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CONFIGURATION OPTIMIZATION FOR SWARM SPACECRAFT BASED ON PSO-SQP ALGORITHM

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This paper proposes a method for optimizing the configuration for swarm spacecraft. Based on relative orbital elements, the description parameters of the cluster configuration is presented, and the cluster stability is analyzed in the presence of the J_2 perturbation. Three main constraints are considered for the design of the cluster configuration, including station-keeping ΔV requirements, passive safety requirements and inter-module communication range requirements; we also formulate the constraints in terms of relative orbital elements. Then, the hybrid particle swarm optimization algorithm integrated with sequential quadratic programming is adopted to handle the optimization issue, and a scenario of 8 modules is used as an example. The results show that the proposed approach is valid and the hybrid PSO-SQP algorithm is very effective in optimizing the cluster configuration.

INTRODUCTION

Distributed spacecraft has been well concerned in the past decade because of its unique technical merits and good prospects in applications. Many small satellites can form a virtual large satellite, which possess advantages such as increased instrument resolution, reduced cost, reconfigurability, and overall system robustness, which can in turn enhance the scientific return[1]. Swarm spacecraft has been focused on in the last few years. Cluster configuration design, cluster control architecture, and cluster control strategy are three basic components for the swarm control system, and the cluster configuration design is a fundamental problem, which should be solved firstly. Swarm spacecraft usually consist of tens and scores of modules and the modules often operate in close proximity, which make the cluster configuration design issue more challenging.

Generally, the configuration design methods can be categorized into two kinds: one is based on Cartesian states and the other is based on relative orbital elements [2]. The advantages of relative orbital elements are that they are more intuitive and easier to address J_2 perturbation. Relative orbital elements were demonstrated during the GRACE, PRISMA and TanDEM-X missions [3]. Therefore, the current study will concentrate on the cluster configuration design based on relative orbital elements.

The configurations of swarm spacecraft can affect the performance characteristics of the cluster in multiple phases [4]. There are several requirements that must be satisfied, such as the re-

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quirements of the collision free conditions, which ensure the safety of the modules; the requirements of the inter-module communication range, which make the inter-module communication possible; the requirements of the station-keeping ΔV, which minimize the fuel consumption during the lifetime of the spacecraft. This paper will mainly consider the above-mentioned three kinds of constraints to optimize the cluster configuration.

The organization of this paper is as follows: Section 2 presents the general cluster configuration description parameters and analyzes the stability under J_2 perturbation. Section 3 formulates the constraints in terms of relative orbital elements. In section 4, we establish the optimization model. Section 5 gives an optimization example of 8 modules, and analyzed the stability of the designed cluster configuration. The current paper ends with the conclusions in Section 6.

RELATIVE MOTION MODEL

Description of the cluster configuration

The relative reference frame used is the Hill frame. The origin of the coordinate system is placed at the center of mass of the master satellite; the x axis is aligned in the radial direction, the z axis is aligned with the angular momentum vector and the y axis completes the right-handed system.

The Keplerian orbital elements are a, e, i, Ω, ω, and u, which correspond to the semi-major axis, eccentricity, inclination, right ascension of the ascending node, argument of perigee, and mean argument of latitude ($u=\omega+M$, where M is the mean anomaly), respectively. Spacecraft-1 is the master satellite, and Spacecraft-2 is the deputy satellite. For near-circular satellite orbits, the relative eccentricity vector can be defined as follows [5]:

$$\Delta\boldsymbol{e}=\begin{bmatrix}\Delta e_x\\ \Delta e_y\end{bmatrix}=\delta e\begin{bmatrix}\cos\theta\\ \sin\theta\end{bmatrix}=e_2\begin{bmatrix}\cos\omega_2\\ \sin\omega_2\end{bmatrix}-e_1\begin{bmatrix}\cos\omega_1\\ \sin\omega_1\end{bmatrix} \tag{1}$$

$$\theta=arc\tan(\Delta e_y,\Delta e_x) \tag{2}$$

where δe represents the amplitude of $\Delta\boldsymbol{e}$ and θ defines the initial phase angle of the in-plane motion.

The inclination vector $\Delta\boldsymbol{i}$ can be defined using the law of sines and cosines for the spherical triangle [8]:

$$\Delta\boldsymbol{i}=\begin{bmatrix}\Delta i_x\\ \Delta i_y\end{bmatrix}=\delta i\begin{bmatrix}\cos\varphi\\ \sin\varphi\end{bmatrix}\approx\begin{bmatrix}\Delta i\\ \Delta\Omega\sin i_1\end{bmatrix} \tag{3}$$

$$\varphi=arc\tan(\Delta i_y,\Delta i_x) \tag{4}$$

where $\Delta i=i_2-i_1$, $\Delta\Omega=\Omega_2-\Omega_1$, δi represents the amplitude of $\Delta\boldsymbol{i}$, and φ defines the initial phase angle of the cross-track plane motion.

For a near-circular reference orbit, the relative motion of the cluster flying modules can be described by the following equations [6]:

$$\begin{cases}x=\Delta a-p\cos(u-\theta)\\ y=2p\sin(u-\theta)+l\\ z=s\sin(u-\varphi)\end{cases} \tag{5}$$

where $\{p,s,\alpha,\theta,l\}$ are the five general cluster configuration description parameters; $p=a\delta e$ represents the semi-minor axis of the relative in-plane ellipse; $s=a\delta i$ denotes the cross-track amplitude; $\alpha=\theta-\varphi$ defines the relative initial phase angle between the in-plane and cross-track plane motions; and θ is the initial phase angle of the in-plane motion. $\Delta u = u_2 - u_1$, $l = a(\Delta u + \Delta\Omega\cos i) - \frac{3}{2}(u - u_0)\Delta a$, u_0 is the initial mean argument of latitude of the deputy satellite, and l represents the along-track offset of the center of the in-plane motion. An example trajectory is shown in Figure 1.

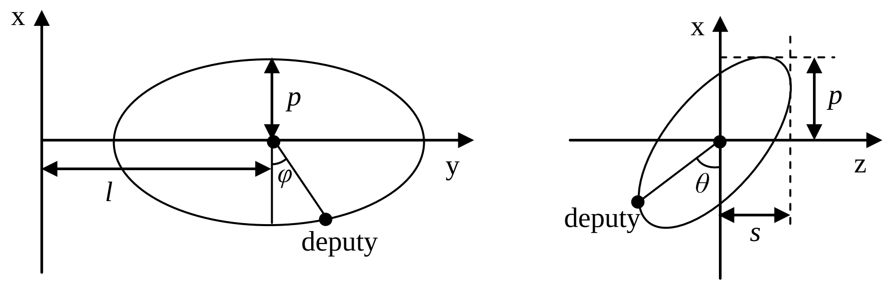

Figure 1. Example of a relative motion in a near-circular reference orbit.

Cluster stability analysis

The J_2 perturbation and atmospheric drag are the main perturbations experienced by swarm spacecraft in LEO. For a near-circular orbit, when the short periodic perturbations are ignored, the relative eccentricity vector rotates in the e-vector plane with an angular velocity expressed by

$$\dot{\theta} \approx \frac{3}{2}(\pi / T)(R_e^2 / a^2) J_2 (5\cos^2 i - 1) \tag{6}$$

where a represents the semi-major axis, i represents the inclination, and T is the orbit period.

The influences of the J_2 perturbation on the relative inclination vector are described as follows:

$$\Delta \boldsymbol{i} = \begin{Bmatrix} \Delta i \\ \Delta\Omega \cdot \sin i \end{Bmatrix} = \begin{Bmatrix} \Delta i_x \\ \Delta i_y + \frac{d\Delta i_y}{dt} \cdot t \end{Bmatrix} \tag{7}$$

where

$$\frac{d\Delta i_y}{dt} \approx \frac{3\pi}{T}(R_e^2 / a^2) J_2 \sin^2 i \cdot \Delta i \tag{8}$$

Eq. (8) shows that Δi determines the changes in the relative inclination vector. The J_2 perturbation does not affect the relative inclination vector of the clusters with the same inclinations.

CONFIGURATION DESIGN PROBLEM

Before establishing the optimization model, we should clarify the requirements of the cluster configuration. The first one is the station-keeping ΔV requirements, which minimizes the fuel consumption due to J_2 perturbation. The second one is the passive safety requirements, which prevent the collisions among the modules even in the presence of the along-track drift. The last one is the inter-module communication range requirements, which ensure that the modules are in the restriction range of inter-module communication. In the following part, we will express the three constraints in terms of the cluster configuration parameters.

Station-keeping ΔV requirements

We assume that the nominal configuration parameters in the orbital plane are p_1 and θ_1, and the current configuration parameters in the orbital plane are p_2 and θ_2. According to Eq. (5), the relative position in the orbital plane can be described as:

$$\begin{cases} x = -p_2\cos(u-\theta_2) + p_1\cos(u-\theta_1) \\ y = 2p_2\sin(u-\theta_2) - 2p_1\sin(u-\theta_1) \end{cases} \tag{9}$$

which is equal to

$$\begin{cases} x = -p_0\cos(u-\theta_0) \\ y = 2p_0\sin(u-\theta_0) \end{cases} \tag{10}$$

where

$$\begin{cases} p_0 = \sqrt{p_1^2 + p_2^2 - 2p_1p_2\cos(\theta_2-\theta_1)} \\ \theta_0 = arc\tan(p_2\sin\theta_2 - p_1\sin\theta_1, p_2\cos\theta_2 - p_1\cos\theta_1) \end{cases} \tag{11}$$

The problem of controlling the current configuration to achieve the nominal configuration is equivalent to the problem of setting p_0 to zero. According to the Gauss variation equation, the variances in relative orbital elements can be expressed by the along-track Δv_T:

$$\begin{cases} \Delta\Delta a = (2a/v)\Delta v_T \\ \Delta\Delta l = -(3t)\Delta v_T \\ \Delta\Delta e_x = (2/v)\Delta v_T\cos u \\ \Delta\Delta e_y = (2/v)\Delta v_T\sin u \end{cases} \tag{12}$$

where v is the orbital velocity.

The relative orbital element and the configuration parameters have the following relationship:

$$\begin{bmatrix} \Delta e_{x0} \\ \Delta e_{y0} \end{bmatrix} = \frac{p_0}{a}\begin{bmatrix} \cos\theta_0 \\ \sin\theta_0 \end{bmatrix} \tag{13}$$

Setting p_0 to zero is equivalent to setting Δe_{x0} and Δe_{y0} to zero. Therefore,

$$\begin{cases} (2/V)\Delta v_T\cos u = -(p_0/a)\cos\theta_0 \\ (2/V)\Delta v_T\sin u = -(p_0/a)\sin\theta_0 \end{cases} \Rightarrow \begin{cases} \Delta v_T = np_0/2 \\ u = \theta_0 + \pi \end{cases} \tag{14}$$

Eq. (14) shows that the total Δv needed for the in-plane configuration control can be calculated once the initial and nominal configuration parameters are provided.

We can also derive the relative configuration in the cross-track plane according to the nominal configuration parameters s_1 and φ_1, and the current configuration parameters s_2 and φ_2:

$$\begin{cases} s_0 = \sqrt{s_1^2 + s_2^2 - 2s_1s_2\cos(\varphi_2-\varphi_1)} \\ \varphi_0 = \arctan(s_2\sin\varphi_2 - s_1\sin\varphi_1, s_2\cos\varphi_2 - s_1\cos\varphi_1) \end{cases} \tag{15}$$

According to the Gauss variation equation, we can obtain the Δv_N for the cross-track plane configuration control:

$$\Delta v_N = ns_0 \tag{16}$$

Therefore, the station-keeping ΔV can be expressed as:

$$\Delta V = \Delta v_T + \Delta v_N \tag{17}$$

Passive safety requirements

The distance in the cross-track plane can be expressed as:

$$r_{XOZ} = \sqrt{x^2 + z^2} \tag{18}$$

By substituting Eq. (5) into Eq. (18), we obtain r_{XOZ}:

$$r_{XOZ} = \sqrt{p^2 \cos^2(u-\theta) + s^2 \sin^2(u-\varphi)} = \sqrt{\frac{p^2 + s^2 + p^2 \cos 2(u-\theta) - s^2 \cos 2(u-\varphi)}{2}} \tag{19}$$

where

$$[p^2 \cos 2(u-\theta) - s^2 \cos 2(u-\varphi)]^2 + [p^2 \sin 2(u-\theta) - s^2 \sin 2(u-\varphi)]^2 = p^4 + s^4 - 2p^2 s^2 \cos 2(\theta-\varphi) = p^4 + s^4 - 2p^2 s^2 \cos 2\alpha \tag{20}$$

so that

$$\left| p^2 \cos 2(u-\theta) - s^2 \cos 2(u-\varphi) \right| \le \sqrt{p^4 + s^4 - 2p^2 s^2 \cos 2\alpha} \tag{21}$$

Minimum distance r_{XOZ} in the cross-track plane is

$$r_{XOZ}^{\min} = \sqrt{\frac{p^2 + s^2 - \sqrt{p^4 + s^4 - 2p^2 s^2 \cos 2\alpha}}{2}} \tag{22}$$

Eq. (22) shows that $r_{XOZ}^{\min} = 0$ when $\alpha = \pi/2$ or $\alpha = 3\pi/2$, $r_{XOZ}^{\min} = \min(p, s)$ when $\alpha = 0$ or $\alpha = \pi$.

Inter-module communication range requirements

The three-dimensional distance between any two modules can be expressed as:

$$r_{XYZ} = \sqrt{x^2 + y^2 + z^2} \tag{23}$$

By substituting Eq. (5) into Eq. (23), and using the same technique as that of deriving Eq. (22), we can obtain the maximum distance between any two modules:

$$r_{XYZ}^{\max} = \sqrt{\frac{5p^2 + s^2 + \sqrt{9p^4 + s^4 + 6p^2 s^2 \cos 2\alpha}}{2}} \tag{24}$$

OPTIMIZATION MODEL

Based on relative orbital elements, an optimization model for N-module cluster is provided.

Design variables

The goal of the cluster configuration design is to design the cluster configuration description parameters $\{p,s,\alpha,\theta,l\}$ for all the member modules. The along-track offset l can be assumed to equal zero for the nominal configuration. In order to eliminate the secular drift caused by the differences due to J_2 perturbation, we can adopt the J_2 invariant relative orbit. However, the J_2 invariant relative orbit requirements are too strict, which may result in impractically large relative orbits [7]. Therefore, we adopt the partly J_2 invariant relative orbit to reduce the main influences of the J_2 perturbations. We force the inclinations of the module spacecraft to be equal, which greatly reduces the influences the J_2 perturbation. In other words, $\varphi_i = \pi/2 \, or \, \varphi_i = 3\pi/2, i = 1,2\cdots N$.

Therefore, the design variables are $\{p_i, s_i, \theta_i\}$, $i = 1,2\cdots N$,where N represents the number of the module.

Objective function

The combination of the fuel consumptions, the relative distances in the *xoz* plane and the three-dimensional relative distances are chosen as the optimization performance index.

$$J = \sum_{\substack{j=1 \\ j\neq i}}^{N} \sum_{i=1}^{N} \left[C_1 \Delta V^2 + C_2((r_{XOZ}^{\min})_{ij} - d_{XOZ}^{safe})^2 + C_3((r_{XYZ}^{\max})_{ij} - d_{XYZ}^{\max})^2 \right] \quad (25)$$

where C_1 , C_2 and C_3 are the weighting coefficients, ΔV is the fuel consumption for the station-keeping of the modules , $r_{XOZ}^{\min}$ is the minimum relative distance in the *xoz* plane, d_{XOZ}^{safe} is the minimum safe distance predefined to guarantee the passive relative orbit, $r_{XYZ}^{\max}$ is the maximum relative distance between any two modules, $d_{XYZ}^{\max}$ is a predefined maximum distance which guarantee the module satellites can communicate with each other regularly. N is the number of the module satellites.

Constraints

Two types of constraints are considered:

(1) Collision avoidance constraints:

$$(r_{XOZ}^{\min})_{ij} \geq d_{XOZ}^{safe} \qquad (i, j = 1,2,\ldots,N, i \neq j) \quad (26)$$

(2) Maximum relative distance constraints:

$$(r_{XYZ}^{\max})_{ij} \leq d_{XYZ}^{\max} \qquad (i, j = 1,2,\ldots,N, i \neq j) \quad (27)$$

Hybrid PSO-SQP Algorithm

The PSO algorithm is one of the evolutionary computation techniques introduced by Kennedy and Eberhart in 1995. The basic PSO algorithm is [8]:

$$\begin{aligned} v_{id}^{k+1} &= \omega \cdot v_{id}^{k} + c_1 \cdot rand() \cdot (P_{id} - x_{id}^{k}) + c_2 \cdot rand() \cdot (P_{gd} - x_{id}^{k}) \\ x_{id}^{k+1} &= x_{id}^{k} + v_{id}^{k+1} \end{aligned} \quad (28)$$

The SQP is a nonlinear programming method, which is very fit for the nonlinear constraint programming issue [9]:

$$\min_{d\in R^n} \frac{1}{2} d^T H_k d + \nabla f(x_k)^T d$$
$$s.t. \nabla g(x_k)^T d + g(x_k) \le 0 \quad (29)$$

We propose a hybrid optimization approach, which utilize the PSO to find the initial guess value for the SQP, the SQP technique is used to reduce computation time and to improve convergence performance. We use a linear decay rule to adapt the inertia weight, and use SQP technique when the convergent value satisfies the predefined threshold. The operational flow of the PSO-SQP algorithm is shown as Figure 2.

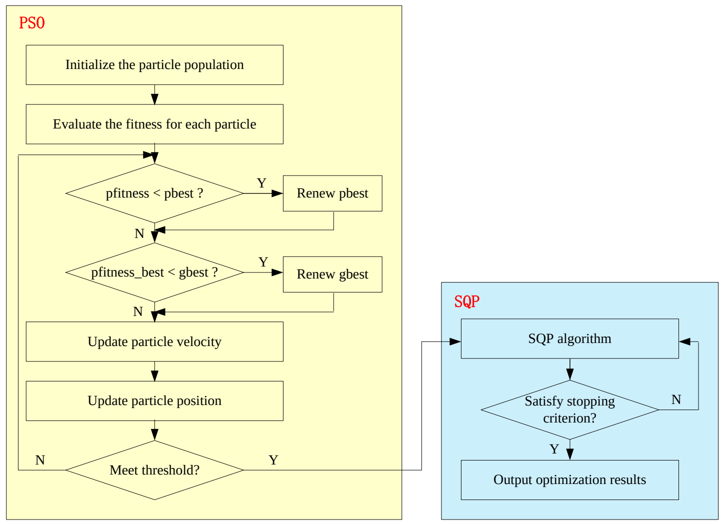

Figure 2. Operational flow of the PSO-SQP algorithm.

NUMERICAL SIMULATIONS AND RESULTS ANALYSIS

This section presents an optimization example of 8 modules to illustrate the performance of the proposed optimization approach, and analyses the stability of the designed cluster configuration.

Optimal Cluster Configurations Design Results

When describing the cluster configurations, the orbits of the modules are usually specified as orbits relative to a reference trajectory which generally passes through the geometry center of the cluster. There may or may not be a module on the reference trajectory, in this paper, we assign one module as the reference module. The orbital elements of the reference orbit are as follows:

a =6892937 m, e =0.00117, i =97.4438°, Ω =100°, ω =90°, and M=0°

The number of the modules in the cluster is 8. The relative configurations between any two modules can be calculated according to Eq. (11) and Eq. (15). C_1, C_2 and C_3 are 1/3, the control period of the station-keeping is 5 days, the minimum safe distance in the *xoz* plane d_{XOZ}^{safe} is 100 m, the maximum restriction distance $d_{XYZ}^{\max}$ is 2000 m. The particle number is 40, the inertial weight vary from 0.9 to 0.4, the learn factors are 2, 2, respectively. The threshold for the switching of PSO to SQP is 0.01.

The optimal configuration design results of the 8 modules are shown in Table 1.

Table 1 Optimal configuration design results of the 8 modules.

	Sat1	Sat2	Sat3	Sat4	Sat5	Sat6	Sat7	Sat8
p /m	0	790.725	993.215	412.476	846.295	820.516	346.112	264.142
s /m	0	540.435	272.836	685.163	162.732	1212.518	1094.515	877.388
θ / °	0	54.072	58.163	61.893	38.447	64.686	36.702	23.394

Figure 3 shows the three-dimensional cluster configuration, Figure 4 shows the projected cluster configuration in the *xoz* plane, Figure 5 shows the relative distances among the 8 modules in the *xoz* plane, while Figure 6 shows the three-dimensional relative distances among all modules.

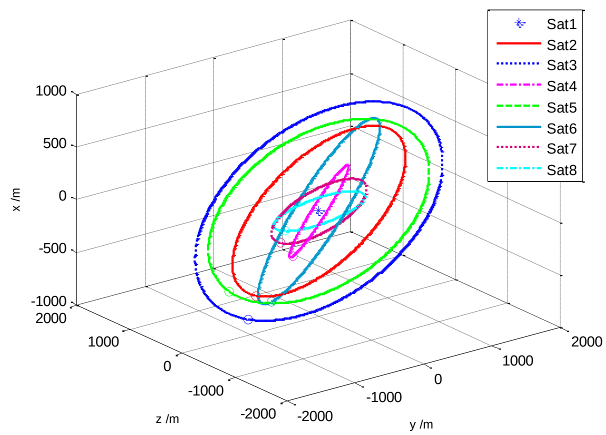

Figure 3. Three-dimensional cluster configuration.

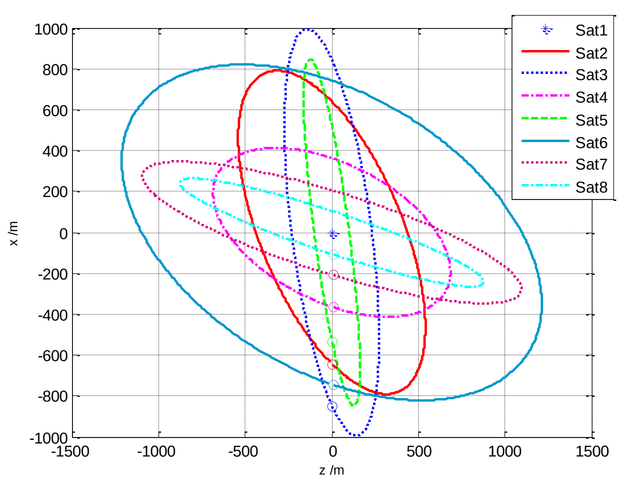

Figure 4. The projected cluster configuration in the *xoz* plane.

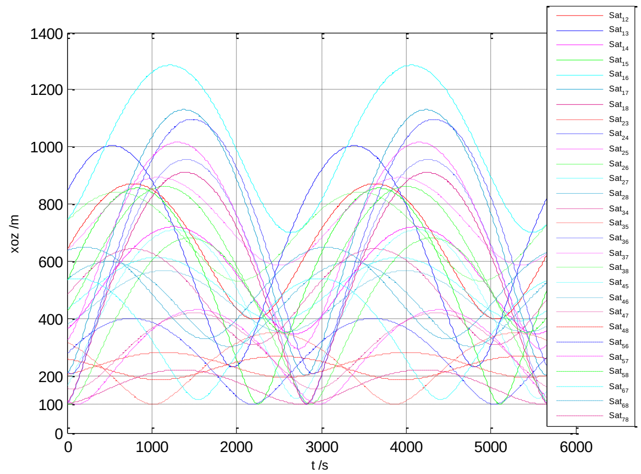

Figure 5. The relative distances in the *xoz* plane.

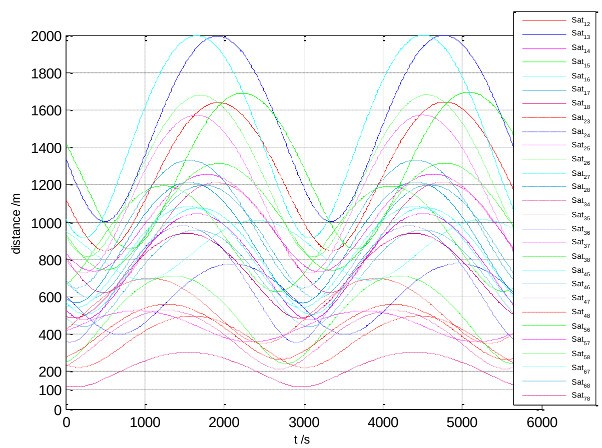

Figure 6. The three-dimensional relative distances.

As shown in Figure 5, the minimum distance among all modules in the *xoz* plane is always larger than 100m. Figure 6 shows that the maximum distances among all modules satisfy the maximum communication range constraint.

Cluster Configuration Stability Analysis

After obtaining the optimal cluster configuration, we analyze the stability of the cluster configuration during the station-keeping period. From Figure4, we can see that Sat5 has the smallest geometry in the *xoz* plane; therefore, we will take the configuration of Sat5 for example. The station-keeping period is 5 days.

Figure 7 and Figure 8 show the in-plane and cross-track plane relative motion that considers the J_2 perturbation in a five-day evolution, respectively. Figure 9 and Figure 10 show the variations of the relative distances in the *xoz* plane and 3D, respectively.

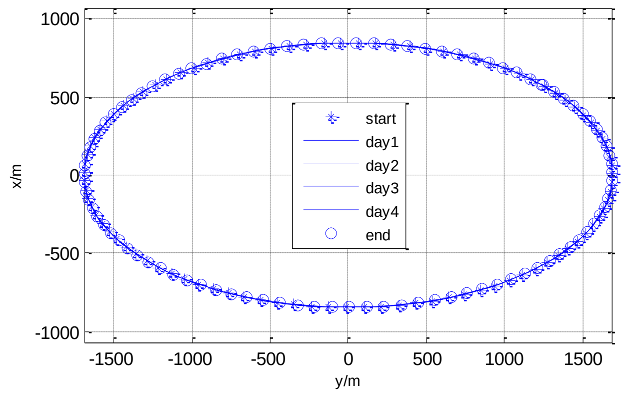

Figure7. In-plane motion caused by the J_2 perturbation.

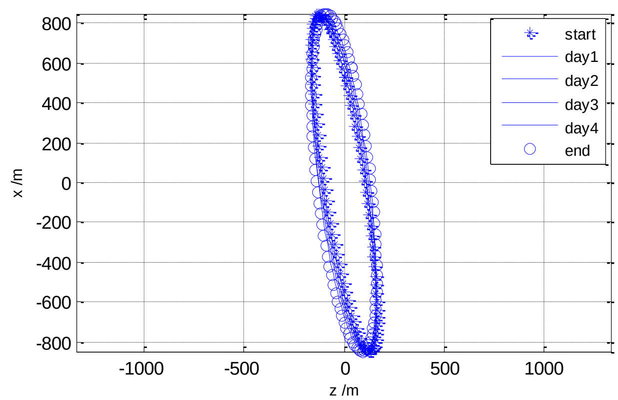

Figure 8. Cross-track motion caused by the J_2 perturbation.

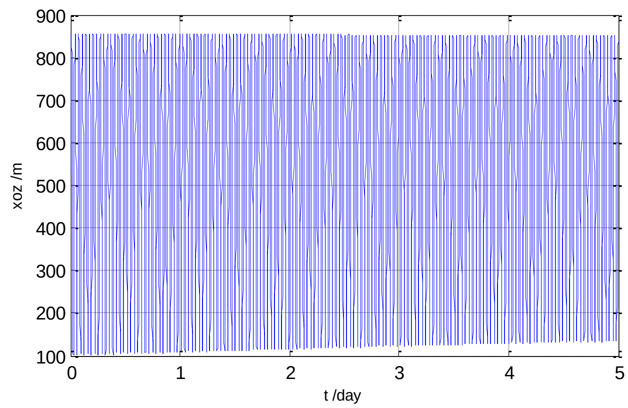

Figure 9. The relative distances in the *xoz* plane.

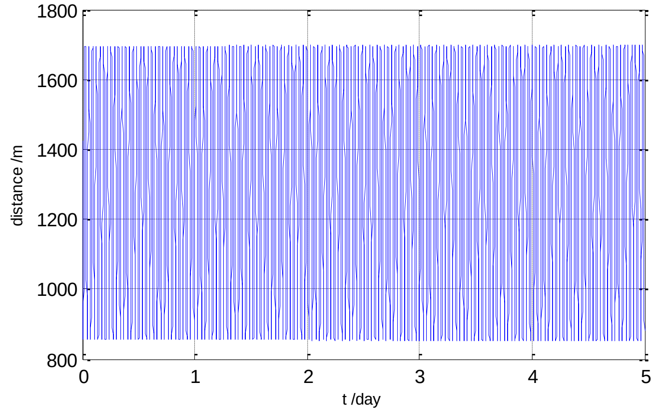

Figure 10. The three-dimensional relative distance.

As seen in Figure 7, the J_2 perturbation does not affect the amplitude of the in-plane motion. However, Eq. (6) shows that the J_2 perturbation influences the in-plane phase angle. For sun-synchronous swarm spacecraft at altitudes of 500 km, the in-plane phase angle θ decreases by 3.5° per day. Figure 8 shows a slight change in the cross-track motion amplitude. However, the J_2 perturbation affects the in-plane phase angle, thereby increasing the minimum distance in the cross-track plane from 100.644 m to 133.881 m. This increased distance ensures the safety of the cluster. From Figure 9 and Figure 10, we can see that the changes of the configuration of Sat5 still satisfy the collision avoidance constraints and the maximum relative distance constraints during a five-day station-keeping time span. The stability of the designed configuration is good.

CONCLUSIONS

The goal of this paper was to solve the optimization of the cluster configuration for swarm spacecraft with practical constraints by hybrid PSO-SQP algorithm. Based on relative orbital elements, we have developed the optimization model for optimizing the cluster configuration with considering the minimum fuel consumption, collision avoidance constraints and maximum relative distance constraints. The main constraints are reformulated in terms of the represented description parameters of the cluster configuration. The hybrid PSO-SQP algorithm is employed to solve the optimization problem. Our model and approach have been verified effectively by designing the cluster configuration of 8 modules. The numerical experiments show that the hybrid PSO-SQP algorithm is very effective in optimizing the cluster configuration, and the performance of the optimal configuration is good.

REFERENCES

[1] M. Hu, G.Q. Zeng. “Autonomous Decentralized Coordination Control for Fractionated Spacecraft Based on Cyclic Pursuit Strategy”. *1st IAA Conference on Dynamics and Control of Space Systems*, 19-21 March, 2012. Porto Portugal.

[2] J.H. Wang, S. Nakasuka. “Cluster Flight Orbit Design Method for Fractionated Spacecraft”. *Aircraft Engineering and Aerospace Technology*, 2012, 84(5):330-343.

[3] G.Q. Zeng, M. Hu, H. Yao. “Relative Orbit Estimation and Formation Keeping Control of Satellite Formations in Low Earth Orbits”. *Acta Astronautica*. 2012, 76: 164-175.

[4] B. Yaglioglu. “A Fractionated Spacecraft Architecture For Earth Observation Missions”. Luleå University of Technology, January 2011.

[5] S. D'Amico, O. Montenbruck. “Proximity Operations of Formation-Flying Spacecraft Using an Eccentricity/Inclination Vector Separation”. *Journal of Guidance, Control, and Dynamics*, 2006, 29(3):554-563.

[6] J.S. Ardaens, S. D'Amico. "Spaceborne Autonomous Relative Control System for Dual Satellite Formations". *Journal of Guidance, Control, and Dynamics*, 2009, 32(6): 1859-1870.
[7] G.Q. Zeng, M. Hu, J.L. Song. "Collision Monitoring and Optimal Collision Avoidance Maneuver for Formation Flying Satellites". *Aircraft Engineering and Aerospace Technology*. 2012, 84(6):413-422.
[8] J. Kennedy, R. Eberhart. "Particle Swarm Optimization". *Proceedings of the IEEE International Conference on Neural Network*, Perth, Australia, 1995: 1942-1948.
[9] R.B. Wilson. "A simplicial Algorithm for Concave Programming". Boston: Graduate School of Business Administration, Harvard University, 1963.