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LYAPUNOV BASED ATTITUDE STABILIZATION OF AN UNDERACTUATED SPACECRAFT WITH FLEXIBILITIES.

G. Mattei,[*] P. Di Giamberardino,[†] S. Monaco,[‡] D. Normand-Cyrot [§]

In this work the attitude control problem for an underactuated flexible spacecraft is investigated. Underactuation and the presence of flexibilities are two challenging problems for space applications even considered separately. Underactuation can be overcome using control torques supplied by gas-jet actuators about two of the principal axes of inertia only if the uncontrolled principal axis of the satellite is not a symmetry axis. This ensures, as well known, that the rigid dynamics is small-time locally controllable. However, the spacecraft cannot be globally asymptotically stabilized using a time-invariant continuous feedback. Using Lyapunov direct criterion, we propose a time-varying feedback stabilizing the rigid-body dynamics and adding, if necessary, the appropriate damping to the flexible motions. Moreover, the control law obtained is robust against uncertainties in the coupling matrix. The stabilization property achieved being only local, we propose a digital multirate steering law to bring the system in the neighbourhood of the origin and then apply the Lyapunov-based stabilizer. We compare our controller with a simpler one that does not take into account the flexible dynamics. The results show the effectiveness of the proposed stabilizing control strategy.

INTRODUCTION

In this work the attitude control problem for an underactuated spacecraft with flexibilities is investigated.

Underactuation and the presence of flexibilities are two challenging problems for space applications even considered separately.

The underactuation condition in attitude control problems, and more in general in all the reconfiguration and reorientation problems in space multibody systems, usually corresponds to deal with faults on the actuators or in some design constraints. It is then clear that the possibility to maintain the full controllability of the satellite in such conditions is a crucial aspect for the success of the mission.

This fact motivates the presence of a large number of results in literature which propose different techniques for facing such a problem. In fact, it is well known[4] that underactuation corresponds to the presence of a non-holonomic constraint on the system dynamics which makes traditional control design techniques fail.

[*]Ph.D. Candidate, Department of Computer, Control and Management Engineering “Antonio Ruberti”, Sapienza - University of Rome, mattei@dis.uniroma1.it

[†]Researcher, Department of Computer, Control and Management Engineering “Antonio Ruberti”, Sapienza - University of Rome, digiamberardino@diag.uniroma1.it

[‡]Professor, Department of Computer, Control and Management Engineering “Antonio Ruberti”, Sapienza - University of Rome, salvatore.monaco@uniroma1.it

[§]Research Director, Laboratoire des Signaux et Systmes - CNRS, Paris, cyrot@lss.supelec.fr

The problem of controlling nonholonomic dynamics has been widely investigated during the last two decades for its importance not only in space applications but, mainly, in robotic planning and motion problems.

To get feasible and effective solutions, one must deal with time-varying, discontinuous or discrete-time control laws[4,2,16,5,11,13] making use of more sophisticate control design approaches. For example, in the context of space applications, the problem can be overcome using control torques supplied by gas-jet actuators about only two of the principal axes of inertia, provided that the uncontrolled principal axis of the satellite is not a symmetry axis. This ensures, as well known, that the rigid dynamics is small-time locally controllable even if the spacecraft cannot be globally asymptotically stabilized using a time-invariant continuous feedback, since Brockett's condition is not fulfilled.

On the other hand, flexibilities, mainly due to satellite appendages like antennas or solar arrays, represent a perturbation source that, when not explicitly considered in the control design may deteriorate the system performances.

Early contributions to improve performances in presence of such kind of perturbations date back to some years ago, like in[7] as an example; in subsequent years the problem has been faced by several researchers and, as the theoretic studies provided new methodologies, new different approaches have been proposed like in[15] or, more recently, in[3] .

In this paper a solution to the control design problem under the combined presence of underactuation and flexibilities is proposed. The technique adopted makes use of Lyapunov direct criterion, proving that it is possible to compute a time-varying stabilizing feedback law with the property of maintaining, during manoeuvres, the passivity of the flexible dynamics by damping injection. This allows us to design, with only one shot, a controller able to get stabilization of the angular velocity dynamics in an underactuated configuration, at the same time attenuating the disturbances coming from flexible appendages and exploiting their residual effects on the rigid motion. Furthermore, the Lyapunov design framework makes it possible to modify the control law in order to improve robustness and introduce further dynamics for adaptation or observation objectives. In particular, we have introduced uncertainty into the coupling matrix between flexible and rigid dynamics in order to show that a slight robust redesign of the control law already obtained is able to counteract its effects.

Since the stabilization property achieved is only local, we propose a digital multirate steering law to bring the system in the neighbourhood of the origin and then apply the Lyapunov-based stabilizer. As a matter of fact, the multirate steering law can bypass, at least virtually, the underactuation condition by means of dividing the sampling interval into two sub-intervals, so "doubling" the effect on the dynamics of one of the two control inputs. Moreover, it provides feasible trajectories to be tracked since it is based on the nonlinear model of the spacecraft.

The so obtained control law is finally compared, through numerical simulations, with a simplified version which does not take into account flexibility. The results show the effectiveness of the proposed stabilizing strategy and the improvements in the overall performance.

DYNAMICAL MODEL

The dynamical model of the rigid satellite with flexible appendages is built using the extended Euler laws, as done in[12,3] . Using the conservation of angular momentum, it is possible to write the coupled rigid and flexible equations:

$$J\dot{\omega} + N^T\ddot{\eta} = S(\omega)\left[J\omega + N^T\psi\right] + u$$

$$\ddot{\eta} + C\dot{\eta} + K\eta = N\dot{\omega}$$

where ω is the angular velocity vector of the spacecraft, $u = (u_1\; u_2\; 0)^T$ is the control inputs vector and η, $\psi \in \mathbb{R}^M$ are the elastic coordinates such that $\psi = \dot{\eta}$. The matrix $N \in \mathbb{R}^{M\times 3}$ is the coupling matrix between the attitude dynamics and the M elastic modes considered, $C \in \mathbb{R}^{M\times M}$ is the damping matrix, $K \in \mathbb{R}^{M\times M}$ is the stiffness matrix. The inertia matrix is taken along the principal axes, hence it is the diagonal matrix:

$$J = \begin{pmatrix} J_x & 0 & 0 \\ 0 & J_y & 0 \\ 0 & 0 & J_z \end{pmatrix}$$

The angular velocity equation can be written as

$$\dot{\omega} = J_{eq}^{-1}\left[G + N^T(C\psi + K\eta - CN\omega) + u\right]$$

where $J_{eq} = J + N^T N$ is the equivalent inertia matrix and $G = S(\omega)\left[J_{eq}\omega + N^T\psi\right]$ is the gyroscopic term. Hence the overall dynamical model can be written as:

$$\begin{aligned} \dot{\omega} &= J_{eq}^{-1}\left[S(\omega)\left(J_{eq}\omega + N^T\psi\right) + N^T\left(C\psi + K\eta - CN\omega\right) + u\right] \\ \dot{\eta} &= \psi - N\omega \\ \dot{\psi} &= -\left(C\psi + K\eta\right) + CN\omega \end{aligned} \tag{1}$$

The coupling matrix is assumed to be unknown, while we assume to have good estimates of the values of C and K. To take into account the uncertainty in N, we separate it into its nominal part and its (deterministic) uncertain part in this way:

$$N = N_n + \Delta N$$

The uncertain matrix ΔN is given by the element-wise Hadamard product, identified by the symbol $\circ$, of a matrix νN, whose elements are measures of the percentage of uncertainty, with a matrix W, whose elements are sinusoidal functions of time at the different frequencies w_{ij}, representing the speed of variation of the actual values of the elements of N, n_{ij}, around their nominal values.

$$\Delta N = \begin{pmatrix} \nu n_{11} & \nu n_{12} & \nu n_{13} \\ \vdots & \vdots & \vdots \\ \nu n_{M1} & \nu n_{M2} & \nu n_{M3} \end{pmatrix} \circ \begin{pmatrix} \sin w_{11}t & \sin w_{12}t & \sin w_{13}t \\ \vdots & \vdots & \vdots \\ \sin w_{M1}t & \sin w_{M2}t & \sin w_{M3}t \end{pmatrix} = \nu N \circ W$$

The elements of the matrix W are called deviation functions[8,9] . We assume to know upper and lower bounds for ΔN, namely

$$
\begin{aligned}
\|\Delta N\|_2 &= \sqrt{\lambda_{\max}(\Delta N^T \Delta N)} \leq \rho_N \\
\|\Delta N\|_\circ &\leq \begin{pmatrix} \rho n_{11} & \rho n_{12} & \rho n_{13} \\ \vdots & \vdots & \vdots \\ \rho n_{M1} & \rho n_{M2} & \rho n_{M3} \end{pmatrix} = R_{\circ N}
\end{aligned} \tag{2}
$$

where $\rho n_{ij} \geq \nu n_{ij}$. For consistency of notation in the following calculations, we use two different kind of norms, the first being the classical induced $\mathcal{L}_2$ norm and the second being an element-wise norm suitable when dealing with Hadamard products.

MULTIRATE STEERING LAW

To begin with, a large angle maneuver is performed in order to bring the attitude of the spacecraft near the origin. This is done by using a multirate logic based on the sampled-data equivalent model of the satellite. This model can be computed in a finite form if we consider the system without flexibilities

$$
\dot{\omega} = J^{-1}\left[S(\omega)J\omega + u\right] = \begin{pmatrix} f_1(\omega) + \dfrac{1}{J_x}u_1 \\ f_2(\omega) + \dfrac{1}{J_y}u_2 \\ \dfrac{Jx - Jy}{Jz}\omega_1\omega_2 \end{pmatrix}
$$

and perform a partially linearizing feedback, renaming suitably the term $\dfrac{Jx - Jy}{Jz}$

$$
\begin{aligned}
u &= \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} -f_1(\omega) + J_x v_1 \\ -f_2(\omega) + J_y v_2 \end{pmatrix} \\
a &= \frac{Jx - Jy}{Jz}
\end{aligned} \tag{3}
$$

We obtain the simplified dynamic model

$$
\begin{aligned}
\dot{\omega}_1 &= v_1 \\
\dot{\omega}_2 &= v_2 \\
\dot{\omega}_3 &= a\,\omega_1\omega_2
\end{aligned} \tag{4}
$$

This model admits a finite sampled representation, which according to the results in[14] can be computed using the series expansion

$$
x(k+1) = x(k) + \sum_{i=1}^{\infty} \frac{\delta^i}{i!} x^{(i)}(k) \tag{5}
$$

where k is a non negative integer, δ is the sampling period and $x^{(i)}(k)$ represents the i-th time derivative of $x(t)$ computed at time $t = k\delta$.

Hence, the sampled equivalent model for the partially linearized rigid spacecraft can be obtained:

$$\begin{aligned}
\omega_1(k+1) &= \omega_1(k) + \delta v_1(k) \\
\omega_2(k+1) &= \omega_2(k) + \delta v_2(k) \\
\omega_3(k+1) &= a\left(\omega_3(k) + \delta\omega_1(k)\omega_2(k) + \frac{\delta^2}{2}\left(v_1(k)\omega_2(k) + \omega_1(k)v_2(k)\right) + \frac{\delta^3}{3}v_1(k)v_2(k)\right)
\end{aligned} \tag{6}$$

To apply the multirate steering strategy, we need to write the expression of the sampled equivalent at $t = (k+2)\delta$. For the sake of simplicity, let us drop the dependence on k and simply underline the fact the on the right-hand side we are dealing with delta dependent terms which are two sampling instants ahead of those on the left-hand side. We obtain:

$$\begin{aligned}
\omega_1(2\delta) &= \omega_1 + \delta v_1^1 + \delta v_1^2 \\
\omega_2(2\delta) &= \omega_2 + 2\delta v_2 \\
\omega_3(2\delta) &= a\left(\omega_3 + \delta\omega_1\omega_2 + \frac{\delta^2}{2}\left(v_1^1\omega_2 + \omega_1 v_2\right)\right. \\
&\quad \left. + \,\delta\left[\omega_1 + \delta v_1^1\right)(\omega_2 + \delta v_2) + \frac{\delta^2}{2}\left(v_1^2(\omega_2 + \delta v_2) + \left(\omega_1 + \delta v_1^1\right)v_2\right] + \frac{\delta^3}{3}v_1^2 v_2\right)
\end{aligned} \tag{7}$$

Take the sampling time $T = 2\delta$. In the first δ-half of the sampling period we apply the control v_1^1, while in the second half we apply v_1^2. With this in mind, setting the desired final values of angular velocities and defining the steering error as $\Delta\omega = \omega_f - \omega_i$, we obtain the expression of v_2

$$v_2 = \frac{\Delta\omega_2}{T}$$

which is constant along the whole sampling interval. Setting $\omega_f = 0$, it is possible to invert the relationship (7) in order to determine the steering law for the remaining control input.

$$\begin{pmatrix} v_1^1 \\ v_1^2 \end{pmatrix} = -B^{-1}A \tag{8}$$

where:

$$B = \begin{pmatrix} \dfrac{T}{2} & \dfrac{T}{2} \\ a\dfrac{7}{48}T^2\omega_2 & a\dfrac{1}{48}T^2\omega_2 \end{pmatrix} \quad A = \begin{pmatrix} \omega_1 \\ a\left(\omega_3 + T\omega_1\omega_2 - \dfrac{T}{2}\omega_1\omega_2\right) \end{pmatrix} \tag{9}$$

Remark 1. Note that the matrix B is invertible provided that $\omega_2 \neq 0$, since $a \neq 0$ by assumption. However, it is possible to develop a dual version of this steering law in which is v_2 that changes its value at half the sampling interval. In this way, we obtain a steering law singular in $\omega_1 = 0$, thus accommodating initial conditions with $\omega_2 = 0$. The case in which the $(\omega_1, \omega_2) = (0, 0)$ clearly cannot be faced using this planning strategy.

The final values toward which the multirate steering law brings the system can be adjusted in order to avoid the singularity in the inversion of B and to accommodate the intervention of the local system stabilizer designed in the next section. As a matter of fact, the multirate steering can be used to perform large angle maneuvers since it bypass in a certain sense the issue of underactuation by means of doubling the action of one of the two control inputs during a sampling interval. After the spacecraft has been taken near the desired value of angular velocity, the Lyapunov-based local stabilizer intervenes to bring the error to zero and to counteract the effects of the flexible motions on the maneuver.

LYAPUNOV STABILIZER

The Lyapunov direct criterion is useful to find a control law which achieves the local asymptotic stability of the angular velocity dynamics. The overall system with flexibilities (1) is first of all feedback linearized, assuming we have good knowledge of the values of inertia moments, damping and stiffness matrix and of the nominal value of the coupling matrix. Keeping in mind the fact that the value of N is uncertain, the following input transformation

$$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} -f_1(\omega,\eta,\psi) \\ -f_2(\omega,\eta,\psi) \end{pmatrix} + \begin{pmatrix} j_{eq}^{11} & j_{eq}^{12} \\ j_{eq}^{21} & j_{eq}^{22} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \tag{10}$$

with f_1 and f_2 being respectively first and second component of the term $S(\omega)\left(J_{eq}\omega + N^T\psi\right) + N^T\left(C\psi + K\eta - CN\omega\right)$ and the terms j_{eq}^{ij} being elements of the equivalent inertia matrix, yields

$$\begin{aligned} \dot{\begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix}} &= \begin{pmatrix} v_1 \\ v_2 \\ a\,\omega_1\omega_2 + \tilde{f}_3(\omega,\eta,\psi) \end{pmatrix} + \begin{pmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{pmatrix} + \begin{pmatrix} \Delta_{v_1} \\ \Delta_{v_2} \\ 0 \end{pmatrix} \\ \dot{\eta} &= \psi - N\omega \\ \dot{\psi} &= -\left(C\psi + K\eta\right) + CN\omega \end{aligned} \tag{11}$$

where with Δ we identify the uncertain terms in the dynamics

$$\begin{aligned} \Delta = \begin{pmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{pmatrix} &= S(\omega)\left(\Delta J_{eq}\omega + N^T\psi\right) + N^T\left(C\psi + K\eta - CN\omega\right) \\ &\quad + S(\omega)\Delta N^T\psi + \Delta N^T\left(C\psi + K\eta - C\Delta N\omega\right) \\ \Delta_v = \begin{pmatrix} \Delta_{v_1} \\ \Delta_{v_2} \\ 0 \end{pmatrix} &= \begin{pmatrix} \Delta\tilde{J}_{eq}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \\ 0 \end{pmatrix} \end{aligned} \tag{12}$$

More in detail, we have

$$\begin{aligned} \Delta J_{eq} &= \Delta\left(J - N^T N\right) = -\Delta N^T \Delta N \\ \Delta\tilde{J}_{eq} &= \Delta\begin{pmatrix} j_{eq}^{11} & j_{eq}^{12} \\ j_{eq}^{21} & j_{eq}^{22} \end{pmatrix} = -\Delta\tilde{N}^T\Delta\tilde{N} \end{aligned} \tag{13}$$

with

$$\Delta\tilde{N} = \begin{pmatrix} \nu n_{11} & \nu n_{12} \\ \vdots & \vdots \\ \nu n_{M1} & \nu n_{M2} \end{pmatrix} \tag{14}$$

The function $\tilde{f}_3(\omega, \eta, \psi)$ is the residual effect of the flexible dynamics on the ω_3 equation, which could not be linearized due to underactuation.

We would like to impose an asymptotically stable rigid dynamics for ω_3: this can be obtained if ω_1 and ω_2 reach the desired values

$$\begin{aligned} \omega_{1d} &= -\omega_3 \\ \omega_{2d} &= \omega_3^2 \end{aligned} \tag{15}$$

It is now possible to define the errors:

$$\begin{aligned} e_1 &= \omega_{1d} - \omega_1 = -\omega_3 - \omega_1 \\ e_2 &= \omega_{2d} - \omega_2 = \omega_3^2 - \omega_2 \end{aligned} \tag{16}$$

This approach is slightly inspired by that used by Byrnes and Isidori in.[6] However, while they perform a change of coordinates and a cancellation, in our work we apply Lyapunov direct criterion since we need to make the control law robust against the effect of flexible motions.

Remark 2. The desired values ω_{1d} and ω_{2d} should satisfy two regularity assumptions in order for the stabilizer to work properly. They should be zero in zero and have a continuous time-derivative in the origin. For more details, the reader is referred to.[6]

Using a mixed notation, the system can be written in terms of the error $e = (e_1\ e_2)$ in this way

$$\begin{aligned} &\dot{\begin{pmatrix} e_1 \\ e_2 \\ \omega_3 \end{pmatrix}} = \begin{pmatrix} r_1(e, \omega_3) + v_1 \\ r_2(e, \omega_3) + v_2 \\ -\omega_3^3 + r_3(\omega_3, e) + \tilde{f}_3(\omega, \eta, \psi) \end{pmatrix} + \begin{pmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{pmatrix} + \begin{pmatrix} \Delta_{v_1} \\ \Delta_{v_2} \\ 0 \end{pmatrix} \\ &\dot{\eta} = \psi - N\omega \\ &\dot{\psi} = -\left(C\psi + K\eta\right) + CN\omega \end{aligned} \tag{17}$$

where we have, more in detail

$$\begin{aligned} r_1(e, \omega_3) &= -(e_1 + \omega_3)(\omega_3^2 - e_2) \\ r_2(e, \omega_3) &= -2(e_1 + \omega_3)(\omega_3^2 - e_2) \\ r_3(e, \omega_3) &= e_1 e_2 + e_2 \omega_3 - \omega_3^2 e_1 \end{aligned} \tag{18}$$

Before starting the design of the stabilizer, we make the following assumption, which substantially states that the drift of the flexible dynamics is globally exponentially stable and we know a Lyapunov function for it.

Assumption 1. *There exists a unique, symmetric and positive definite, solution P of the equation*

$$P\begin{pmatrix} 0 & I \\ -K & -C \end{pmatrix} + \begin{pmatrix} 0 & I \\ -K & -C \end{pmatrix}^T P = -2Q \tag{19}$$

for each fixed matrix Q, symmetric and positive definite.

We also assume to know a suitable bound $\rho_{\tilde{N}}$ for $\Delta\tilde{N}$ in order to define proper bounds for the uncertainties in the dynamics, namely

$$\begin{aligned} \|\Delta J_{eq}\| &\le -\rho_N^2 = \rho_{eq} \\ \left\|\Delta \tilde{J}_{eq}\right\| &\le -\rho_{\tilde{N}}^2 = \tilde{\rho}_{eq} \\ \|\Delta\| &\le \left\|S(\omega)\left(\rho_{eq}\omega + N^T\psi\right) + N^T\left(C\psi + K\eta - CN\omega\right)\right\| \\ &\quad + \|S(\omega)\rho_N\psi + \rho_N\left(C\psi + K\eta - CR_{\circ N}\omega\right)\| \le \rho_\Delta \\ \|\Delta_v\| &\le \left\|\begin{pmatrix} \tilde{\rho}_{eq}\begin{pmatrix} v_{1_M} \\ v_{2_M}\end{pmatrix} \\ 0 \end{pmatrix}\right\| \le \rho_{\Delta_v} \end{aligned} \tag{20}$$

where v_{1_M} and v_{2_M} are estimates of the maximum values assumed by the nominal control inputs developed in the following.

Furthermore, we need to define the robust control function[8] that will be used to compensate the uncertainties coming from the flexible motion.

Definition 1 (Robust control function)**.** We call *robust control function* the following sigmoid function:

$$v = \rho_u \mathrm{sign}(s)(1 - e^{-\sigma_s|s|}) = \rho_u \mathrm{sigm}(s, \sigma_s) \tag{21}$$

Its structure emulates a discontinuous function, but it can also be used in a two-step recursive procedure as virtual control input, since it has a continuous first derivative:

$$\frac{\partial v}{\partial s} = \rho_u \sigma_s e^{-\sigma_s|s|} \tag{22}$$

We now define the candidate Lyapunov function V as

$$V = \frac{1}{2}\left(e_1^2 + e_2^2 + \omega_3^2\right) + \frac{1}{2}\left(\eta \;\; \psi\right)^T P \left(\eta \;\; \psi\right) \tag{23}$$

and evaluate its derivative along the trajectories of (17)

$$\begin{aligned} \dot{V} &= e_1\left(r_1(e,\omega_3) + v_1 + \Delta_1 + \Delta_{v_1}\right) + e_2\left(r_2(e,\omega_3) + v_2 + \Delta_2 + \Delta_{v_2}\right) + \omega_3\left(-\omega_3^3 + r_3(e,\omega_3)\right) \\ &\quad + \left(\eta \;\; \psi\right)^T P\left[\begin{pmatrix} 0 & I \\ -K & -C\end{pmatrix}\begin{pmatrix}\eta \\ \psi\end{pmatrix} - \begin{pmatrix} I \\ -C\end{pmatrix} N\omega\right] \end{aligned} \tag{24}$$

As first step, we set

$$\begin{aligned} v_1 &= -r_1(e,\omega_3) - (\rho_\Delta + \rho_{\Delta_v})\mathrm{sigm}(e_1,\sigma_1) - k_1 e_1 + v_{f_1} \\ v_2 &= -r_2(e,\omega_3) - (\rho_\Delta + \rho_{\Delta_v})\mathrm{sigm}(e_2,\sigma_2) - k_2 e_2 + v_{f_2} \end{aligned} \tag{25}$$

obtaining

$$\begin{aligned}\dot{V} &\leq -e_1^2 - e_2^2 - \omega_3^4 + \omega_3 r_3(e,\omega_3) + \mathrm{res}_1(e,\sigma_1,\sigma_2) \\ &+ (\eta \ \ \psi)^T P \left[\begin{pmatrix} 0 & I \\ -K & -C \end{pmatrix}\begin{pmatrix}\eta \\ \psi\end{pmatrix} - \begin{pmatrix} I \\ -C\end{pmatrix} N\omega\right] + e^T \begin{pmatrix} v_{f_1} \\ v_{f_2}\end{pmatrix}\end{aligned} \tag{26}$$

Setting now:

$$\begin{pmatrix} v_{f_1} \\ v_{f_2}\end{pmatrix} = \frac{1}{2}\tilde{e}_{inv}\,(\eta \ \ \psi)^T P \begin{pmatrix} I \\ -C\end{pmatrix} N\omega \tag{27}$$

where

$$\tilde{e}_{inv} = \begin{pmatrix} \dfrac{e_1}{e_1^2 + e^{-\gamma t}} \\ \dfrac{e_2}{e_2^2 + e^{-\gamma t}} \end{pmatrix} \tag{28}$$

we finally obtain

$$\begin{aligned}\dot{V} &\leq -e_1^2 - e_2^2 - \omega_3^4 + \omega_3 r_3(e,\omega_3) + \mathrm{res}_1(e,\sigma_1,\sigma_2) \\ &- 2\,(\eta \ \ \psi)^T Q \begin{pmatrix}\eta \\ \psi\end{pmatrix} + \mathrm{res}_2(e,\omega,\eta,\psi,\gamma)\end{aligned} \tag{29}$$

The expressions of the residual terms are the following:

$$\begin{aligned}\mathrm{res}_1(e,\sigma_1,\sigma_2) &= (\rho_\Delta + \rho_{\Delta_v})\left(e^{-\sigma_1|e_1|}|e_1| + e^{-\sigma_2|e_2|}|e_2|\right) \\ \mathrm{res}_2(e,\omega,\eta,\psi,\gamma) &= \left(\frac{1}{2}\frac{e^{-\gamma t}}{e_1^2 + e^{-\gamma t}} + \frac{1}{2}\frac{e^{-\gamma t}}{e_2^2 + e^{-\gamma t}}\right)(\eta \ \ \psi)^T P \begin{pmatrix} I \\ -C\end{pmatrix} N\omega\end{aligned} \tag{30}$$

It is possible to show[10,17] that setting properly the design parameters σ_1, σ_2 and γ we achieve *uniform ultimate boundedness* of the closed-loop trajectories. From the practical point of view, this means that the angular velocity and the flexible variables can be brought into a neighbourhood of the origin that can be made arbitrarily small by increasing the values of the design parameters. This is of course of primary interest in real applications, in which of course the requisite of attaining an exact zero error at steady-state is unlikely.

Remark 3. Note that, because of the term $\omega_3 r_3(e,\omega_3)$ in the derivative, the stabilization obtained is only local. Thus, the planning made using the multirate strategy is of primary importance in steering near the origin the state of the spacecraft making a large-angle maneuver.

Remark 4. Note that the controller developed needs the knowledge of the entire state, flexible variables included. While usually angular velocities are available through measurements from gyroscopes or IMU, this is not the case for the state variables of the flexible dynamics. However, they can be obtained using a simple observer, as done in[3] where the output feedback problem is solved in the fully-actuated scenario.

Remark 5. The complete problem of attitude stabilization, including the kinematic equations, can be solved by cascading the obtained controller with the one developed in[6] .

SIMULATIONS AND RESULTS

Simulations are performed to test the controller and the steering law developed in the previous section with a simpler controller not taking into account flexible dynamics and not exploiting a motion planning system for large angle maneuvers. More in detail, we have simply taken the control laws (10)-(25)-(27) and we have dropped the terms compensating the flexible dynamics and the uncertainties in the coupling matrix, obtaining the simpler expressions:

$$\begin{aligned} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} &= \begin{pmatrix} -f_1(\omega,\eta,\psi) \\ -f_2(\omega,\eta,\psi) \end{pmatrix} + \begin{pmatrix} j_{11} & j_{12} \\ j_{21} & j_{22} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \\ v_1 &= -r_1(e,\omega_3) - k_1 e_1 \\ v_2 &= -r_2(e,\omega_3) - k_2 e_2 \end{aligned} \tag{31}$$

with j_{ij} being an element of the inertia matrix J. These expressions substantially recover the Byrnes-Isidori controller.

For the sake of simplicity, we consider the case of two flexible modes, with matrices

$$N = \begin{bmatrix} 8.8080 & -8.8090 & 3736.3 \\ -8.8090 & 8.8089 & 295.1091 \end{bmatrix} 10^{-3} \left[\mathrm{kg}^{\frac{1}{2}} \cdot \mathrm{m}\right]$$

$$K = \begin{bmatrix} 6.652 & 0 \\ 0 & 13.376 \end{bmatrix} \left[\frac{\mathrm{rad}^2}{\mathrm{s}^2}\right]$$

$$C = \begin{bmatrix} 0.0005 & 0 \\ 0 & 0.0007 \end{bmatrix} \left[\frac{\mathrm{rad}}{\mathrm{s}}\right]$$

natural frequencies and damping values

$$\omega_n = \begin{bmatrix} 2.5792 & 3.6574 \end{bmatrix} \left[\frac{\mathrm{rad}}{\mathrm{s}}\right]$$

$$\zeta = \begin{bmatrix} 0.001 & 0.0008 \end{bmatrix}$$

In all the figures, we show the values of the angular velocities ω_1, ω_2 and ω_3.

Complete controller

The complete controller developed in our work is tested in two scenarios. In the first scenario, the switching between the multirate steering law and the Lyapunov stabilizer is made at the very end of the sampling interval, i.e. 0.01 seconds before its ending, as it is shown in Fig. 1. Stabilization is achieved with a very fast response and good precision.

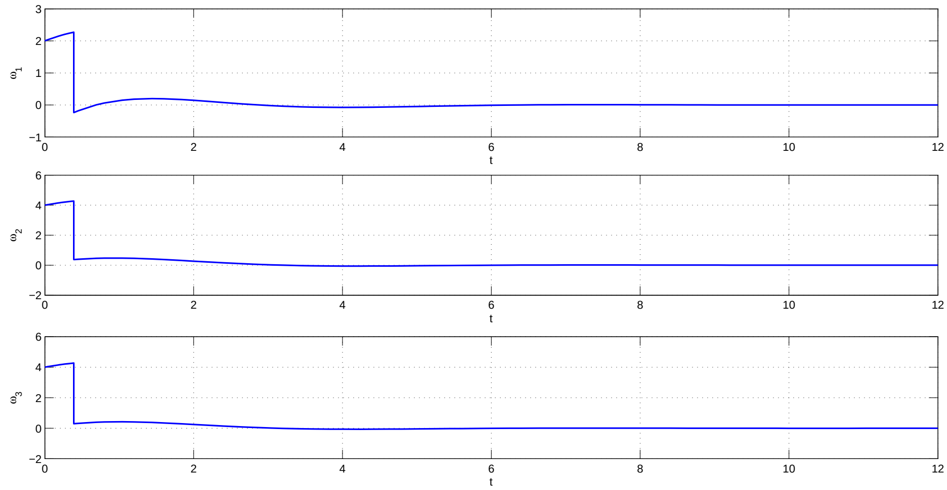

Figure 1. Steering maneuver and local stabilization of the spacecraft dynamics in the case when the stabilizer intervenes 0.01 seconds before the end of the sampling interval.

In the second scenario, the switching is made 0.1 seconds before the end of the sampling interval, as shown in Fig. 2. The performance is still good but the transient is longer and the effect of flexibility on the attitude dynamics is more evident. This is a consequence of the fact that the local stabilizer starts working when the trajectories are still far from the origin, thus nonlinearities and couplings have a heavier impact on the closed-loop response.

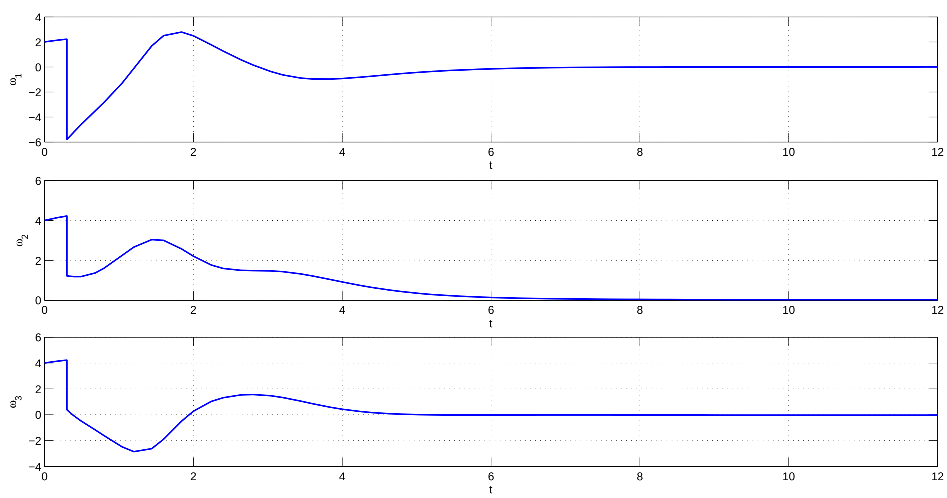

Figure 2. Steering maneuver and local stabilization of the spacecraft dynamics in the case when the stabilizer intervenes 0.1 seconds before the end of the sampling interval.

Simplified controller

The simplified controller is tested in two scenarios. In the first, the damping of the flexible dynamics has been radically increased to the following values:

$$\zeta = \begin{bmatrix} 0.1 & 0.08 \end{bmatrix}$$

The results are still acceptable in this case, even if the transient is very long and the oscillations of the flexible appendages cannot be limited anyway (Fig. 3)

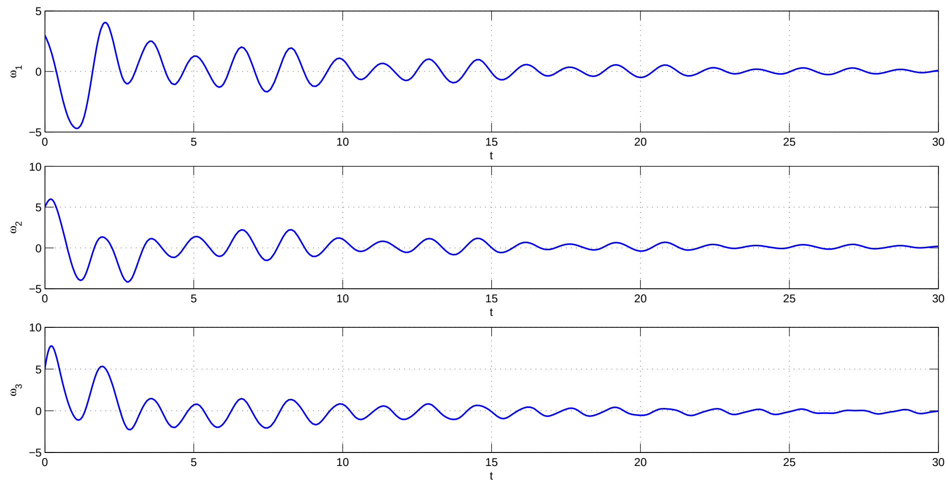

Figure 3. Local stabilization of the spacecraft dynamics using a simplified controller in the case of highly damped flexible motions.

However, when tested on the original, less damped, flexible dynamics, this controller shows a radically worse performance, not acceptable in an attitude control system. In this case, not only the transient response is very nervous and long, but there is also an unacceptable steady-state error, as shown in Fig. 4.

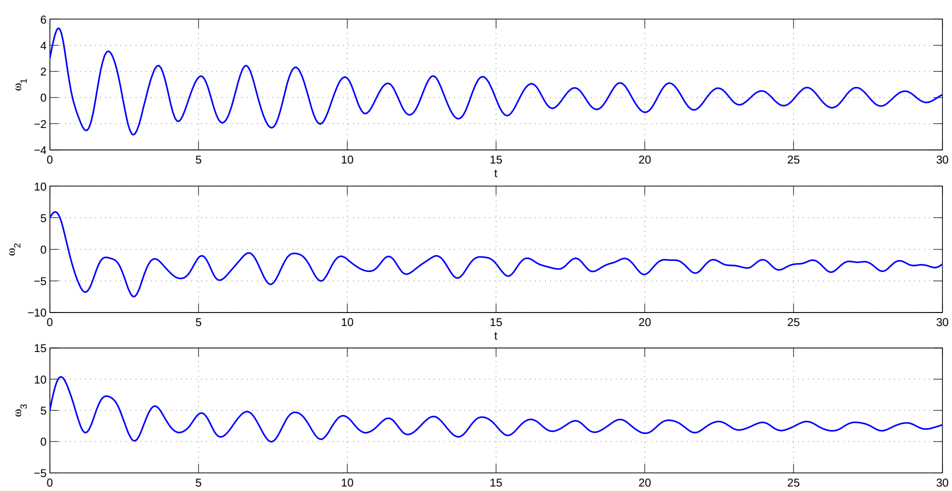

Figure 4. Local stabilization of the spacecraft dynamics using a simplified controller in the presence of standard values of the damping for the flexible motions.

CONCLUSIONS

In this work, the problem of attitude dynamics stabilization for a spacecraft with flexible appendages has been considered. The solution proposed consists in using a digital multirate steering strategy to bring the state of the system near the origin and then applying a local Lyapunov-based stabilizer to achieve the control objectives. The stabilizer is conceived to be robust against flexible disturbances and related uncertainties. The simulation results show the effectiveness of the proposed controller in comparison with its simplified version, implemented without a steering strategy and not considering flexibility in control design. As a result, the performance of the developed controller overcomes that of the simplified one. Future works should consider the solution of the full-attitude control problem, including the kinematics, and the under-sensed problem in which the flexible states are not directly available for feedback, but have to be estimated using an observer.

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