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QUASIOPTIMAL ROTATION DECELERATION OF A DYNAMICALLY ASYMMETRIC BODY IN A RESISTIVE MEDIUM

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The problem of quasioptimal deceleration of rotations of a rigid body in a resistive medium was studied analytically and numerically. The asymptotic approach made it possible to determine the control, time (Bellman's function), evolutions of the magnitude of the elliptic functions modulus, and dimensionless kinetic energy and kinetic moment. Investigation of quasistationary motions of a body was conducted. The qualitative properties of the quasioptimal motion were found.

INTRODUCTION

The progress in studies of dynamics and control of rigid body motion is associated with taking into account the fact that bodies are not absolutely rigid and that the ideal models are just approximations of real ones. The effect of imperfections can be revealed by means of asymptotic methods of nonlinear mechanics (singular perturbations, averaging, and the like). This effect manifests itself in additional perturbation torques in the Euler equations of motion for some fictitious rigid body. Much attention was paid to the analysis of passive motion of rigid bodies carrying a movable mass elastically attached to the body, which moves in a resistant medium, in the presence of viscous friction [1–5]. Rotation control of "quasirigid" bodies by means of concentrated torques applied to the body received much less attention. A class of systems with smooth controls was identified for which singular perturbation methods do not result in accumulation of errors of the "boundary layer" type [6–8].

Below we consider a quasioptimal problem of deceleration of rotations of a dynamically asymmetric rigid body. In addition, the rigid body is subjected to a retarding torque generated by linear medium resistance forces. The rotation is controlled by a torque of bounded magnitude. Components of control torque are presented in the form of products $\varepsilon b_i u_i$ ($i=1,2,3$), where expressions $b_{1,2,3}$ have the dimension of the torque, ε - is small parameter, $u_{1,2,3}$ - are dimensionless control functions to be determined.

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OPTIMAL CONTROL PROBLEM STATEMENT

We consider a dynamically asymmetric rigid body with moments of inertia satisfying, for definiteness, the inequalities $A_1 > A_2 > A_3$. Based on the approach described in [7], the equations of controlled rotations projected on the axes of the body-related coordinate system (the Euler equations) can be expressed as [1, 4, 5, 7]

$$\dot{\mathbf{G}} + \boldsymbol{\omega} \times \mathbf{G} = \mathbf{M}^u + \mathbf{M}^r . \qquad (1)$$

Here, $\mathbf{G}$ is the kinetic moment of the body, $\boldsymbol{\omega} = (\omega_1, \omega_2, \omega_3)^{\mathrm{T}}$ is the vector of absolute angular velocity; $\mathrm{J} = \mathrm{diag}(A_1, A_2, A_3)$ is the tensor of body inertia, $\mathbf{M}^u$ is the vector of control torque; and $\mathbf{M}^r$ is the dissipation torque.

The kinetic moment of the body is determined in the standard way as

$$\mathbf{G} = J\boldsymbol{\omega},\ \mathbf{G} = \left(G_1, G_2, G_3\right)^{\mathrm{T}},\ G_i = A_i\omega_i,\ i = 1,2,3,\ G = \left|\mathbf{G}\right| = \left(G_1^2 + G_2^2 + G_3^2\right)^{1/2} . \qquad (2)$$

It is also believed that the diagonal tensor of the external resistance torque is proportional to the moment of inertia tensor; i.e., the dissipation torque is proportional to the kinetic moment:

$$\mathbf{M}^r = -\lambda' \mathbf{G} . \qquad (3)$$

Here, λ' is a constant coefficient depending on the medium properties. The resistance acting on the body is assumed to be: $\lambda' = \varepsilon\lambda, \varepsilon << 1$, where ε is a small parameter [2, 3].

Components of control torque are presented in the form of products

$$M_i^u = \varepsilon b_i u_i,\ (i = 1,2,3) \qquad (4)$$

where expressions $b_{1,2,3}$ have the dimension of torque, $u_{1,2,3}$ are dimensionless control functions to be determined.

The products εb_i, $i = 1,2,3$ characterize the effectiveness of the control system in the appropriate axis of the related coordinate system.

Equations of control motion Eq. (1) projected on the principal central axes of inertia in the formulation of the problem are

$$\begin{aligned} &\dot{G}_1 + G_3\omega_2 - G_2\omega_3 = \varepsilon b_1 u_1 - \varepsilon\lambda G_1 , \\ &\dot{G}_2 + G_1\omega_3 - G_3\omega_1 = \varepsilon b_2 u_2 - \varepsilon\lambda G_2 , \\ &\dot{G}_3 + G_2\omega_1 - G_1\omega_2 = \varepsilon b_3 u_3 - \varepsilon\lambda G_3 . \end{aligned} \qquad (5)$$

It is required to find an optimal control for the system Eq. (5) $u_i = u_i\left(t, \omega_1, \omega_2, \omega_3\right)$, $i = 1,2,3$, that are satisfied the constraint

$$u_1^2 + u_2^2 + u_3^2 \le 1 \qquad (6)$$

and transfer the system Eq. (5) for a minimum time of an arbitrary initial state $\boldsymbol{\omega}(t_0) = \boldsymbol{\omega}^0$ to the resting state $\boldsymbol{\omega}(T) = 0$.

It is of interest for applications to the motion of a rigid body with a given control law in a quite simple form [7, 9]:

$$M_i^u = \varepsilon b_i u_i, \; u_i = -\frac{G_i}{G}, \; i = 1, 2, 3. \tag{7}$$

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SOLUTION OF THE QUASIOPTIMAL DECELERATION PROBLEM

Multiplying the first equation in Eq. (5) by G_1, the second equation by G_2, and the third equation by G_3, and summing them (scalar product $\dot{\mathbf{G}} \cdot \mathbf{G}$), we obtain a scalar equation to be integrated:

$$\dot{G} = -\varepsilon\lambda G - \frac{\varepsilon}{G^2}\sum_{i=1}^{3} b_i G_i^2 . \tag{8}$$

Multiplying the first equation in Eq. (5) by ω_1, the second equation by ω_2, and the third equation by ω_3, and summing them. The resulting expression for the derivative of the kinetic energy is

$$\dot{H} = -2\varepsilon\lambda H - \frac{\varepsilon}{G}\sum_{i=1}^{3} b_i A_i \, \omega_i^2 . \tag{9}$$

Consider an undisturbed motion ($\varepsilon = 0$).In the absence of perturbations, the rotation of the rigid body is a Euler–Poinsot motion. The variables G and H become constant and φ, ψ, and θ are functions of time t. The slow variables in the perturbed motion are G and H, and the fast variables are the Euler angles φ, ψ, and θ.

Consider a motion under the condition $2HA_1 \geq G^2 > 2HA_2$ corresponding to the trajectories of the kinetic moment vector, which envelope the major torque axis Oz_1. Define

$$k^2 = \frac{(A_2 - A_3)\left[2H(t)A_1 - G^2(t)\right]}{(A_1 - A_2)\left[G^2(t) - 2H(t)A_3\right]} \quad \left(0 \leq k^2 \leq 1\right), \tag{10}$$

which is the modulus of elliptic functions describing this motion and is a function of the kinetic moment G and the kinetic energy H (in the case of unperturbed motion, it is a constant).

To construct the averaged first-approximation system of equations, we substitute the solution of the unperturbed Euler–Poinsot motion into the right-hand side of Eq. (8) and (9) and average over the period. Here, we retain the notation for the slow variables G and H. As a result, we obtain

$$\frac{dH}{d\tau} = -2\lambda H - \frac{G}{S(k)}\left\{ b_1 (A_2 - A_3)\frac{E(k)}{K(k)} + b_2 (A_1 - A_3) W(k) + b_3 (A_1 - A_2)\left(k^2 - W(k)\right)\right\}, \tag{11}$$

$$\frac{dG}{d\tau} = -\lambda G - \frac{1}{S(k)}\left\{ b_1 A_1 \left(A_2 - A_3\right)\frac{E(k)}{K(k)} + b_2 A_2 \left(A_1 - A_3\right) W(k) + b_3 A_3 \left(A_1 - A_2\right)\left(k^2 - W(k)\right) \right\},$$

$$S(k) = A_1\left(A_2 - A_3\right) + A_3\left(A_1 - A_2\right)k^2,\; W(k) = 1 - \frac{E(k)}{K(k)}.$$

Here, $K(k)$ and $E(k)$ are the complete elliptic integrals of the first and second kind, respectively [11]. Equation (11) implies that the resistance of the medium and the torque of the viscous fluid in the body cavity as well the control moment cause the evolution of the kinetic energy H of the body. The expression in the braces on the right-hand side of Eq. (11) is positive (for $A_1 > A_2 > A_3$), because of the inequalities[11]

$$(1-k^2)K \le E \le K. \tag{12}$$

Consequently, $dH/d\tau < 0$ because $H > 0$, i.e., H is a strictly decreasing variable for any $k^2 \in [0,1]$. Similarly, it is shown that the kinetic moment also decreases.

NUMERICAL CALCULATION

For numerical construction of the system Eq. (11) to give a dimensionless form, we use the deceleration time T, as the characteristic parameters of the problem, the projection coefficient of the control torque b_3 and the value of the kinetic moment at the initial time G_0. Dimensionless variables are

$$\tilde{t} = \frac{\tau}{T},\; \tilde{\lambda} = \lambda T,\; \tilde{H} = \frac{H}{b_3},\; \tilde{A}_i = \frac{A_i}{G_0 T},\; \tilde{G} = \frac{G}{G_0}.$$

Let us introduce the characteristic number

$$\sigma = \frac{b_3 T}{G_0}, \tag{13}$$

that defines the basic process of the problem– deceleration process of the rigid body under the action of the control torque for a minimum period of time T.

The equations of motion in dimensionless form are

$$\frac{d\tilde{H}}{d\tilde{t}} = -2\tilde{\lambda}\tilde{H} - \frac{\tilde{G}}{\tilde{S}(k)}\left\{ \chi_1\left(\tilde{A}_2 - \tilde{A}_3\right)\frac{E(k)}{K(k)} + \chi_2\left(\tilde{A}_1 - \tilde{A}_3\right)W(k) + \left(\tilde{A}_1 - \tilde{A}_2\right)\left(k^2 - W(k)\right) \right\},$$

$$\frac{d\tilde{G}}{d\tilde{t}} = -\tilde{\lambda}\tilde{G} - \frac{\sigma}{\tilde{S}(k)}\left\{ \chi_1\tilde{A}_1\left(\tilde{A}_2 - \tilde{A}_3\right)\frac{E(k)}{K(k)} + \chi_2\tilde{A}_2\left(\tilde{A}_1 - \tilde{A}_3\right)W(k) + \tilde{A}_3\left(\tilde{A}_1 - \tilde{A}_2\right)\left(k^2 - W(k)\right) \right\},$$

$$\frac{dk^2}{d\tilde{t}} = \frac{2\sigma}{\tilde{G}}\left\{ \chi_1 k^2 \frac{E(k)}{K(k)} + \chi_2(k^2 - 1)W(k) + \left(W(k) - k^2\right) \right\},$$

$$\tilde{S}(k) = \tilde{A}_1\left(\tilde{A}_2 - \tilde{A}_3\right) + \tilde{A}_3\left(\tilde{A}_1 - \tilde{A}_2\right)k^2,\; W(k) = 1 - \frac{E(k)}{K(k)}. \tag{14}$$

The integration was carried out for the initial conditions $k^2(0)=0.9999$, $\tilde{G}(0)=1$, kinetic energy at the initial time was determined from the relation

$$\tilde{H}(0)=\frac{\tilde{G}^2(0)\left(\tilde{A}_2-\tilde{A}_3+(\tilde{A}_1-\tilde{A}_2)k^2(0)\right)}{2\sigma\tilde{S}\left(k^2(0)\right)}. \qquad (15)$$

The third equation of the system Eq. (14) describes the law of changes in the magnitude of k^2, consequently, in the initial condition $k^2\approx 1$ right-hand side of the equation must be negative. Second and third terms in the curly brackets are negative, therefore, must be satisfied conditions for the dimensionless coefficients of the control torque:

$$\chi_1<\frac{\chi_2\left(1-k^2\right)W(k)+k^2-W(k)}{k^2F(k)}. \qquad (16)$$

The calculations were performed for various values of χ_1, χ_2 and σ. For different values of the characteristic number σ there are values of the dimensionless coefficients of the control torque χ_1 and χ_2, at which occurs quasioptimal deceleration of the rigid body. The nature of the deceleration has a different view.

Let us carry out the research in $\sigma=1.3$. Figure 1 illustrates the numerical analysis for kinetic moment of the body. The moments of inertia have the values: $\tilde{A}_1=8$, $\tilde{A}_2=6$, $\tilde{A}_3=4$, for the value of the dimensionless moment coefficient of resistance of the medium $\tilde{\lambda}=0.1$. Curve *1* corresponds to the values $\chi_1=0.5$ and $\chi_2=0.8$, curve *2* – $\chi_1=0.6$ and $\chi_2=1$, curve *3* – $\chi_1=0.7$ and $\chi_2=1.4$.

Figure 1 shows the higher values of χ_1 and χ_2, the greater curvature of the curve; in the first case, the function $\tilde{G}=\tilde{G}(\tilde{t})$ is almost linear. This character has a function of angular momentum for other values of the characteristic number σ.

Figure 2 shows the result of numerical integration for a rigid body with the same geometry mass, in the same resistive medium for the characteristic number $\sigma=1.4$. Curve *1* corresponds to the dimensionless coefficients of the control torque $\chi_1=0.6$ and $\chi_2=0.8$, Curve *2* – $\chi_1=0.7$ and $\chi_2=1.1$, curve *3* – $\chi_1=0.8$ and $\chi_2=1.8$.

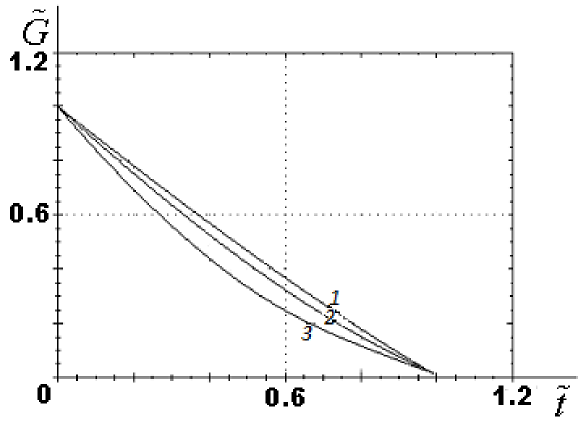

Figure 1.

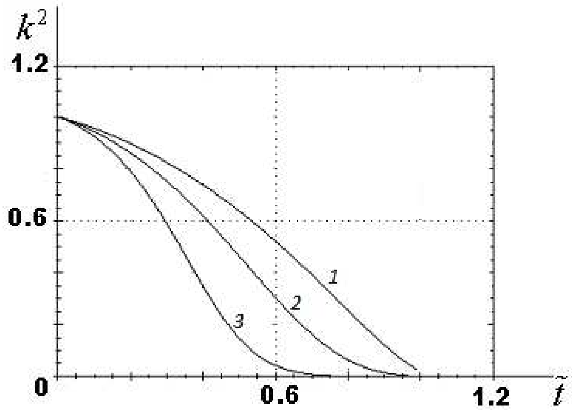

Figure 2.

Behavior of the function of the kinetic energy in the case of the quasioptimal deceleration of the rigid body is shown in Figure 3. Numerical investigation was carried out for a rigid body with the same geometry of mass in the same resisting medium. The figure illustrates that in all cases the body comes in the state of rest in a quasioptimal time of deceleration. Curve *1* is for dimensionless coefficients of the control moment $\chi_1 = 0.2$ and $\chi_2 = 1.6$, and for the characteristic number $\sigma = 0.7$. The second curve is for the characteristic number $\sigma = 0.8$ in coefficients of the control moment $\chi_1 = 1.0$ and $\chi_2 = 1.2$. We make a numerical integration for the value $\sigma = 1.1$ and get quasioptimal deceleration for $\chi_1 = 0.5$ and $\chi_2 = 1.1$, that corresponds to the curve *3*. Curve *4* shows the function $\tilde{H} = \tilde{H}(\tilde{t})$, that corresponds to the curve *2* Figure 3. Note, that for each case, the function is monotonically decreasing to zero over quasioptimal time T.

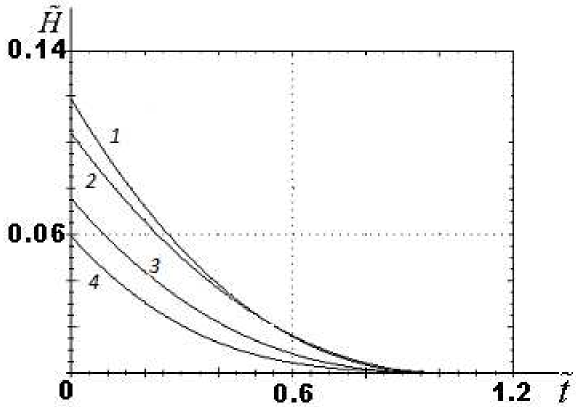

Figure 3.

CONCLUSION

The problem of quasioptimal deceleration of rotations of a dynamically asymmetric rigid body in a resistive medium was studied analytically and numerically. The asymptotic approach made is possible to determine the control, time (Bellman's function), evolutions of the magnitude of the elliptic functions modulus k^2, and dimensionless kinetic energy and kinetic moment.

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