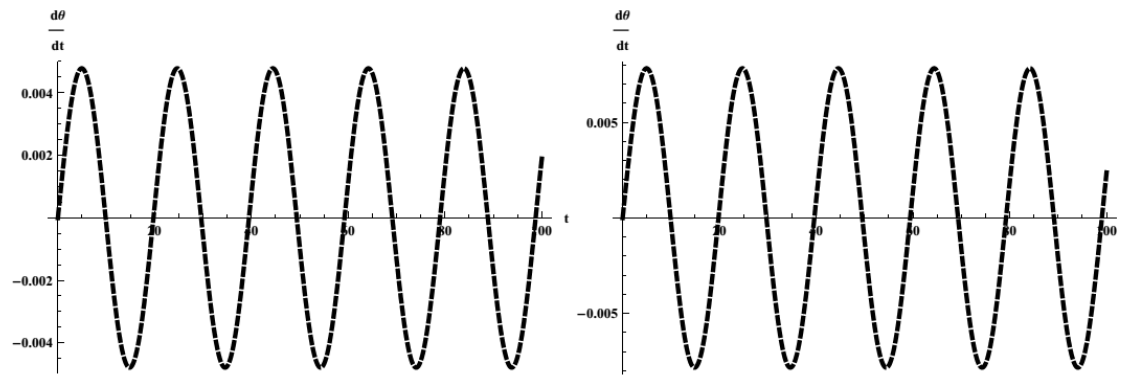

Figure 5. (Left) Oscillation in angular velocity of LAGEOS with the orbital elements (Right) Oscillation in angular velocity of LARES with the orbital elements

Figure 5, shows the comparison between the oscillation in angular velocity for an artificially charged LAGEOS satellite (5-left) and oscillation in angular velocity for an artificially charged LARES satellite (5-right) due to Lorentz torque. It is clear from the figure that the oscillation in LARES is two time the oscillation in LAGEOS for.

EXISTENCE AND STABILITY OF EQUILIBRIUM SOLUTIONS

In this section, we discuss the existence and stability of equilibrium positions of a general shape spacecraft under the influence of gravitational torque and Lorentz torque. We will identify regions in the phase space in which a continuous family of stable equilibrium positions exists.

Existence of equilibrium positions

To find the equilibrium positions, take the right hand side of equation (14) equal to zero which reduces to the following equation when .

(30)

If we solve, *we will obtain all the equilibrium positions for all values of.For*, after some algebraic manipulation *is reduced to the following equation.*

(31)

The above equation is quartic in and can theoretically be solved but is very complicated and beyond the scope of this study. Therefore, we will study it numerically for fixed values of . As is apparent from equation (31), the values of *will have a significant e*ffect on the existence equilibrium solutions. For example, when *there are four equilibrium positions for* all values of except when *and when* and, *the number of equilibrium positions reduces to two.* Two typical examples are given in figure (5). In figure (5, left), when,there are four equilibrium positions for all values of except when. For , the equilibriums occur around. For the first equilibrium, occurs around and converges to zero as the value of decreases to -1. The other three equilibriums occur around. In figure (5, right), when, there are four equilibrium positions for very small values of and two when *. These equilibrium positions* occur at when the spacecraft is negatively charged and at for a positively charged spacecraft

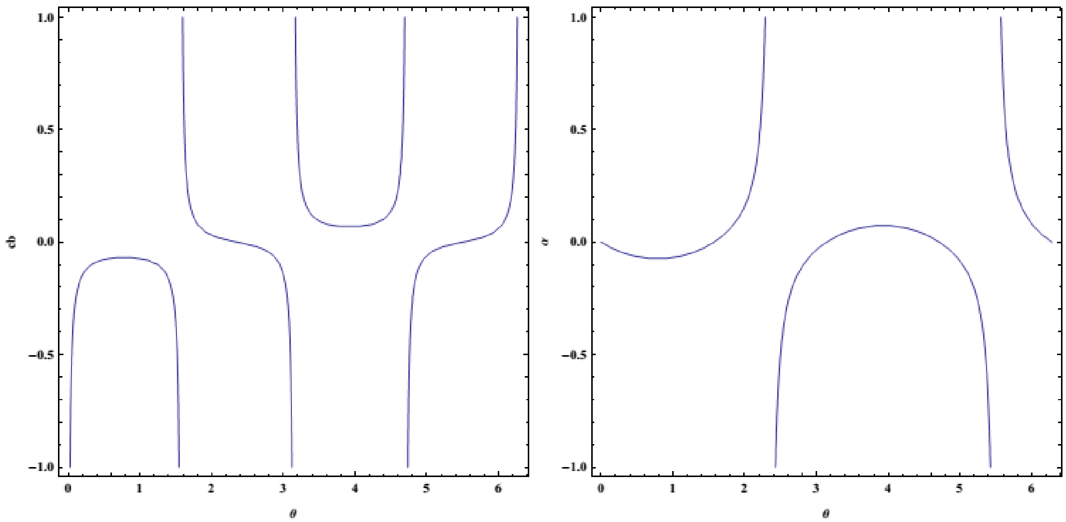

Figure 6. Family of equilibrium positions when (left) *and (right)* **.**

Stability analysis of equilibrium solutions

To discuss the stability of the equilibrium position identified we convert equation (14) to a system of two first order equations.

After linearization, we can write the Jacobian of the above system as below.

Let be an eigenvalue of *The* characteristic equation is given by *It is well known that if at least one of the eigenvalues is positive then the corr*esponding equilibrium will be unstable. If both the eigenvalues are pure imaginary then corresponding equilibrium will be a center. In our case *will imply real eigenvalues with one of them positive and hence instability and will imply a stable center.* Therefore, to study the stability of all the equilibrium positions we need to write

.

Before we give a complete picture of the stability and unstable regions, we give specific examples of the artificial satellites LARES and LAGEOS 1. We assume that these two satellites are artificially charged. The orbital elements of LARES are

Let it has a charge to mass ratio of*. There are two equilibrium positions for LARES at* It is a straightforward exercise to show that *is positive and hence* is an unstable equilibrium. The value of *is negative and hence this equilibrium is a stable center. To support this conclusion we also* give a phase diagram in *in figure* (7, left) which confirms both the conclusions. The orbital elements for LAGEOS 1 satellite are

Let it has a charge to mass ratio of *.There are two equilibrium positions for LARES* at . The equilibrium position at is stable and is unstable as the eigenvalues are imaginary and positive respectively.

To completely analyze the stability and its dependence on *, we provide the stability diagrams for various values of . In figures (8 to 10), the blue lines correspond to the family of equilibrium pos*itions and the shaded regions are the regions where the equilibrium positions will be stable. Figure (8, left) and figure (8, right) are given for *. The locations of the stable regions are nearly unaffected by the changing values of the charge to mass ratios but the locations and hence stability of equ*ilibrium positions are significantly affected. For example in the upper half of both parts of figure 8, there are four equilibriums each and two of them stable but for *, the first equilibrium is at 0 which is stable and for there is no such equilibrium. In fact there is an at 0 but for negative values of* Similar behavior is shown by the equilibrium There is no change in the behavior of the equilibrium close to . The only change in this case is when cb is very close to 0.

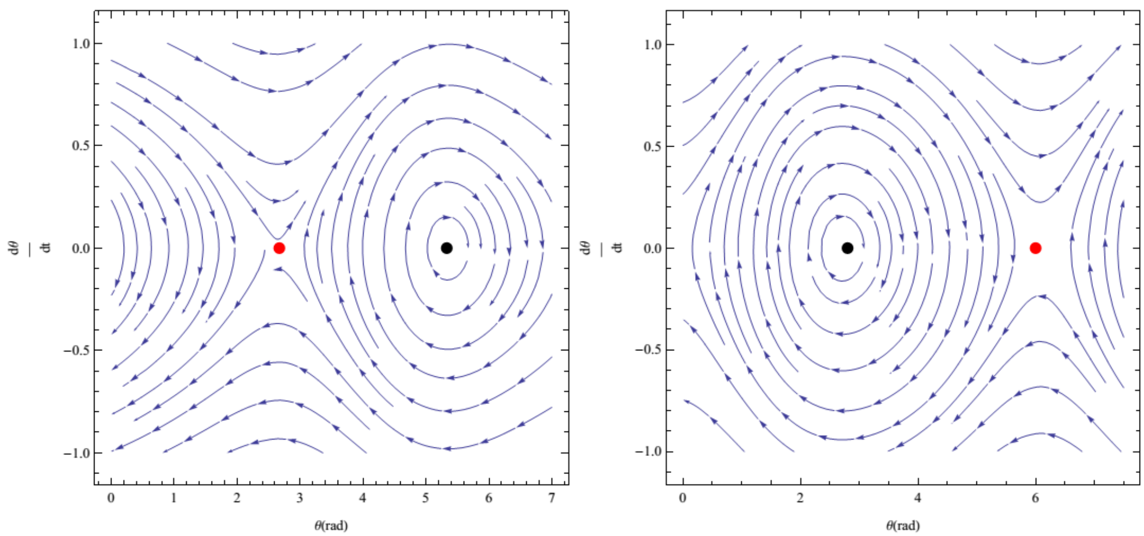

Figure 7. Phase diagram in *of (left) LARES satellite with and (right) LAGEOS satellite* with *. The red dots in both figures correspond to the unstable orbit and the black dot correspond*s to the stable orbit.

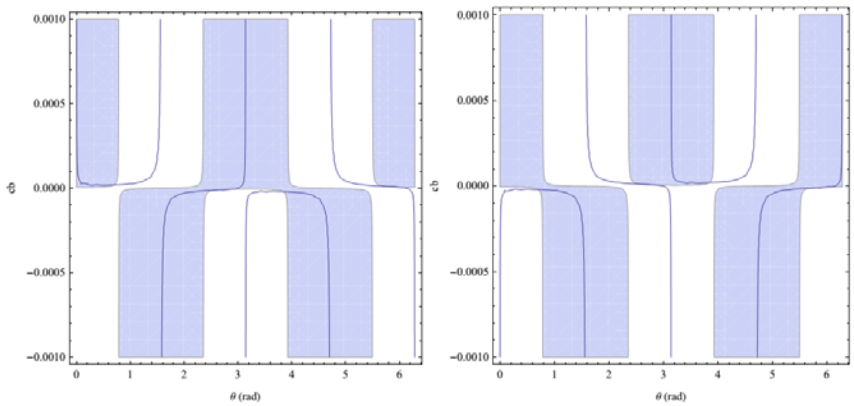

Figure 8. Stability regions when *and* **(Left)** *and (right)*

Figure (9, left) and figure (9, right) are given for *. It* is readily noticed that, both the locations of the stable regions and locations of equilibrium positions are revered by the changing values of the charge to mass ratios. When *, there are four equilibriums when* and there equilibriums when with two and one of them lying in the stable regions respectively. Similarly, when*, there are four equilibriums when* and three equilibriums when with two and one of them lying in the stable regions respectively. Similar conclusions can be drawn from figures (10) which is given for *.*

In summary, we can conclude from figures 8,9 and 10 that *and* significantly affect both the stability and locations of the equilibrium solutions and hence can be used as semi-passive control.

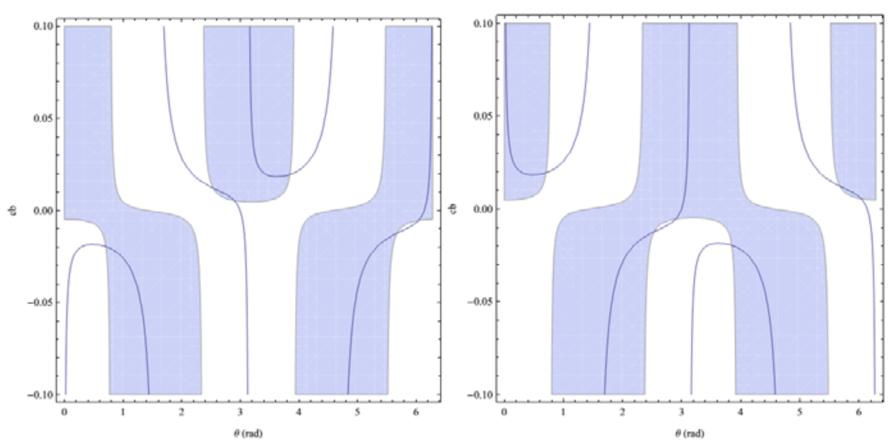

Figure 9. Stability regions when *and* (Left) *and (right)*

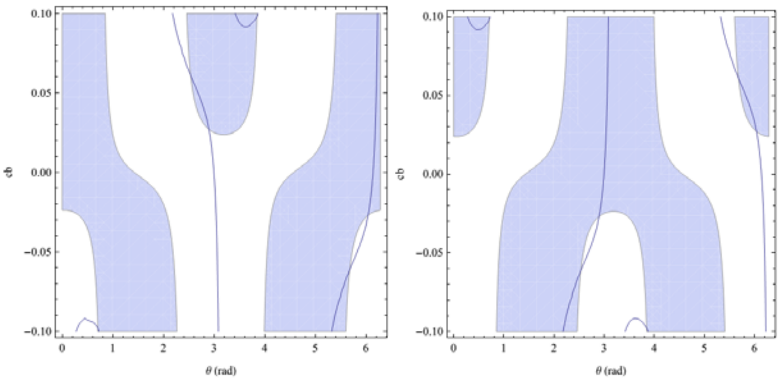

Figure 10. Stability regions when *and* (Left) *and (right)*

CONCLUSIONS

In this work, we analyzed the problem of attitude dynamics of an electrostatically charged spacecraft under the effects of gravity gradient and Lorentz torque in pitch direction. The effects of orbital elements on the magnitude of Lorentz torque is analyzed for various value of charge to mass ratio. The oscillation in angular velocity in pitch direction for various values of charge to mass ratio is considered for different operational satellite (LARES and LAGEOS 1).The stable and unstable regions concerning the effects of Lorentz toreque for different satellite orbits are identified. Both stable and unstable equilibrium poitisions are identified for LARES and LAGEOS 1 satellite. It is shown that for an artificialy charged LARES and LAGEOS 1 satellite there are two possible equilibrium positions with one each stable and one unstable. As from the general analysis it is possible to chnge the location of the equilibrium positions by varying the values of charge to mass ratio *or the moment of inertia. This is demonstrated by various numerical studies performed for various values of orbital elements. Depending on the values of and moment of inertia the number of equilibrium positions vary between two and four.*

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