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ADAPTIVE ROBUST REDESIGN OF FEEDBACK LINEARIZATION FOR A SATELLITE WITH FLEXIBLE APPENDAGES

G. Mattei*, A. Carletti†, P. Di Giamberardino‡, S. Monaco§, D. Normand-Cyrot ¶

The paper deals with the attitude control problem for a satellite with flexible appendages in the presence of perturbations. Satellite dynamics is nonlinear and depends on poorly known parameters and environmental disturbances; flexible appendages increase complexity. In this work an adaptive robust Lyapunov redesign of feedback linearization is applied to the uncertain nonlinear system to achieve large-angle manoeuvres. While gravity-gradient and aerodynamic drag are considered partially structured uncertain terms and are counteracted with the robust part of the control law, the structured unknown parameters characterizing the flexible dynamics become naturally part of the adaptation laws. Not relying completely on a worst-case design has the affect of reducing conservatism and enhancing performance. Simulation results show the superiority of the robust adaptive approach compared to all-robust and nominal versions.

INTRODUCTION

Satellite dynamics exhibits in general nonlinear behavior with environmental disturbances and poorly known parameters. Flexible appendages increase the complexity of control design. First of all, the design of control laws can not ignore the intrinsic nonlinearity of the system considered. Furthermore, we also need to deal with uncertainties in the parameters of the dynamic model (e.g. the inertia matrix), with the effect of unmodeled dynamics (flexible dynamics in our case) on system's behaviour and with external disturbances like the gravity gradient and aerodynamic drag.

The first attempts to apply nonlinear control such as feedback linearization for satellite attitude control hail from the mid Eighties, where in[1] [2] and[3] a rigid model is considered. General studies on the problem are presented in[4] and.[5] During the same years, this classical nonlinear methodology is employed in the continuous time context[6] and under sampling[7] for the purpose of designing large-angle manoeuvres attitude control laws for spacecraft with flexible appendages. Other nonlinear controllers are proposed in[8] and the references therein,[9] and the more recent.[10] Adaptive versions of feedback linearization for flexible satellites are then successfully developed in[11] [12] and,[13] while a robust modification using μ-synthesis is employed in[14] and a robust backstepping approach is introduced in.[15] An approach both robust and adaptive developed in a nonlinear context is presented

*Ph.D. Candidate, Department of Computer, Control and Management Engineering "Antonio Ruberti", Sapienza - University of Rome, mattei@dis.uniroma1.it
†Post-graduate student, Sapienza - University of Rome, next86@libero.it
‡Researcher, Department of Computer, Control and Management Engineering "Antonio Ruberti", Sapienza - University of Rome, salvatore.monaco@uniroma1.it
§Professor, Department of Computer, Control and Management Engineering "Antonio Ruberti", Sapienza - University of Rome, salvatore.monaco@uniroma1.it
¶Research Director, Laboratoire des Signaux et Systmes - CNRS, Paris, cyrot@lss.supelec.fr

in[16] while inverse optimal stabilization is achieved in[17] Ȧ robust output feedback control law is presented, for the rigid case, in,[18] while an output feedback passivity-based controller is developed in[19] in the presence of flexibility.

The applied literature introduced above is mostly based on the geometric theory of control design, but there are also results which encompass Lyapunov theory and even Hamiltonian energetic concepts, as in[20] and passivity concepts in[21]Ȧdaptive versions of robust controllers using Lyapunov direct method and achieving uniform ultimate boundedness of the trajectories are developed, in several steps, in[22] [23] [25] and[26] while in[27] an adaptive robust nonlinear controller is developed for systems with nonlinearly parametrized uncertainties. The robust adaptive nonlinear control design presented in[29] is the most similar to that developed in the present work, but it is focused on triangular systems and uncertainties and on a backstepping nominal control law.

In our work we would like to propose a control design approach consisting of three steps. The first step is that of performing the feedback linearization of the nominal system. At the second step a control Lyapunov function is associated to the nominal control law found in order to develop a robust redesign which compensates the uncertainties in the parameters and the external disturbances. The third step consists in augmenting the Lyapunov function with a term depending on the uncertainties coming from the flexible dynamics, in order to design adaptive control using the certainty equivalence principle. While the uncertainties dealt with the robust redesign are lumped into unstructured terms, those compensated through the adaptation law are linearly parametrized. The resulting control law guarantees almost-global practical stabilization of the satellite about the desired equilibria and trajectories. The theoretical results are validated through extensive simulations of point-to-point large angle maneuvres, with or without a prescribed attitude trajectory to be followed. The adaptive robust redesign is compared to the nominal design of feedback linearization and respectively to its all-robust, more conservative, redesign. The effectiveness of the proposed approach is not only confirmed by the precision of tracking and regulation achieved, but also by the reduction of the control effort employed in the maneuvres.

As far as we know, the introduction of a robust adaptive redesign in the applicative context here considered is a novelty. Nonlinear controllers plus estimators usually fail since it is hard to capture good estimates of the flexible dynamics matrices. On the other hand, the introduction of robust observers increases too much the complexity of the design, often slowing down the response of the control system. Furthermore, the difficulty in estimating upper and lower bounds for the flexibility uncertainties entails a high level of conservatism of standard *worst-case* robust redesigns, often leading to chattering and instability due to resonance conditions at the frequencies of the bending modes. In addition to this, the control effort produced by these conservative all-robust redesigns is usually very high. Control laws are excessively nervous in the first seconds of transient, pushing actuators far beyond their physical limits. Our idea is to use adaptation to deal with flexibility uncertainties using the certainty equivalence principle and integrating with it the standard robust redesign of feedback linearization. In this way we use worst-case design only for parametric uncertainties whose upper and lower bounds estimates are well known and not even so far from the nominal values, thus decreasing the level of conservatism of the design together with its bad outcomes. The adaptive redesign is also essential to deal with the strong couplings between the rigid and the flexible behaviours, otherwise difficult to handle in the robust nonlinear context.

In the following section the mathematical model that describes the dynamic behaviour of the satellite is introduced and then the uncertainties due to parameter variations, disturbances and unmodeled dynamics effects are defined, together with their nominal values and bounding functions.

In the section Control, the complete control law is derived and, as a result, stability of uniform ultimate boundedness is shown. Finally, the performance of the adaptive robust controller is tested, validated through simulations and compared with that of other control strategies.

THE MATHEMATICAL MODEL

The rotation matrix A represents the orientation of the satellite body reference frame with respect to the inertial reference frame. The parameters that characterize the attitude are the unit quaternions $q_0 \in \mathbb{R}$, $q \in \mathbb{R}^3$, defined as

$$q_0 = \cos \frac{\phi_E}{2}, \;\; q = \vec{\epsilon}_E \sin \frac{\phi_E}{2}$$

where the Euler angle ϕ_E defines the rotation that brings the satellite reference frame into the inertial frame and $\vec{\epsilon}_E$ is the unit vector that identifies the axis around which the rotation is performed, also called Euler vector. The attitude matrix $A(q)$ can be expressed as:

$$R(q) = \begin{bmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1 q_2 + q_0 q_3) & 2(q_1 q_3 - q_0 q_2) \\ 2(q_1 q_2 - q_0 q_3) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2 q_3 + q_0 q_1) \\ 2(q_1 q_3 + q_0 q_2) & 2(q_2 q_3 + q_0 q_1) & q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix}$$

The kinematic relationship between the angular velocity and the attitude is given by:

$$\begin{bmatrix} \dot{e}_0(t) \\ \dot{e}(t) \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -e^T \\ R(e_0, e) \end{bmatrix} \omega_e \tag{1}$$

$$R(e_0, e) = e_0 I_{3\times 3} + S(e)$$

where (e_0, e) is the error quaternion, the error between the desired attitude and the actual one. The relationship with the desired attitude q_{r0}, q_r is given by:

$$\begin{bmatrix} e_0 \\ e \end{bmatrix} = \begin{bmatrix} q_0 & q^T \\ -q & q_0 I + S(q) \end{bmatrix} \begin{bmatrix} q_{r0} \\ q_r \end{bmatrix} \tag{2}$$

where $S(\cdot)$ is the skew symmetric matrix

$$S(\omega) = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}$$

The dynamic equations of a satellite consisting of a rigid mean body and flexible appendages, such as solar arrays and antennas, can be obtained following the Newton-Euler approach used in several works[6] [7] .[13]
According to the conservation of angular momentum we can write the coupled rigid and flexible equations:

$$J\dot{\omega} + N\ddot{\eta} = S(\omega)\left[J\omega + N\psi\right] + \tau$$

$$\ddot{\eta} + C\dot{\eta} + K\eta = N^T\dot{\omega}$$

where ω is the angular velocity vector, $\tau \in \mathbb{R}^3$ are the torque inputs and η, $\psi \in \mathbb{R}^M$ are the elastic coordinates such that $\psi = \dot{\eta}$. The matrix $N \in \mathbb{R}^{M\times 3}$ is the coupling matrix between the attitude dynamics and the M elastic modes, $C \in \mathbb{R}^{M\times M}$ is the damping matrix, $K \in \mathbb{R}^{M\times M}$ is the stiffness matrix. The angular velocity equation can be simplified to

$$\dot{\omega} = J_{eq}^{-1}\left[G + N(C\psi + K\eta) + \tau\right]$$

where $J_{eq} = J + NN^T$ is the equivalent inertia matrix and $G = S(\omega)\left[J\omega + N\psi\right]$ is the gyroscopic term.

Hence the mathematical model of the satellite with flexible appendages can be written in a form suitable to describe a point to point realignment or a tracking maneuver, once assigned a desired trajectory in quaternions q_r, see equation (2), or angular velocity ω_r, where $\omega_e = \omega_r - \omega \in \mathbb{R}^3$.

$$\begin{bmatrix}\dot{e_0}\\ \dot{e}\\ \dot{\omega}_e\\ \dot{\eta}\\ \dot{\psi}\end{bmatrix} = \begin{bmatrix}-\frac{1}{2}e^T\omega_e\\ \frac{1}{2}R(e_0,e)\omega_e\\ \dot{\omega}_r + J_{eq}^{-1}\left[G + N(C\psi + K\eta)\right]\\ \psi\\ -C\psi - K\eta + N^T\dot{\omega}\end{bmatrix} + \begin{bmatrix}0\\0\\J_{eq}^{-1}\\0\\0\end{bmatrix}u_b \tag{3}$$

where $u_b = \tau$ is the control inputs vector. In addition to this model, we need to define the total disturbance $d = d_a + d_g$ as the sum of the aerodynamic torque and the gravity gradient. We know that gravity gradient is not negligible as in most cases, because the asymmetric distribution of the mass interacts with the Earth gravity field generating the gravity gradient torque d_g

$$d_g = \frac{3\mu}{R_0^3} r \times Jr \tag{4}$$

where $\mu = GM_E \in \mathbb{R}$ is the gravitational Earth constant, $R_0 \in \mathbb{R}$ the distance to the Earth center and $r(q_0, q) \in \mathbb{R}^3$ identifies the direction towards the Earth center. Furthermore, since we consider the experimental class of satellites with solar electric propulsion exploiting large solar array to provide the necessary power for LEO-GEO transfer, the aerodynamic drag torque cannot be disregarded, since it affects importantly the attitude dynamics in the first part of the transfer. In low Earth orbit the presence of gas interacts with the surface of the spacecraft and generates drag. Since generally the center of pressure (c.o.p.) does not coincide with the center of mass (c.o.m.), this phenomenon generates the aerodynamic torque d_a.

$$d_a = -\frac{1}{2}c_d\rho\|V_0\|^2\left(A_{mb}S(d_{p,0}) + A_{sa}n^T r_v S(\Delta_p)\right)r_v \tag{5}$$

where $n(q) \in \mathbb{R}^3$ is the second row of the attitude matrix, normal to the solar panels

$$n(q) = A(q)\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}2(q_1q_2 + q_0q_3)\\ q_0^2 - q_1^2 + q_2^2 - q_3^2\\ 2(q_2q_3 + q_0q_1)\end{bmatrix}$$

$r_v(q) \in \mathbb{R}^3$ is a unit vector identifying the spacecraft velocity direction. It depends on both attitude and orbital parameters, and we assume it perfectly known. $d_{p,0} \in \mathbb{R}^3$ is the distance between the c.o.p. and the c.o.m. of the mean body, $\Delta_p = d_{p,1} - d_{p,2} \in \mathbb{R}^3$ is the difference between the distances of the solar arrays c.o.p.'s from the center of mass. The mean body area is $A_{mb} \in \mathbb{R}$ and the solar array area is $A_{sa} \in \mathbb{R}$, $\rho \in \mathbb{R}$ the "gas" density, $V_0 \in \mathbb{R}^3$ the orbital velocity and $c_d \in \mathbb{R}$ the drag coefficient.

UNCERTAINTY MODELING

The presence of disturbances and errors in parameters knowledge requires adequate compensation. We have uncertainties on the inertia matrix J, on the coupling matrix N, on the stiffness matrix K and on the damping matrix C. Together with the external disturbances, these uncertain terms are considered only *partially structured*, since they are, in general, nonlinear functions $f(x, p)$ of the state variables x and the parameters p. In fact, flexible dynamics is nonlinearly coupled with the attitude motion thanks to the gyroscopic effect term. However, we highlight that the other flexible terms which directly affect the angular motion of the satellite show a linear parametrization between state variables and parameters of the form

$$f(x, p) = \theta(p)x$$

In particular this is the case of the term

$$N(C\psi + K\eta) = \begin{bmatrix} NC & NK \end{bmatrix} \begin{bmatrix} \psi \\ \eta \end{bmatrix} = \theta(p)x$$

which is linear in the flexible states but allows products between parameters. As a consequence, these uncertainties are considered *fully structured*, thus reducing the conservatism of the uncertain model of the system. In the uncertain system the elastic motions are considered as an exogenous dynamics which interacts with the attitude motion together with the gravitational and aerodynamic disturbances. that is used to redesign the nominal nonlinear control law already found with the purpose of making it robust against the partially unstructured uncertainties and adaptive with respect to the linearly parametrized terms.

The parameter estimates $\hat{p}$ are made up by the sum of the nominal part p and a deviation term $\delta_p p$

$$\begin{aligned} \hat{J} &= (1 + \delta_J)J \qquad \hat{N} = (1 + \delta_N)N \\ \hat{K} &= (1 + \delta_K)K \qquad \hat{C} = (1 + \delta_C)C \\ \hat{J}_{eq} &= (1 + \delta_{J_{eq}})J_{eq} \end{aligned} \tag{6}$$

Starting from the system (3) and using the model of the uncertainties described in (6) the system can be written as

$$\begin{bmatrix} x_{e0} \\ x_e \\ x_{\omega_e} \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} x_e^T x_{\omega_e} \\ \frac{1}{2} R(x_e) x_{\omega_e} \\ \dot{x}_{\omega_{ra}} + J_{eq}^{-1} \left[S(x_\omega) J x_\omega \right] \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ J_{eq}^{-1} \end{bmatrix} \begin{bmatrix} u + \theta\zeta + \Delta \end{bmatrix} \tag{7}$$

where we have redefined the state variables as follows: $e_0 = x_{e0}$, $e = x_e$ and $\omega_e = x_{\omega_e}$. In this model two kinds of uncertainties are shown, the linearly structured *parametric uncertainty*, which arises owing to the unknown $\theta \in \mathbb{R}^{3 \times 2M}$, and the nonlinear unstructured *state-dependent uncertainty*, which arises owing to $\Delta \in \mathbb{R}^3$. The structured uncertain terms, defined as follows

$$\theta\zeta := \begin{bmatrix} NK & NC \end{bmatrix} \begin{bmatrix} \eta \\ \psi \end{bmatrix} = \begin{bmatrix} \theta_1 & \theta_2 \end{bmatrix} \begin{bmatrix} \zeta_1 \\ \zeta_2 \end{bmatrix}$$

are going to be compensated using a Lyapunov-based adaptive control law. Note that these terms come from the flexible dynamics, which is considered as an exogenous dynamics with measurable (or at least observable) state variables. On the other hand, the unstructured terms in Δ are counteracted through the robust part of the control law. Redefining suitably the right-hand side of (7) as follows:

$$f_N(x) = \begin{bmatrix} -\frac{1}{2}x_e^T x_{\omega_e} \\ \frac{1}{2}R(x_e)x_{\omega_e} \\ \dot{x}_{\omega_{ra}} + J_{eq}^{-1}\left[S(x_\omega)Jx_\omega\right] \end{bmatrix} \; ; \; g(x) = \begin{bmatrix} 0 \\ 0 \\ J_{eq}^{-1} \end{bmatrix}$$

system equations can be written in the more compact form

$$\dot{x} = f_N(x) + g(x)[u + \theta\zeta + \Delta] \tag{8}$$

Different contributions can be distinguished in the Δ term

$$\Delta = \left[\Delta_{f_N} + \Delta_\psi + \Delta_\eta + \Delta_{Dis} + \Delta_u\right]$$

The term due to the estimation error on the nominal plant is Δ_{f_N},

$$\Delta_{f_N} = S(x_\omega)J\delta_J x_\omega + \delta_{J_{eq}} S(x_\omega)J(1+\delta_J)x_\omega$$

The terms Δ_ψ and Δ_η are the errors in flexible dynamics estimates

$$\Delta_\psi = \{S(x_\omega)(1+\delta_N)N + \delta_{J_{eq}}^{-1} S(x_\omega)(1+\delta_N)N +$$

$$+N\delta_C C + \delta_N N(1+\delta_C)C + \delta_{J_{eq}}^{-1}[(1+\delta_N)N(1+\delta_C)C]\}\psi$$

$$\Delta_\eta = \{N\delta_K K + \delta_N N(1+\delta_K)K + \delta_{J_{eq}}^{-1}[(1+\delta_N)N(1+\delta_K)K]\}\eta$$

Moreover, we have the disturbance due to the gravity gradient d_g (4) and aerodynamic torque (5)

$$\Delta_{Dis} = (1+\delta_{J_{eq}})\left[d_a + d_g\right]$$

and the error caused by an erroneous estimation of the inertia matrix

$$\Delta_u = \delta_{J_{eq}} u$$

The Δ parameters need to be bounded by more or less structured functions of the state in order for the robust control to work. We start by giving a definition which is going to be exploited in the following.

Definition 1 *The quantity $||\cdot||_v$ is the matrix (or vector) of absolute values, with $a \in \mathbb{R}^{n\times m}$*

$$||a||_v := \begin{bmatrix} |a_{11}| & \dots & |a_{1m}| \\ \vdots & \ddots & \vdots \\ |a_{n1}| & \dots & |a_{nm}| \end{bmatrix}$$

•

It is then possible to define the bounding functions for the unstructured uncertain terms as follows

$$||\Delta_{f_N}||_v \leq S(x_\omega)\beta_1 x_\omega + \beta_2 S(x_\omega)\beta_3 x_\omega$$
$$||\Delta_\psi||_v \leq \beta_4 S(x_\omega)\beta_5 x_\psi + S(x_\omega)\beta_6 x_\psi + \beta_7 x_\psi$$
$$||\Delta_\eta||_v \leq \beta_8 x_\eta$$
$$||\Delta_{Dis}||_v \leq \beta_9 S(r(q))\beta_{10} r(q) + \beta_{11} r_v(q) + \beta_{12} n^T(q) r_v(q)\beta_{13} r_v(q)$$

where u_M is an estimate of the maximum values assumed by the control inputs.

$$\left.\begin{aligned} \beta_1, \beta_2, \beta_3, \beta_4, \beta_7, \beta_{10}, \beta_{13}, \beta_{14} \in \mathbb{R}^{3\times 3} \\ \beta_5, \beta_6, \beta_7, \beta_8 \in \mathbb{R}^{3\times M} \\ \beta_9, \beta_{11}, \beta_{12} \in \mathbb{R} \end{aligned}\right\} \begin{array}{r} \text{appropriate bounds} \\ \text{a priori known} \end{array}$$

CONTROL

The design of the control system is based on the computation of a control law that is mainly composed by three contributions, each of them obtained handling the three different effects that have been considered. Then, in a compact way, it is possible to represent the control law $u(t)$ as composed by three terms

$$u(t) = u_N(t) + u_R(t) + u_A(t)$$

where $u_R(t)$ represents the nominal control once all perturbations and uncertainties are neglected, $u_R(t)$ represents the contribution for getting robustness with respect to disturbances and $u_A(t)$ contains the terms necessary to compensate the parametric uncertainties.

The nominal control $u_N(t)$

The control design starts computing

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} L_{f_N} x_e \\ L_g L_{f_N} x_e [u + \theta\zeta + \Delta] + L^2_{f_N} x_e \end{bmatrix}$$

obtained applying the Lie derivative to system (8) and rewriting it in the more compact form

$$\dot{z} = Dz + B[A(x_e)[u + \theta\zeta + \Delta] + b(x_e)] \tag{9}$$

where Dz is a linear term, $B = \begin{bmatrix} 0_{3\times 3} \\ I_{3\times 3} \end{bmatrix}$, $A(x_e) \in \mathbb{R}^{3\times 3}$ and $b(x_e) \in \mathbb{R}^3$ given by:

$$A(x_e) = L_g L_{f_N} x_e = \frac{1}{2} R(x_e) J^{-1}$$

$$\begin{aligned} b(x_e) = L^2_{f_N} x_e &= \frac{\partial f_{N2}}{\partial x_{e_0}} f_{N1} + \frac{\partial f_{N2}}{\partial x_e} f_{N2} + \frac{\partial f_{N1}}{\partial x_{\omega_e}} f_{N3} \\ &= \frac{1}{2} x_{\omega_e} f_{N1} - \frac{1}{2} S(x_{\omega_e}) f_{N2} + \frac{1}{2} R(x_e) f_{N3} \end{aligned}$$

Assuming a full knowledge of the model, and neglecting the presence of both θ and Δ, a simple feedback law can be computed by inversion, so getting

$$u = -A^{-1}(x_e)\left[Kz + b(x_e)\right] \tag{10}$$

where K is the 6×6 matrix that sets the poles of $(D - BK)$, i.e. at closed loop, in the left-half complex plane. The control given in 10 represents the nominal control $u_N(t)$.

Once the presence of θ and Δ is considered, the control in 10 is changed in

$$u = -A^{-1}(x_e)\left[Kz + b(x_e)\right] - \theta\zeta - \Delta \tag{11}$$

and, due to the uncertainties on Δ and the lack of knowledge of the parameters θ, in 11 the terms $\theta\zeta$ and Δ have to be changed with the two previously introduced terms $u_A(t)$ and $u_R(t)$ respectively, in the form

$$u_A(t) = -\hat{\theta}\zeta$$

$$u_R(t) = -v_R(x, \psi, \eta)$$

where $\hat{\theta}$ is an estimation of the actual value θ and $v_R(\cdot)$ has to be computed to get robustness with respect to Δ, so getting

$$u = -A^{-1}(x_e)\left[Kz + b(x_e)\right] - \hat{\theta}\zeta - v_R(x, \psi, \eta) \tag{12}$$

Remark 1 *The number of state variables of system* (8) *is 7, but since the quaternions fulfil a constraint that guarantees the unit norm, $q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1$, the state of the system has dimension 6.*

Remark 2 *In view of the expression of the control law* (12) *it is necessary that $A(x)$ is a non-singular matrix in x; this condition is met if e_0 is different from zero. This corresponds to a pointing displacement with an Euler angle error of $\phi_E = 180\,\text{deg}$.*

Getting robustness with term $u_R(t)$

Following,[28] the robust control law is developed along a unit norm smoothing function, so the discontinuity is transformed into a sigmoid with derivative in zero equal to $\sigma \in \mathbb{R}^+$, a design constant. Let W be a Lyapunov function for a nominal system $\dot{x} = f(x) + g(x)\left(u + \Delta(x,t)\right)$, where $\Delta(x,t)$ is a matched uncertainty,

$$v_R = \beta(x,t)sgn(\omega)(1 - e^{-\sigma||\omega||_1}) \tag{13}$$

where

$$\omega^T = \frac{\partial W}{\partial x} g(x)$$

$\beta(x,t) \in \mathbb{R}^3$ is a bounding function,

$$\beta(x,t) \geq \frac{\rho(t,x)}{1 - k_0(x)}$$

obtained from

$$||\Delta(x,t)|| \leq \rho(t,x) + k_0||v_R||_\infty$$

Remark 3 *The robust terms are structured to erase only the possibly destabilizing disturbances, such as, for instance, the gravity gradient and the aerodynamic drag. The uncertainties coming from flexible appendages are only partially considered in the robust redesign, since they will be compensated through the adaptive control term.*

Remark 4 *The robust control helps avoiding the phenomenon of chattering, because it yields differentiable control laws. We extend the methodology described in*[26, 25] *for backstepping-based robust control laws.*

Including parameters adaptation (term $u_A(t)$)

The adaptive design is based on the "certainty equivalence principle".[29] The aim is to obtain an adaptive control law which offset the flexible dynamics effects on the attitude dynamics.

The expression of the adaptation laws, one for each set of parameters, can be assumed as

$$\dot{\hat{\theta}}_j = \gamma_j \left(\zeta_j \left(BA(x_e) \right)^T z - K_\theta (\hat{\theta}_j - \theta_j^0) \right) \qquad \gamma_j > 0, \quad j = 1 \dots 2M \tag{14}$$

where $\zeta_j \in \mathbb{R}$, $j = 1 \dots 2M$, are the element of ζ and $K_\theta \in \mathbb{R}^{3\times 3}$ is the gain of the adaptive control law.

The complete control law

Proposition 1 *The control law* (12)*, with* (13) *and* (14)*, guarantees that $z(t)$ and $\tilde{\theta}(t) = \hat{\theta}(t) - \theta$ are globally uniformly ultimately bounded.*

Proof 1 *Using* (9) *and* (12)*, one has*

$$\dot{z} = (D - BK)z - \tilde{B}(\tilde{\theta}\zeta - (v_R - \Delta)) \tag{15}$$

where $K_H = (D - BK) \in \mathbb{R}^{7\times 7}$ is Hurwitz and $\tilde{B} = BA(x_e)$.

Choosing the candidate Lyapunov function as

$$V(z, \hat{\theta}) = \frac{1}{2} z^T z + \sum_{j=1}^{2M} \frac{1}{2\gamma_j} \tilde{\theta}_j^T \tilde{\theta}_j \tag{16}$$

the derivative along the solution is

$$\dot{V}(z, \hat{\theta}) = z^T [K_H z - \tilde{B}(\tilde{\theta}\zeta - (v_R - \Delta))] +$$

$$+ \sum_{j=1}^{2M} \tilde{\theta}_j^T (\tilde{B}^T z \zeta_j - K_\theta (\tilde{\theta}_j - \theta_j^0) \tag{17}$$

where the equalities

$$z^T \tilde{B}(\tilde{\theta}\zeta) = \sum_{j=1}^{2M} \tilde{\theta}_j^T \zeta_j (\tilde{B}^T z)$$

and

$$-\tilde{\theta}_j^T K_\theta(\tilde{\theta}_j - \theta_j^0) = -\frac{1}{2}\tilde{\theta}_j^T K_\theta \tilde{\theta}_j -$$

$$-\frac{1}{2}(\hat{\theta}_j - \theta_j^0)^T K_\theta(\hat{\theta}_j - \theta_j^0) + \frac{1}{2}(\theta_j - \theta_j^0)^T K_\theta(\theta_j - \theta_j^0)$$

can be considered.

Setting $z^T \tilde{B} = \omega^T$*, it is possible to write*

$$\omega^T(v_R + \Delta) \leq \omega^T v_R + ||\omega||_1 ||\Delta||_\infty \leq$$

$$\leq \omega^T v_R + ||\omega||_1[\rho(x,t) + k_0||v_R||_\infty]$$

and, making reference to relationship (13)*, one can get*

$$\omega^T(v_R + \Delta) \leq -\beta(z,t)(1 + k_0(z,t))||\omega||_1 + \rho||\omega||_1 \leq$$

$$\leq -\rho||\omega_1||_1 + \rho||\omega||_1 = 0$$

So, (17) *assumes the expression*

$$\dot{V}(z,\hat{\theta}) \leq z^T K_1 z - \frac{1}{2}\sum_{j=1}^{2M} \tilde{\theta}_j^T K_1 \tilde{\theta}_j +$$

$$-\sum_{j=1}^{2M} \frac{1}{2}(\hat{\theta}_j - \theta_j^0)^T K_\theta(\hat{\theta}_j - \theta_j^0) +$$

$$+\sum_{j=1}^{2M} \frac{1}{2}(\theta_j - \theta_j^0)^T K_\theta(\theta_j - \theta_j^0) \leq \delta V + \epsilon$$

where

$$\begin{aligned} \delta &= min\{K_H, K_{\theta j}\} \\ \epsilon &= \sum_{j=1}^{2M} \frac{1}{2}(\theta_j - \theta_j^0)^T K_\theta(\theta_j - \theta_j^0) \end{aligned}$$

Let $\rho := \frac{\epsilon}{\delta}$ *so that the Lyapunov function* (16) *satisfies the following differential inequality*

$$0 \leq V(t) \leq \rho[V(0) - \rho]e^{-\delta t} \tag{18}$$

Then, $z(t)$, $\tilde{\theta}(t)$ *are globally uniformly ultimately bounded.*

■

Remark 5 *The terms* θ_j^0 *denotes the initial estimated values of* $\hat{\theta}_j$*. From* (18)*, it is clear that the more* θ_j^0 *is chosen close to* $\hat{\theta}_j$*, the lower is the value of* ρ *and than the faster is the convergence of* $V(t)$*.*

SIMULATIONS AND RESULTS

In this section, numerical results are shown. The parameter of the spacecraft are obtained considering a medium size satellite with large solar arrays. The inertia matrix is:

$$J = \begin{bmatrix} 980 & 29 & 11.5 \\ 29 & 390 & 11.3 \\ 11.5 & 11.3 & 630 \end{bmatrix} [\mathrm{Kg} \cdot \mathrm{m}^2]$$

Four flexible modes characterize the elastic dynamics. The coupling matrix is:

$$N = \begin{bmatrix} 8.8080 & -8.8090 & 3736.3 & -7644.1210 \\ -8.8090 & 8.8089 & -7.4507 & -231.6801 \\ 171.7701 & 295.1091 & 0 & 0 \end{bmatrix} 10^{-3} [\mathrm{kg}^{\frac{1}{2}} \cdot \mathrm{m}]$$

The values of natural frequencies and damping are listed below.

$$\omega_n = \begin{bmatrix} 2.5792 & 3.6574 & 14.816 & 16.001 \end{bmatrix}^T [\frac{\mathrm{rad}}{\mathrm{s}}]$$

$$\xi_n = \begin{bmatrix} 0.001 & 0.0008 & 0.0006 & 0.0004 \end{bmatrix}^T$$

The flexible dynamics matrix are:

$$K = \begin{bmatrix} 6.652 & 0 & 0 & 0 \\ 0 & 13.376 & 0 & 0 \\ 0 & 0 & 219.513 & 0 \\ 0 & 0 & 0 & 275.561 \end{bmatrix} [\frac{\mathrm{rad}^2}{\mathrm{s}^2}]$$

$$C = \begin{bmatrix} 0.0005 & 0 & 0 & 0 \\ 0 & 0.0007 & 0 & 0 \\ 0 & 0 & 0.0025 & 0 \\ 0 & 0 & 0 & 0.0027 \end{bmatrix} [\frac{\mathrm{rad}}{\mathrm{s}}]$$

To evaluate the performance of the controller the integrals of the torque have been taken into consideration, in order to evaluate the *Total Impulse* of the maneuvers

$$I = \int_0^T \left| u \right| dt$$

and the integral of the $e_0 = \cos \frac{\phi_{Ee}}{2}$, the error angle of the maneuver.

$$E = \int_0^T \left| (e_0 - 1) \right| dt$$

For the following comparisons are taken into consideration only the simulations that present similar value of E. The control laws that have better performance presents a lower value of I.

To simulate the behavior of a realistic attitude control system we introduce in all the following simulations a sampling time of the torque delivered, $T_s = 10^{-1}$.

The satellite performs a realignment point to point maneuver; the amplitude of the Euler angle is $\phi_E = 160°$, the Euler axis is $\vec{\epsilon}_E = [\frac{1}{2}, -\frac{1}{3}, -\frac{4}{5}]$.

The following graphic shows the comparison between the controllers, the nominal feedback linearization FL, the control law with robust terms FLR, and the controller with adaptive parameters FLRA.

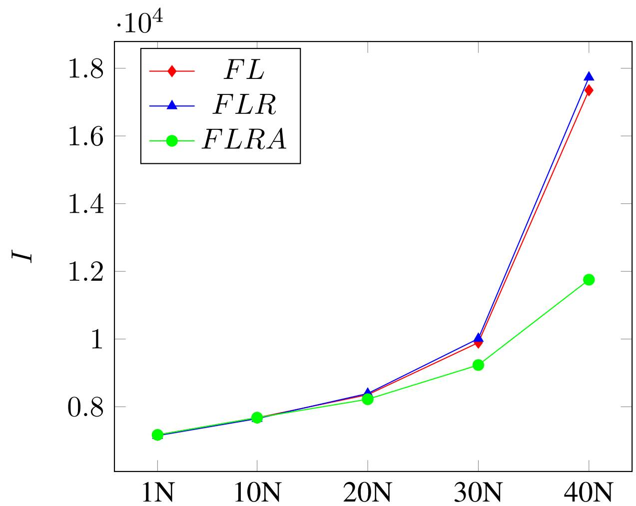

Figure 1: Total Impulse to increase the coupling with elastic dynamics, maneuvers with nominal parameters.

Increasing the coupling with the elastic dynamics the controller FLRA shows better performance in the case it has a perfect knowledge of the parameters. The presence of the robust part FLR degrades performance compared to the nominal controller FL. However, when the parameters of the model have a 10% of uncertainty or more, the robust controller FLR obtains better results than the nominal FL.

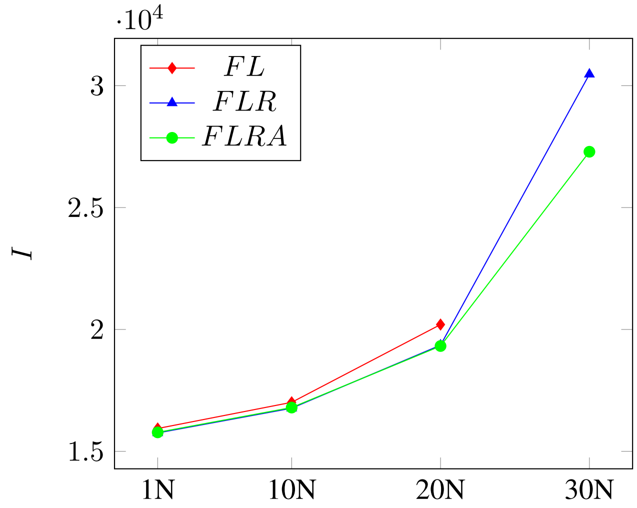

Figure 2: Total Impulse to increase the coupling with elastic dynamics, maneuvers with 10% error on the parameters.

The nominal controller based on feedback linearization FL cannot perform the maneuver when the knowledge of the model is limited by errors on estimates and presence of disturbances.

The gain matrix K assigns the eigenvalues of the feedback-linearized system. The poles are negative real and equal to -1 and -0.1, the simulation windows are respectively $50s$ e $200s$.

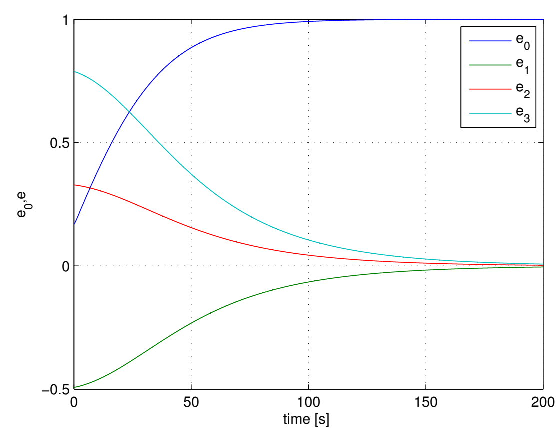

Figure 3: Error quaternion vs time FLRA pole in -0.1, $10N$, $\delta = 0$

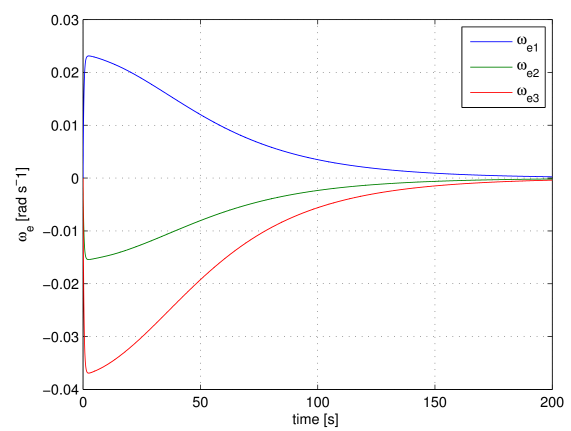

Figure 4: Angular rate vs time FLRA pole in -0.1, $10N$, $\delta = 0$

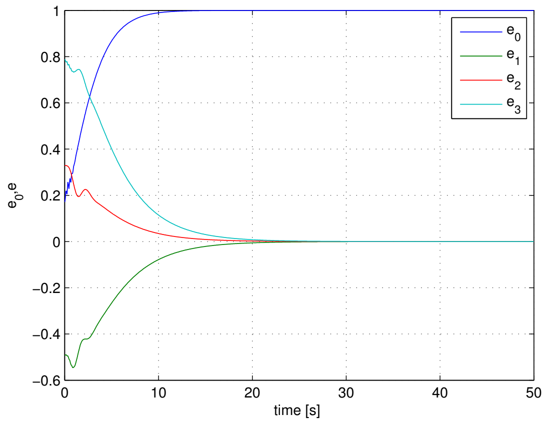

Figure 5: Error quaternion vs time FLRA pole in -1, $20N$, $\delta = 0$

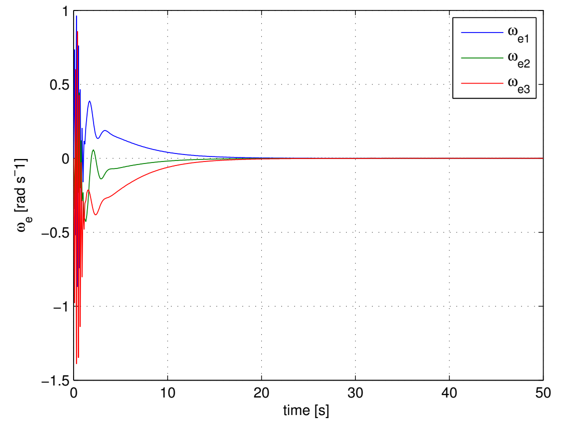

Figure 6: Angular rate vs time FLRA pole in -1, $20N$, $\delta = 0$

The controller FLRA better compensates peaks of elastic energy $E_{el} = \frac{1}{2}\psi^T\psi + \frac{1}{2}\eta^T K\eta$ in the first instants of the maneuver.

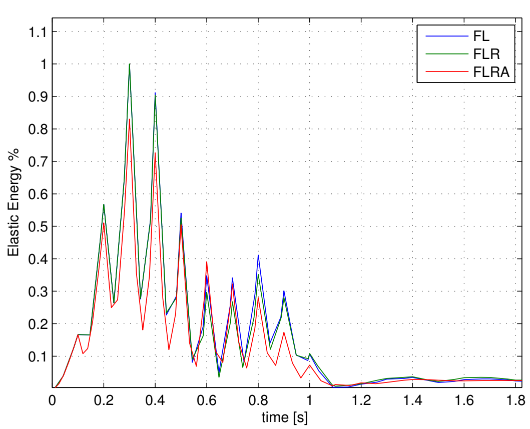

Figure 7: Elastic Energy in first seconds

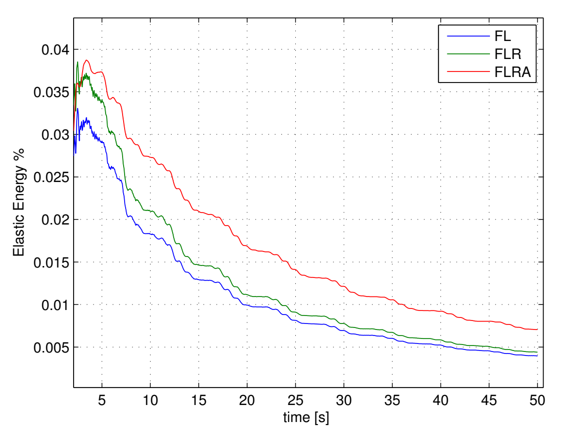

Figure 8: Elastic Energy in last seconds

The following graph shows a direct comparison between the torque at low gain, pole in -0.1, and high gain, pole -1, in the first few seconds of the maneuver.

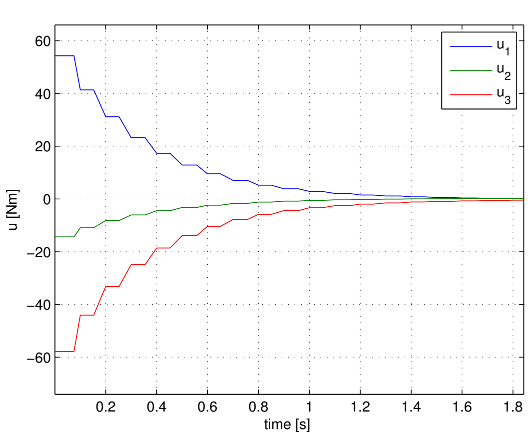

Figure 9: Torque in first second of maneuver vs time FLRA pole in -0.1 with increased disturbances

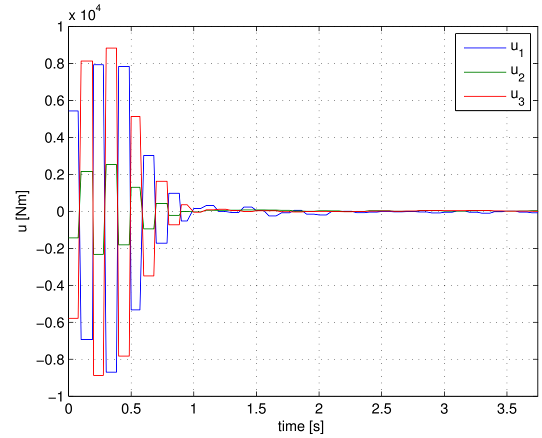

Figure 10: Torque in first second of maneuver vs time FLRA pole in -1 with increased disturbances

The following graphs shows the convergence of the adaptive parameters.

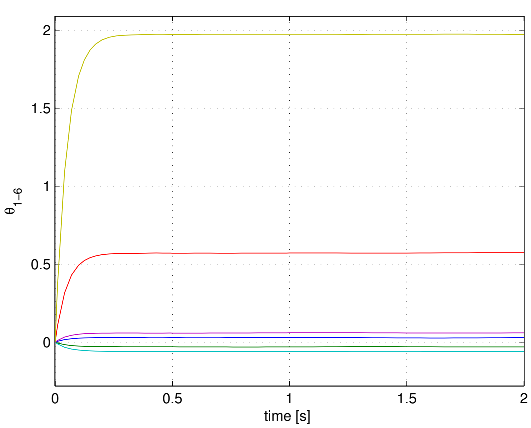

Figure 11: Adaptive parameters proportional to η, θ_1 to θ_6

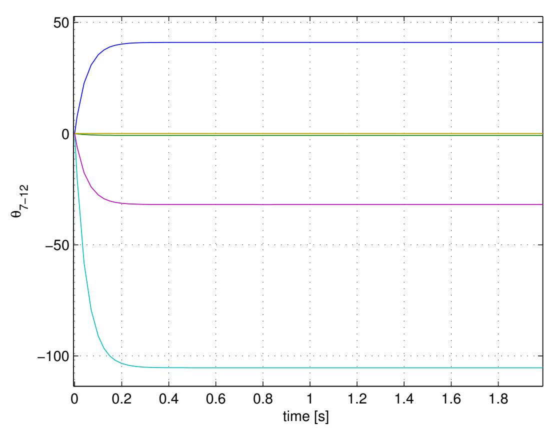

Figure 12: Adaptive parameters proportional to η, θ_7 to $\theta_1 2$

The components proportional to η converge quickly, while the components proportional to ψ present a longer transient.

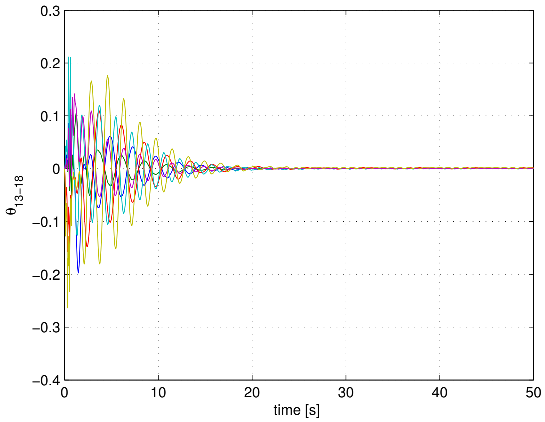

Figure 13: Adaptive parameters proportional to ψ, θ_{13} to θ_{18}

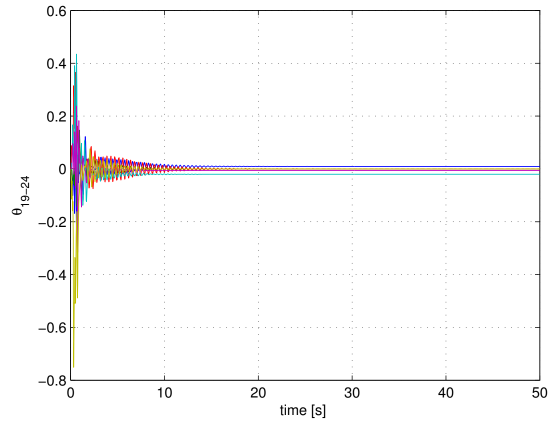

Figure 14: Adaptive parameters proportional to ψ, θ_{19} to θ_{24}

We consider a tracking law assigned a precession motion, where the reference is expressed by

$$q_{r0} = \cos\frac{\phi_r}{2}, \quad q = \vec{\epsilon}_r \sin\frac{\phi_r}{2}$$

where

$$\phi_r = A_{\phi_r}\sin\omega_{\phi_r}t, \quad \epsilon_r = \begin{bmatrix}\cos\omega_\epsilon t\\ \sin\omega_\epsilon t\\ 0\end{bmatrix}$$

Inverting the kinematic relationship (1) we obtain the inverse kinematic expression

$$\omega_r = 2\left[-q_r \quad q_{r0}I_{3\times3} + S(q_r)\right]^T \dot{q}_r = 2Q(q_r)\dot{q}_r$$

It is then possible to express the ratio of change of ω_r

$$\dot{\omega}_r = 2Q(q_r)\ddot{q}_r + 2\dot{Q}(q_r)\dot{q}_r \tag{19}$$

In the figure it is shown the tracking maneuver given trajectory described in (19), an initial error angle ϕ_e of 110 deg.

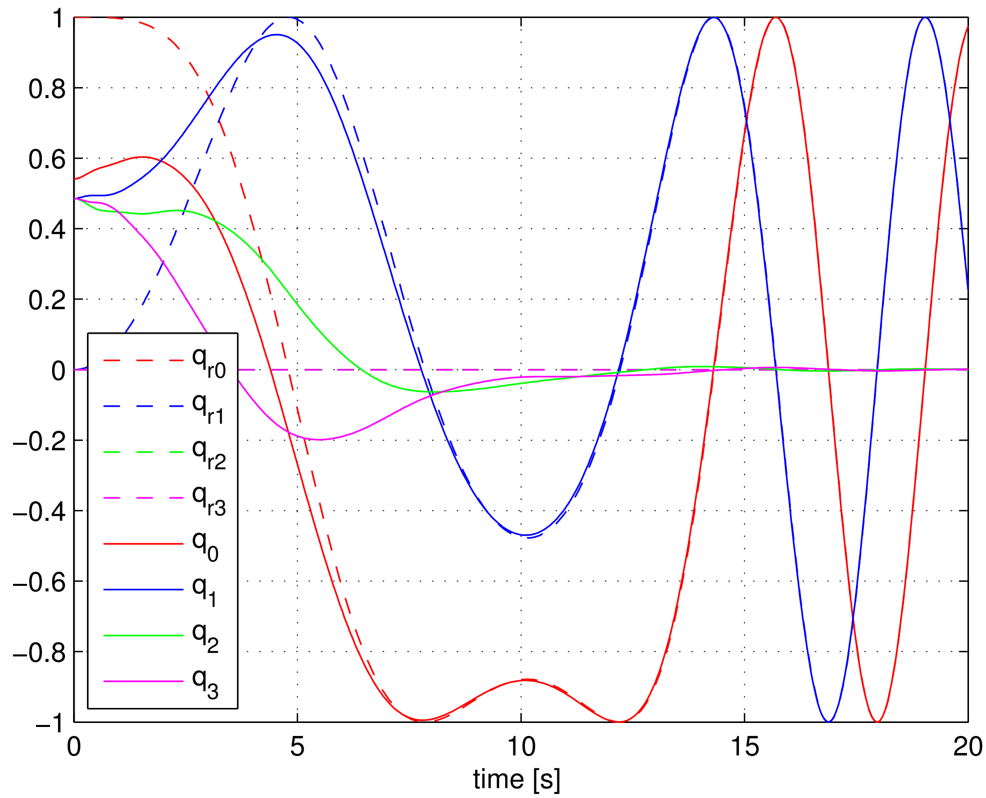

Figure 15: Maneuver assigned a reference q_r

CONCLUSIONS

The problem of attitude stabilization and tracking of a satellite with flexible appendages has been addressed. A complete dynamical model of the satellite, including flexibility, has been derived. Partially structured and linearly parametrized uncertainties are considered in control design, the former being addressed using a robust Lyapunov redesign of feedback linearization, the latter using adaptive control. The effectiveness of the robust adaptive redesign of the classical feedback linearizing controller is shown through several numerical simulations.

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