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COMPARATIVE STUDY OF FSS AND SPT FOR INTERPLANETARY SOLAR SAIL PROPULSION

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With the progress of solar sail interplanetary travel in the last two decades, various alternative concepts and technologies have been developed. Two of the solar sail technologies gaining more popularity are the Flat Solar Sail and Solar Photonic Thruster. Along this frame of thought, the present work is aimed to compare a conventional Flat Solar Sail (FSS) to a modified Solar Photon Thruster (SPT) in order to demonstrate their potentials. To that end, an interplanetary mission is analyzed for a specific trajectory to Mars as an example. The advantages and disadvantages of a SPT are discussed in comparison to a conventional FSS.

Introduction

The concept behind solar sailing is utilizing the vanishingly small momentum of solar photons in space. This can be compared to sail boats that use earth wind vessels to achieve high velocities using sun light as propulsion(References 1 and 2). So far the only successfully deployed solar sail has been the IKAROS Project carried out by JAXA which achieved a successful flyby to Venus in 2010 (Reference 3), depicted in Figure 1. A conventional Flat Solar Sail (FSS) uses a simple extremely large flat surface to reflect and utilize the momentum of solar photons. This means that the entire surface is generating thrust which could lead to a complex problem in controlling the thrust vector, attitude and dynamics of the spacecraft, as also exhibited in Figure 1. However, a Solar Photon Thruster (SPT) uses a large parabolic surface to collect the radiation and directs it to a smaller surface for generating thrust. This means that by controlling a smaller surface in comparison to a FSS, optimizations can be made to the solar sails trajectories. With such backdrop, the present paper analyze the dynamics of both Flat Solar Sail and Solar Photon Thruster for comparative and parametric study. Along this line, two proposed designs for SPT are compared and discussed and finally a Mars mission using a specific type of Trajectory known as Logarithmic Spiral Trajectories is analyzed to illustrate the advantage of SPT.

For the purpose of the present analysis, the Coordinate System used is the heliocentric coordinate system with the origin at the center of the Sun, as also utilized by McInnes

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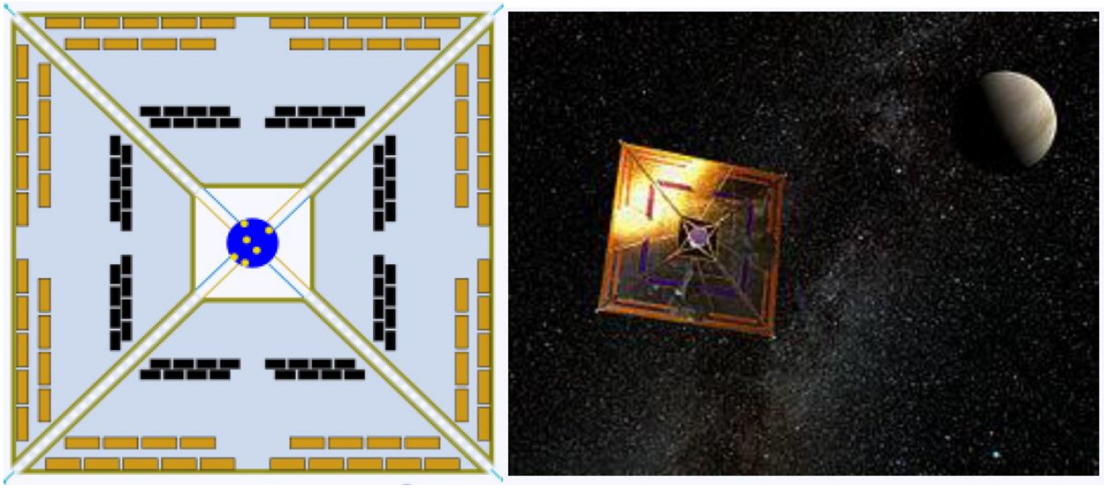
(Reference 1), and shown in Figure 2. In choosing a fundamental plane, the plane of Earth’s orbit around the Sun, also known as the ecliptic[3] plane, is utilized. Such choice of coordinate system, which is conventionally used, will facilitate simplified analysis for a good insight into the problem.

The following assumptions, which will reduce the problem to a simpler one and provide comprehensive insight, will be adopted; these are that only the gravitational field of the Sun is considered as a central Newtonian field, and in all the solar sail configurations considered, they are ideal photon reflectors, in the sense that all photons are perfectly reflected. In addition, the solar sail powered space vehicle and its orbit completely lies in the same orbital plane.



(a)

(b)

Figure 1.(a) Schematic diagram of IKAROS Solar Sail; (b) IKAROS space probe in flight (artist's conception) (Reference 3).

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[3] The ecliptic is the apparent path of the Sun on the celestial sphere, and is the basis for the ecliptic coordinate system. It also refers to the plane of this path, which is coplanar with both the orbit of the Earth around the Sun and the apparent orbit of the Sun around the Earth.

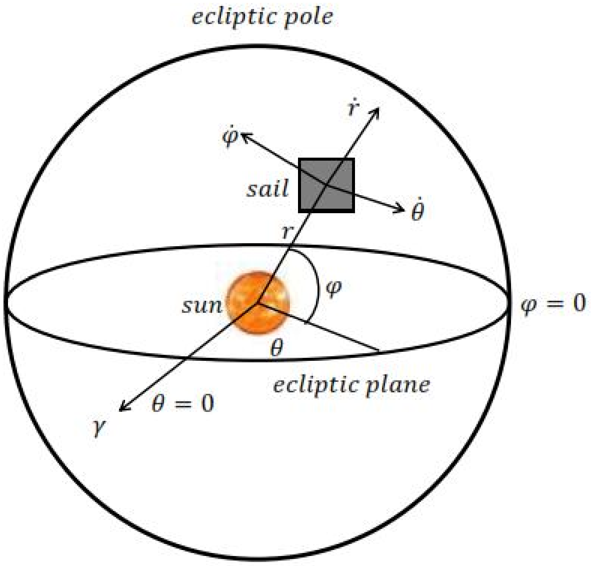

Figure 2.Definition of spherical polar coordinates

Orbital Dynamics of Flat Solar Sailing (FSS)

For convenience in the analysis, the equations of motion can be resolved in a spherical polar coordinate system. The ecliptic plane is known as ϕ=0 plane with ϕ= π/2 directed to the north ecliptic pole. The first point of Aries γ can be defined as the astronomical reference direction in the ecliptic plane.

The analysis carried out in the following sections will be based on fundamental orbital dynamics as can be found in McInnes(Reference 1), Guerman et al (Reference 4). Dachwald (References 5-7), and others (References 7-13). Since solar sails utilize a low thrust propulsion system, their orbital dynamics are similar to a conventional spacecraft using electric propulsion system. The main difference is that an electric propulsion spacecraft may orient its thrust vectors while a solar sail orientation is based on two angles. Sail clock angle δ and the sail cone angle α (Figure 3).

Cone angle α is defined to be the angle between the sail normal and the sun-line while clock angle δ is defined to be the angle between projection of the sail and some reference direction onto a plane normal to the sun-line. Solar radiation force acting on a solar sail can be calculated as follows:

$$F_r = PA(e_r.n)e_r$$

(1)

and

$$F_{r'} = -PA(e_r.n)e_{r'}$$

(2)

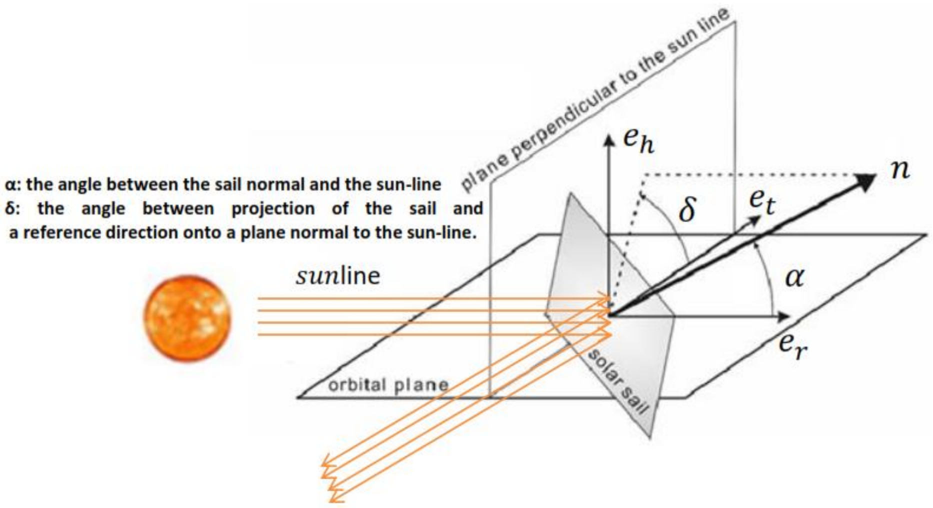

Figure 3.Definition of Sail clock angle δ and the sail cone angle α

Since

$$e_r - e_{r'} = 2(e_r.n)$$

(3)

where e_r and $e_{r'}$ are the unit vectors along the sunline and its reflection respectively, then the total radiation force for a perfectly reflecting solar sail would be

$$F_{SRP} = F_r + F_{r'} = 2PA(e_r.n)^2 n = 2PAcos\alpha^2 n$$

(4)

Since solar radiation pressure has an inverse square variation with radius, Eq. (4) can be rewritten as

$$F_{SRP} = (P_{eff})_{1AU} (\frac{1AU}{r})^2 Acos^2\alpha\, n$$

(5)

where P_{eff} is solar radiation pressure for a perfectly reflecting solar sail at 1 AU distance from the sun which is

$$P_{eff} = 2P_{01AU} = 9.126\mu N / m$$

(6)

characteristic acceleration can also be written in terms of solar gravitational acceleration

$$a_{SRP} = \beta \frac{GM_{\odot}}{r^2} cos^2\alpha\, n$$

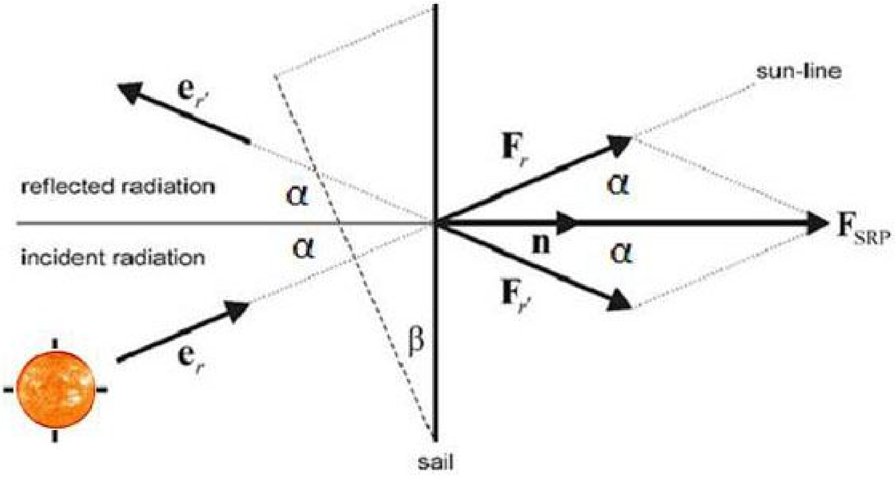

Figure 4. **Schematic of perfect reflection of solar radiation**

(7)

Here β is called the sail loading parameter also known as lightness number which is a dimensionless constant described as the ratio of solar radiation acceleration to solar gravitational acceleration.

Orbital Dynamics of Solar Photon Thruster (SPT)

As seen in Eq. (5), thrust force generated by a Flat Solar Sail is reduced by the cosine squared of sail cone angle. Moreover since it is the transverse component of force tangent to the solar sail orbit which does useful work, only 38% of the available solar radiation pressure force is of use when the sail orientation is optimized(Reference 1). However it can be seen that by having a large parabolic reflective surface for the purpose of collecting solar radiation (Collector) and a smaller flat reflective surface for the purpose of directing the radiation and therefore generating thrust (Controller) , significant optimizations can be made over the conventional FSS. This device is called Solar Photon Thruster, first coined by Robert Forward (Reference 14). Two design cases will be considered. The first case is simple SPT with two reflecting surfaces: the parabolic collector and the controller, as shown in Figure 5 and discussed by Guerman et al (Reference 4).The second case is a more complex SPT elaborated by McInnes (Reference 1), which consists of a parabolic collector, a collimating mirror and the controller, as shown in Figure 6. By tracing a single photon beam through the SPT the equation of radiation force exerted on the sail surface can be derived for both cases. For the first case shown in Figure 5, the photon beam is first reflected by the collector generating a reaction force f_1 and an incident force f_i . After being reflected by the controller the photon beam will cause a reaction force f_r and an inceident force $-f_1$ which means that the total force acting on the sail is the summation of f_i and f_r . for the second case, shown in Figure 6, the incoming solar radiation is directed toward the collimating mirror which will create a uniform beam of radiation director towards the controller.

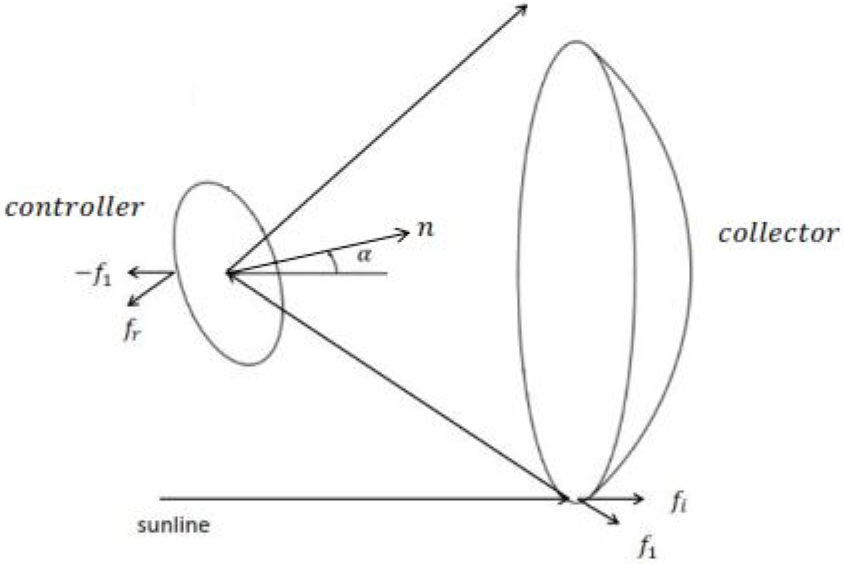

Figure 5. **Solar Photon Thruster analyzed by Guerman et al [3]**

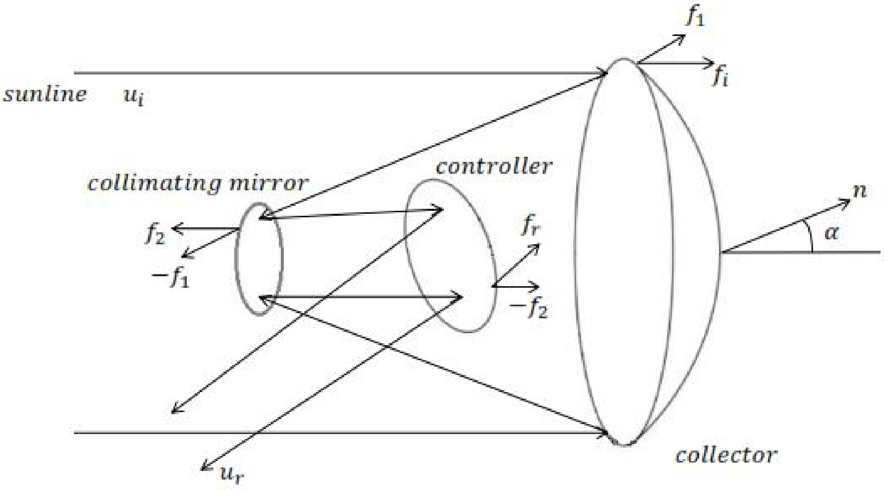

Figure 6. **Solar Photon Thruster elaborated by McInnes [1]**

It can be seen that even for a more complicated optical path such the second case, the reaction forces still cancel each other and therefore the summation of the resultant forces would be identical to the first case which is the summation of f_i and f_r. Asuming that the surfaces are perfectly reflective and the collector has an area of A, the radiation forces are as follows:

$$f_i = PAu_i \tag{8}$$

$$f_r = -|f_i|u_r \tag{9}$$

where p is the radiation pressure, u_i is the unit vector directed along the sunline and u_r is the unit vector directed along the path of photon beams leaving the solar sail. Based on Eq. (3), the total force exerted on the SPT would be:

$$f = 2PA(u_i.n)n \tag{10}$$

which could be rewriten as

$$f = 2PA\cos\alpha\, n \tag{11}$$

Note that the direction of thrust force between these two cases are different yet according to the above equation the magnitute of thrust force would be the same. It can be seen that the magnitute of force acting on the SPT varies with $\cos\alpha$ which compares two the squared cosine that effects the FSS. Figure 7compares the normalized force acting on a solar sail with an area of 100 m^2 at earth distance from sun.

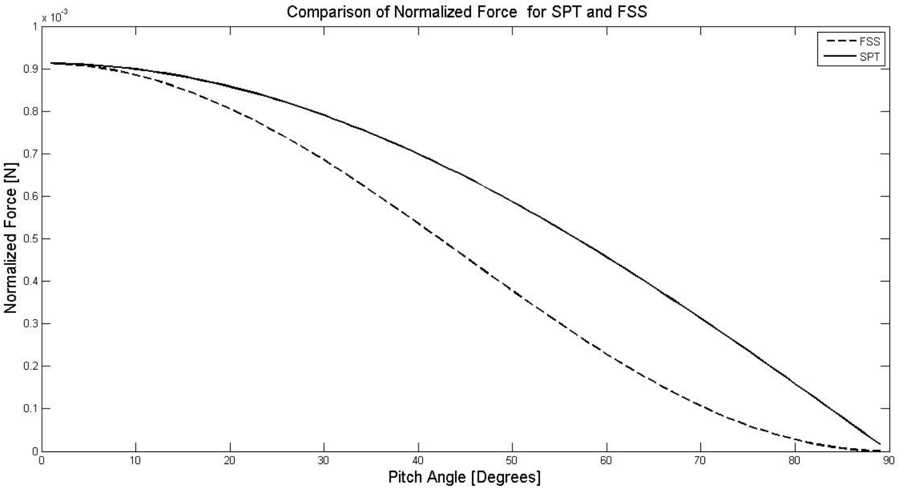

Figure 7.Comparison of Normalized Force for SPT and FSS.

Logarithmic Spiral Trajectories

In this section, a specific type of trajectory for interplanetary mission is discussed and a case study between a FSS and SPT is done revealing the advantages a SPT can have to a conventional sail. Logarithmic Spiral Trajectories are one attractive option for interplanetary travel which requires the spacecraft to utilize a continuous low thrust propulsion system and to have an inverse square variation with distance, as exhibited in Figure 8 (Reference 2).

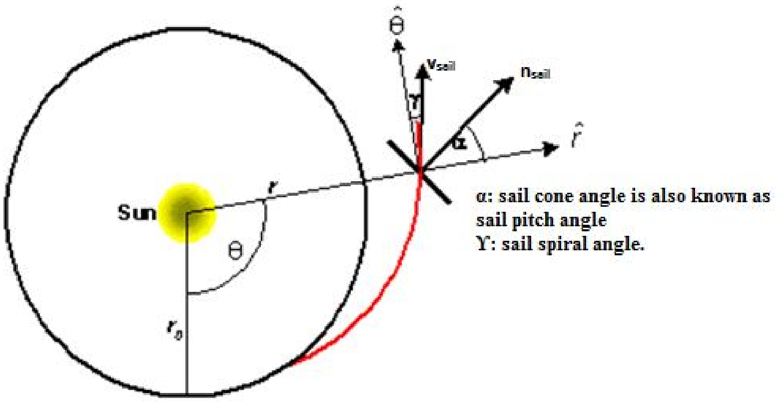

Figure 8. **Logarithmic Spiral Trajectories**

In the consideration of solar radiation for solar sails, the solar radiation pressure has an inverse square variation with the orbital radius,. For Logarithmic Spiral Trajectories, the spacecraft is considered to travel in an ecliptic plane with a clock angle δ of 90^{o}; this means that the sail attitude relies only on the sail cone angle α.

FSS Logarithmic Spiral Trajectories

The formulas derived in this section follow closely the Logarithmic Spiral Trajectories development by McInnes (Reference 1). The equation of motion for a perfectly reflecting flat solar sail can be defined as

$$\frac{d^2r}{dt^2} + \frac{\mu}{r^2}\hat{r} = \beta\frac{\mu}{r^2}(\hat{r}.n)^2 n \qquad (12)$$

where $\mu = G(M_{\odot} + m)$ and since $m << M_{\odot}$ it can be assumed that $\mu = GM_{\odot}$. According to Figure 2, by resolving the above equation to radial $\hat{r}$, transverse $\hat{\theta}$ and normal $\hat{\phi}$ components, the three-dimensional equations of motion for a solar sail can be written as follows

$$\frac{d^2r}{dr^2} - r(\frac{d\phi}{dt})^2 - r(\frac{d\theta}{dt})^2\cos^2\phi = -\frac{\mu}{r^2} + \beta\frac{\mu}{r^2}\cos^3\alpha$$

(13)

$$\frac{1}{r}\cos\phi\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right) - 2r\left(\frac{d\theta}{dt}\right)\left(\frac{d\phi}{dt}\right)\sin\phi = \beta\frac{\mu}{r^2}\cos^2\alpha\sin\alpha\sin\delta$$

(14)

$$\frac{1}{r}\frac{d}{dt}(r^2\frac{d\theta}{dt}) + r(\frac{d\theta}{dt})^2\sin\phi\cos\phi = \beta\frac{\mu}{r^2}\cos^2\alpha\sin\alpha\sin\delta$$

(15)

The transverse and radial components of solar sail velocity vector are derived as follows

$$v_{\theta} = \sqrt{\frac{\mu}{r}}[1 - \beta\cos^2\alpha(\cos\alpha - \tan\gamma\sin\alpha)]^{1/2}\cos\gamma$$

(16)

$$v_r = \sqrt{\frac{\mu}{r}}[1-\beta\cos^2\alpha(\cos\alpha - \tan\gamma\,\sin\alpha)]^{1/2}\ \sin\gamma$$

(17)

and the magnitude of solar sail velocity is

$$v_{(r)} = \sqrt{\frac{\mu}{r}}[1-\beta\cos^2\alpha(\cos\alpha - \tan\gamma\,\sin\alpha)]^{1/2}$$

(18)

by integrating the equation for the radial velocity , transfer time can be obtained from an initial orbit radius r_0 to a distance r

$$t - t_0 = \frac{1}{3}(r^{3/2} - r_0^{3/2})\left(\frac{1-\beta \cos^3\alpha}{\beta^2\mu \cos^4\alpha\, \sin^2\alpha}\right)^{1/2} \tag{19}$$

SPT Logarithmic Spiral Trajectories

Since the force acting on a SPT is effected by $\cos\alpha$ instead of squared cosine that effects the FSS, the equations of motion for a SPT can be rewritten as

$$\frac{d^2r}{dr^2} - r\left(\frac{d\phi}{dt}\right)^2 - r\left(\frac{d\theta}{dt}\right)^2\cos^2\phi = -\frac{\mu}{r^2} + \beta\frac{\mu}{r^2}\cos^2\alpha$$

(20)

$$\frac{1}{r}\cos\phi\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right) - 2r\left(\frac{d\theta}{dt}\right)\left(\frac{d\phi}{dt}\right)\sin\phi = \beta\frac{\mu}{r^2}\cos\alpha\,\sin\alpha\,\sin\delta$$

(21)

$$\frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right) + r\left(\frac{d\theta}{dt}\right)^2\sin\phi\cos\phi = \beta\frac{\mu}{r^2}\cos\alpha\,\sin\alpha\,\sin\delta$$

(22)

Therefore the equations for radial and transverse velocity are

$$v_\theta = \sqrt{\frac{\mu}{r}}[1-\beta\cos\alpha(\cos\alpha - \tan\gamma\,\sin\alpha)]^{1/2}\cos\gamma$$

(23)

$$v_r = \sqrt{\frac{\mu}{r}}[1-\beta\cos\alpha(\cos\alpha - \tan\gamma\,\sin\alpha)]^{1/2}\ \sin\gamma$$

(24)

and the magnitude of solar sail velocity is

$$v_{(r)} = \sqrt{\frac{\mu}{r}}[1-\beta\cos\alpha(\cos\alpha - \tan\gamma\,\sin\alpha)]^{1/2}$$

(25)

Finally the transfer time equation can be rederived to

$$t - t_0 = \frac{1}{3}(r^{3/2} - r_0^{3/2})\left(\frac{1-\beta \cos^2\alpha}{\beta^2\mu \cos^2\alpha\, \sin^2\alpha}\right)^{1/2}$$

(26)

Results and Discussions

For a Solar Photon Thruster with configuration like in Figure 5, the radiation is reflected back towards the parabola. In case the radiation is reflected back on the collector, it could jeopardize the integrity of the sail film and therefore reduce the life span of the collectors sail. Moreover it could result in disturbing torques; therefore the pitch angle of the controller should be adjusted so that the radiation is deflected away from the parabolic surface, as exhibited in Figure 9. For a Solar Photon Thruster elaborated by Guerman et al (Reference 4) the condition below should be met

$$|\tan\alpha| \geq \frac{2aR}{a^2 - R^2} \tag{27}$$

where R is radius of the collector and $a = 2f$, where f is the focus point of the parabola and a is called the parameter of paraboloid. Assuming that $X = \frac{R}{a}$ the equation can be rewritten as

$$|\tan\alpha| \geq \frac{2X}{1 - X^2} \tag{28}$$

Based on the transfer time equations derived for a Logarithmic Spiral Trajectory, it can be seen that for a given spacecraft with a known lightness number β the transfer time between two orbits depends entirely on sail pitch angle α. This means that by finding the optimum sail pitch angle α^* a minimum transfer time can be achieved. Figure 10 shows the transfer time and pitch angle for different values of lightness number for a FSS in an interplanetary Mars mission. Therefore each spacecraft with a known sail lightness number would have a unique optimum sail pitch angle.

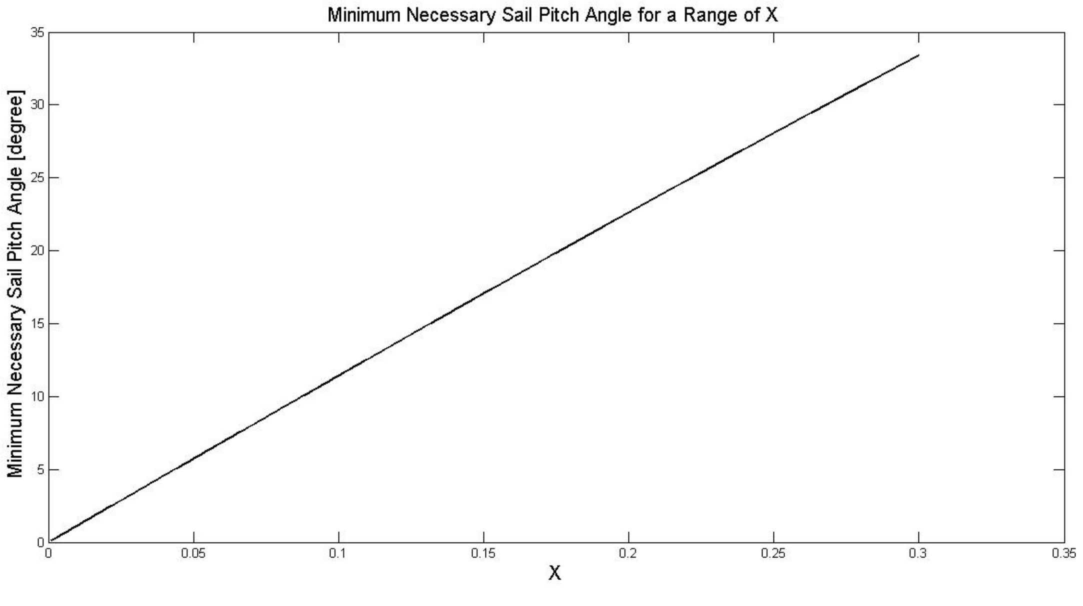

Figure 9. **Minimum necessary sail pitch angle for a wide range of X**

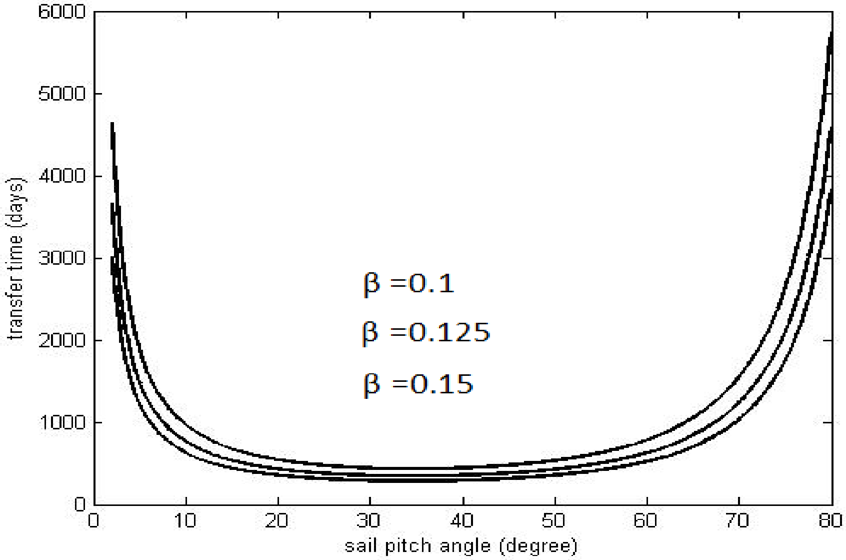

Figure 10. **Transfer time and pitch angle for a FSS with respect to β (reproduced from Reference 2)**

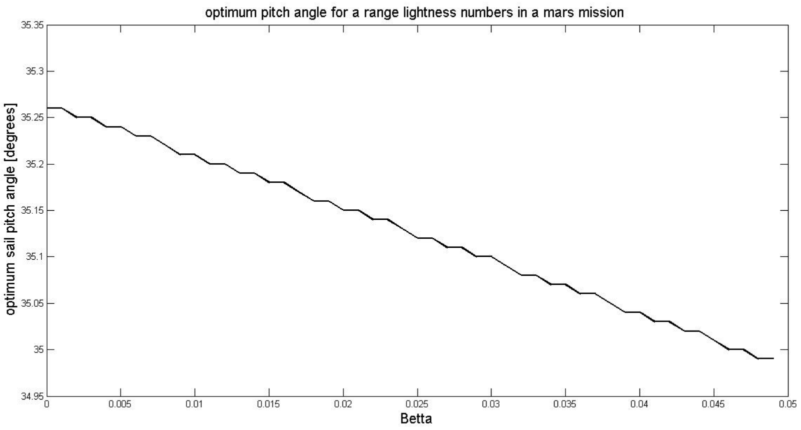

Figure 11.Optimum pitch angle for a range lightness numbers for a FSS in a Mars mission (reproduced from Reference 2)

Figure 11 shows the optimum pitch angle for a range lightness numbers in a Mars mission and it can be concluded that the average α^* for this range of sail lightness numbers is equal to 35.13 degrees. Figure 12 indicates the same procedure carried out for the same range of lightness numbers for a SPT spacecraft. It can be seen that the average α^* for this range of sail lightness numbers is equal to 44.8 degrees. Based on the above Figures 9 and 12 it can be seen that for a Solar Photon Thruster elaborated by Guerman et al (Reference 4) having a Logarithmic Spiral Trajectory, two conditions must be met in order to maximize the performance:

i. An optimum sail pitch angle should be used to minimize the transfer time
ii. Sail pitch angle should meet the condition in Eq. (28)

It should be noted that for a SPT discussed by McInnes (Reference 1) the second condition is not necessary since the radiation is reflected away from the parabolic surface. The SPT configuration to be selected for a specific mission depends entirely on focal distance which dictates the minimum sail pitch angle for the first case SPT.

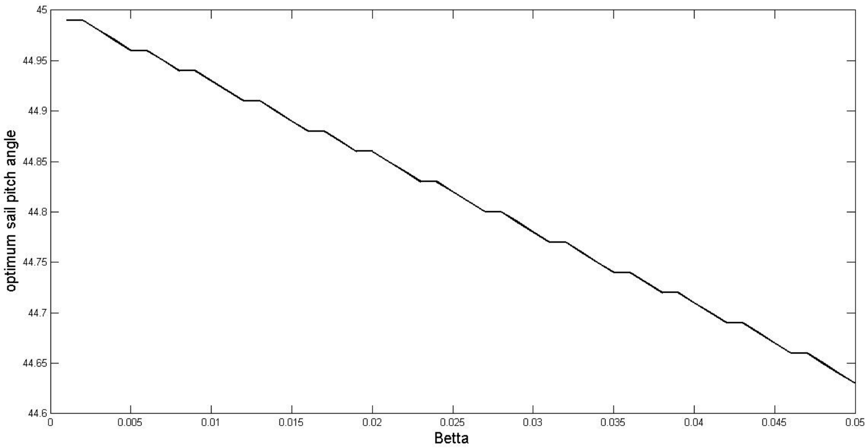

Figure 12.Optimum pitch angle for a range lightness numbers for a SPT in a Mars mission

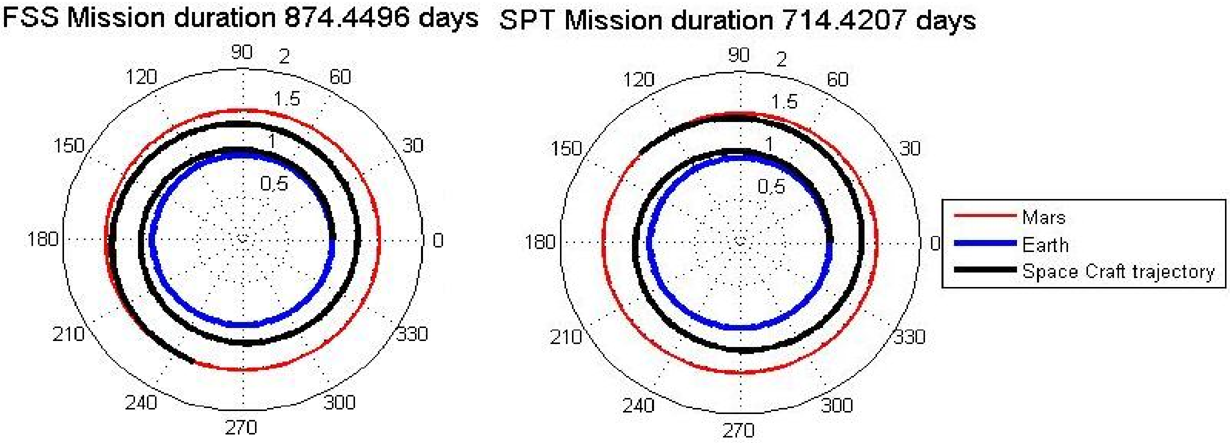

Figure 13. Logarithmic Spiral Trajectories of a FSS and SPT

It is obvious that this design would be preferable given the fact that it is simpler and therefore easier to implement in comparison to the second design. However the minimum sail pitch angle should not exceed the optimum angle so that the spacecraft can achieve a minimum transfer time. It can be seen that the transfer time equation is different for a Solar Photon Thruster compared to a Flat Solar Sail. Figure 13 indicates the Logarithmic Spiral Trajectory of a spacecraft with a sail lightness number of 0.05 for a Mars mission utilizing FSS (left) and SPT (right). Solar Photon

Thruster has shorter transfer time in comparison to a same sized Flat Solar Sail which indicates the significant improvement on SPT. Figure 14 indicates the transfer time difference of these two technologies for a range of sail lightness numbers for a Mars mission. It should be noted that for each sail lightness number, the optimum sail pitch angle has been considered in order to optimize the trajectory.

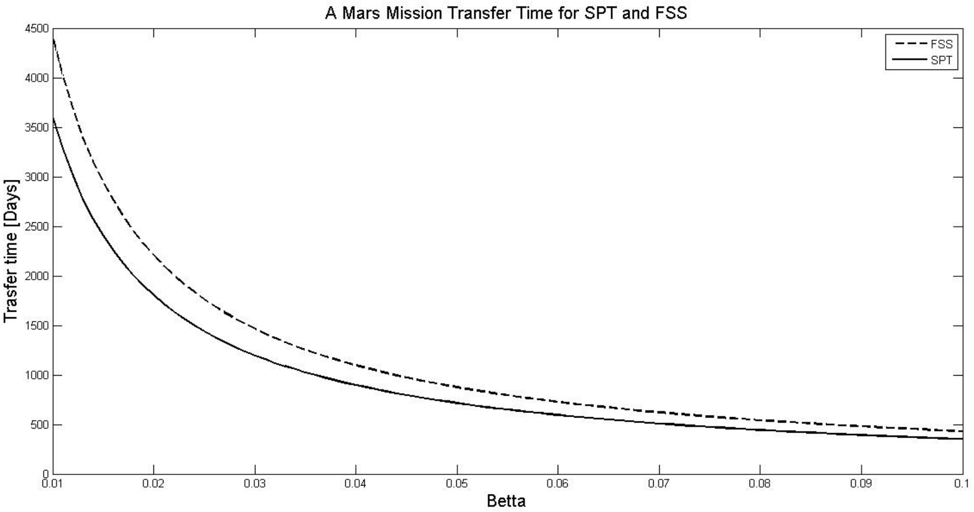

Figure 14. Transfer time difference of these two technologies for a range of sail lightness numbers

Conclusions

In order to investigate the dynamics of Solar Photon Thrusters, two cases were introduced and the equations of radiation pressure force acting on them were derived and their limits and drawbacks were discussed. It was concluded that the thrust generated by both SPT's has the same magnitude and opposite direction given the assumption that no radiation pressure is lost due to lack of reflectivity. Then the dynamics of a Solar Photon Thruster has been investigated in comparison to a conventional Flat Solar Sail for a Logarithmic Spiral Trajectory. According to the results it can be concluded that Solar Photon Thrusters can generate more thrust and leads to shorter transfer time. Moreover since the thrust is generated by a smaller surface, altitude control and dynamics of a SPT would be simpler compared to a FSS where the entire large reflecting surface is responsible for thrust. On the other hand, a Flat Solar Sail has the advantage of simplicity given the fact that it only consists of one flat surface which could be much easier to deploy and geometrically insured in space than a more elaborate parabolic surface. Another common problem with SPT is the elevated sail temperature on the collector's surface due to absorption of concentrated radiation. This is related to the fact that in reality a reflecting surface is not a perfect reflector and a percentage of solar photons are absorbed by the sail.

Further Work

Further work on this work will include a study on the progress and the development of three modes of solar energy based space propulsion: Solar Sail, Magsail and E-sail in selected versions. The first uses solar radiation pressure, while the two latter ones use solar wind. The comparative study may also look into the technology readiness level and complexities related to stowage, deployment and support of the sail structure by utilizing the state of the art for a relatively small spacecraft, and the design a small-scale solar sail deployment and propulsion experiment in low earth orbit.

Nomenclature

F_r = Solar Radiation Force Acting on the Solar Sail
$F_{r'}$ = Solar Radiation Force Reflecting from the Solar Sail
P = Solar Radiation Pressure
A = Sail Area
n = Normal Unit Vector to the Sail Surface
e_r = Unit Vector along the Sunline
$e_{r'}$ = Unit Vector along the Sunline Reflection
F_{SRP} = Total Solar Radiation Force Acting on the Solar Sail
P_{01AU} = Solar Radiation Pressure at 1 AU Distance from the Sun
P_{eff} = Solar Radiation Pressure for a Perfectly Reflecting Solar Sail at 1 AU Distance from the Sun
α = Sail Pitch Angle
δ = Sail Clock Angle
r = Spacecraft's Radial Distance to Sun
a_{SRP} = Total Solar Radiation Acceleration
G = Newton's Gravitational Constant
β = Sail Lightness Number
$M_{\odot}$ = Sun's Mass
f_r = Resultant Force Acting on the Solar Photon Thruster
f_1 = Reaction Force Acting on the Solar Photon Thruster
f_i = Incident Force Acting on the Solar Photon Thruster
u_i = Unit Vector Directed Along the Sunline
u_r = Unit Vector Directed Along the Path of Photons Leaving the Solar Photon Thruster
f = Total Solar Radiation Force Acting on the Solar Photon Thruster
μ = Solar Gravitational Constant
m = Spacecraft's Mass
$\hat{r}$ =Solar Sail Radial Component
$\hat{\theta}$ =Solar Sail Transverse Component
$\hat{\phi}$ = Solar Sail Normal Component
γ = Sail Spiral Angle

v_θ = Transverse Component of Solar Sail Velocity

v_r = Radial Component of Solar Sail Velocity

$v_{(r)}$ = Magnitude of Solar Sail Velocity

$t - t_0$ = Transfer Time

r_0 = Initial Distance From Sun

a = Parameter of Paraboloid

$f_{prabola}$ = Parabola's Focal Distance

R = Radius of Parabola

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