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ACCURACY OF DETERMINING THE ABERRATED COORDINATES OF A RELATIVISTIC INTERSTELLAR SPACECRAFT USING QUASARS' REFERENCE FRAME

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Previous studies showed that the position and velocity of a relativistic interstellar spacecraft can be determined by means of automatic measurements on-board the spaceship of the aberrated angular distances between three quasars, at least. In this paper we report the results of a simulation finalized to optimize the determination of the aberrated coordinates of a relativistic spacecraft during an interstellar space mission. Indeed, it has been shown that the uncertainties of measurements in navigation control can be minimized selecting the set of quasars, suggesting that the quasars used as an inertial reference frame could be changed during an interstellar voyage to maintain the spaceship's trajectory towards its target. In particular, the algorithm used in this study has shown that the accuracy of determining the aberrated apical coordinates of a spacecraft increases significantly ($p < 0.01$) using quasars as an inertial reference frame with aberrated apical latitudes lower than 50°. This result suggests that one or more normal-sized telescopes aboard the spacecraft can carry out feasible maneuvers along the direction of motion to measure velocity and aberrated coordinates of the spacecraft using quasars within a cone with the axis in the direction of motion of the spaceship and an angular aperture of 50°. In addition, it has been demonstrated that assuming that quasars' angular distances can be measured on-board the spacecraft with reasonable accuracies close to 1 mAs, the aberrated apical latitudes of the spacecraft can be determined with relative errors ranging from 10^{-7} to 10^{-9} using quasars' aberrated apical latitude $\theta \leq 50°$.

INTRODUCTION

A representative characteristic of the history of our civilization is the natural tendency of extending the limits of human exploration. This may be considered a primary reason of the exploration of interstellar spaces. Another important reason of space exploration could be represented also by the exponential increase in energy requirements by mankind and by pollution's problem. However, the main aim of the exploration of interstellar spaces is represented by the research of extraterrestrial life. To this aim, several projects to plan a automated spacecraft throwing towards the nearest interstellar systems have been proposed up to now.

The Project Orion proposed a mission towards the star closest to Earth, Alpha Centauri, using nuclear pulse propulsion system, a mission which would take about 140 years[1]. The Project

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Daedalus followed the guidelines that the spacecraft could be designed to allow for a variety of target stars, reaching its destination within a human lifetime, using electron driven D/He[3] fusion reactions, to accelerate the spaceship up to 12% of the velocity of light[2]. The Project Icarus has been recently proposed to revise some aspects of the original Project Daedalus, as the choice of fuel to be used as a propellant[3]. Further possible techniques for propulsion of an interstellar spacecraft have been proposed up to now[4,5,6]. Nevertheless, an upper limit to the velocity of an interstellar spacecraft exists, because hypothesizing a velocity comparable to the light velocity, the spaceship will get a weight more than 2000 ton. Hence, the value of velocity of an interstellar spaceship cannot exceed 0.3c for an interstellar voyage and it is expected to last about 30 years, at least. Such long time forces us to plan navigation and guidance of the spacecraft by means of automated control on-board the spacecraft. Indeed, sending a signal from a spaceship to Earth at a distance of several light years would ask for an extremely long time, as the signal would travel at the velocity of light, making out of the question any Earth-side control of an interstellar mission.

Otherwise, mankind's exploration of space have been characterized by the extraordinary achievements of both robotic and manned space missions. The success of robotic space missions was due to the development of automated space navigation systems, that have enabled the determination of the spacecraft's position and velocity, providing accuracies for traversing interplanetary distances and obtaining precise landings on the surface of the moon and of some planet of the solar system. The required position and velocity of a space mission to support trajectory corrections can be obtained by the current and predicted values of the spacecraft's position and velocity, provided by ground and on-board guidance and control systems.

In contrast, hypothesizing an interstellar voyage, no navigation and guidance control can be carried out by control systems on Earth because the very long distances between Earth and the stars. An interstellar spacecraft have to check automatically its trajectory calculating direction and modulus of its velocity by means of automatic measurements on-board. To this aim, a celestial reference frame is needed so that a fixed coordinate system can provide an instantaneous determination of the spacecraft's position with respect to the celestial reference frame. Hence, the spacecraft's trajectory can be compared with knowledge of the destination stellar object and maneuver control can be applied, determining velocity changes to rectify the spaceship's trajectory.

Previous studies showed that a celestial reference frame constituted by three quasars, at least, can be successfully used to determine the position and velocity of an interstellar spacecraft[7]. Indeed, quasars can be considered a reliable inertial reference frame for an interstellar voyage because they are point-like stellar objects and their proper motion can be neglected due to their extremely long distance[8].

In this paper, it has been shown that the accuracy of determining the aberrated coordinates of an interstellar spacecraft can be improved using a set of quasars whose aberrated apical latitudes are within a cone with the axis in the direction of motion of the spaceship and an angular aperture of 50°.

THE INERTIAL REFERENCE FRAME TO BE USED FOR AN INTERSTELLAR SPACE MISSION

Previous space missions within the Solar Systems have been carried out up to now in a space reference frame associated with the planetary ephemeris represented by a solar system barycentric frame aligned with the planetary ephemeris. In previous space missions, space radio tracking have been performed by means of Doppler and range systems and Very Long Baseline Interferometry (VLBI), so that accurate information regarding corrections to be carried out to the spacecraft's trajectory has been obtained.

Otherwise, automatic measurements on-board an interstellar spacecraft should be carried out to check the prefixed trajectory, comparing computed values of position and velocity with expected values of position and velocity so that the spaceship's trajectory can be automatically rectified towards its target. To this aim, the primary step to plan an interstellar space mission is the choice of a reliable inertial reference frame, using computing and motion sensors on-board, for tracking spacecraft's position, orientation, and velocity to support trajectory corrections.

The discovery of radio pulsars led to the idea of using pulsar timing observations for interstellar navigation[9]. Indeed, pulsars are rotating neutron stars that emit beams of electromagnetic radiation and they are bright enough to be used in a space mission. Nevertheless, some limitations reduce their effectiveness in navigation and guidance of interstellar space missions. Indeed, neighboring celestial objects are broadband radio sources that can obscure weak pulsar signals[10,11]. Furthermore, propagation of radio signals are in phase lags of variable and unpredictable duration so that they set the limitation on accuracy[10]. The most relevant limitation is that at radio frequencies that pulsars emit, radio-based systems on-board would require too large antennas impracticable for a spacecraft[12]. Finally, optical observations of pulsars during interstellar navigation would be impractical because of the small number of detectable optical pulsars[13].

X-ray pulsars were recently considered to overhead these limitations. Indeed, a X-ray telescope of normal-size dimension can be required to detect X-ray pulsars. The basic concept of interstellar space missions using X-ray pulsars was recently described[10,11,14]. Nevertheless, other limitations have to be considered. First, long-term observations of X-ray pulsars highlight irregularities in the pulse rate[15]. Second, irregularities in the spacecraft's clock could cause an error in measurement of time of arrivals of pulsars' beam. Third, the pulse shape may differ between the X-ray and the radio wave bands producing an offset between the time of arrivals measured using the different bands[16]. Finally, the pulsar timing ephemeris obtained from long-term ground-based radio observations may be not reliable because pulsars' proper motion cannot be negligible and reducing uncertainties that arise from pulsar position errors is critical[17].

Otherwise, the position of quasi-stellar objects (quasars) can be considered stationary in the sky because their large distance from the observer, deduced by very high redshift values. Indeed, spectra of the most numerous quasars can be explained only by a cosmological redshift due to the expansion of the Universe. The accretion of material on a central, massive black hole can explain the observed high quasar energy fluxes. Hence, quasars can be considered a reliable inertial reference frame because their proper motion can be neglected due to their extreme distance, bright and point-like appearance. As regards this topic, the International Celestial Reference Frame (ICRF) was proposed and it represents a catalogue of extragalactic radio sources observed with VLBI, the majority of them are quasars and are distributed around the sky[8,18]. The ICRF was successively developed using an extended list of sources that was adopted by the International Astronomical Union in 2009 for a second realization of a new catalogue named ICRF2, which provides absolute coordinates for 3,414 sources with errors within 0.1 mAs (milliarcoseconds) and the orientation of the axes that can be considered fixed within 0.01 mAs [19,20].

Proper motions of sources could be taken into account to improve the reliability of ICRF2. The dominant proper motion of the major parts of sources is related to internal structural changes that can produce apparent motions several hundred µAs/yr (microarcoseconds per year), that is an order of magnitude larger than proper motions due to the secular aberration drift. However, proper motion due to internal structural changes was detected to be relevant only for unstable sources[18,19,21]. A selection of stable sources could be made for a realization of a catalogue of quasars to be used for interstellar space missions. Instead, secular aberration drift is an apparent change in the velocity of distant objects caused by the acceleration of the Solar System barycenter directed towards the Galactic center. This effect may cause apparent proper motion of all quasars by an estimated average value 4-6 µAs/yr and direction towards the points with equatorial coordinates $\alpha = 266°$ and $\delta = -29°$[22,23]. Also this error may be considered negligible, but it could be taken into account for accurate planning of interstellar space mission.

Hence, quasars can represent a reliable inertial reference frame to be used for interstellar space missions, because their proper motions can be neglected. In the optical domain, the Hipparcos catalog is currently used for optical astrometry, due to the launch in 1989 of the ESA space-astrometry satellite Hipparcos, which was aligned to the ICRF to within 0.6 mAs for the orientation at 1991.25[24,25]. Nevertheless, other ambitious space-astrometry projects will provide astrometry measurements in the optical domain. The ESA Gaia mission, which will survey about 10^9 stellar objects brighter than 20 magnitude (mag), with expected accuracies in the 7-25 µAs range down to 15 mag and sub-mAs accuracies at the limit 20 mag. The observations of about 500,000 quasars will provide the Gaia extragalactic reference frame (GCRF), a kinematically non-rotating system close to 0.3 µAs/yr and a positional precision reaching 50 µAs[26]. Of these, only the quasars with the most accurate positions with magnitude lesser than 18 will be used to define a new celestial reference frame in the optical domain, the Large Quasar Reference Frame (LQRF) [27]. The final catalogue is expected around 2021, but with intermediate data that are expected to be available by 2015. An accurate alignment between the two celestial reference frames, the LQRF and the ICRF, can be carried out using only 10% of the current ICRF sources for the alignment with the future Gaia frame[28], but further multi-step VLBI observational projects have been planned to observe new VLBI sources suitable for the alignment with the future Gaia frame[29].

Another space-astrometry project is the Space Interferometry Mission PlanetQuest Light (SIM-Lite), which consists of an optical interferometer system with a baseline of 6 meters and a 30 cm guide telescope that would search 65 nearby stars for planets of masses down to one Earth mass, achieving 8 µAs accuracy on the 19th magnitude objects and 4 µAs for objects up to 14 mag, that would constitute a new astrometric grid[30].

The use of the apical system for an inertial reference frame for interstellar missions

The inertial reference frame for interstellar missions can be represented by a spherical coordinate system where the spacecraft is at rest. This system is termed the “apical system” and is represented in Figure 1, where the origin of the system represents the position of the spaceship, OV is the direction of the motion, P is the North equatorial pole and Q is a quasar. The *apical latitude* θ is measured from the direction of the spacecraft’s velocity OV to the quasar Q. The *apical longitude l* is measured from the plane that contains the direction of velocity and the direction of the equatorial pole to the plane containing the same vector of velocity and the quasar (see Figure 1).

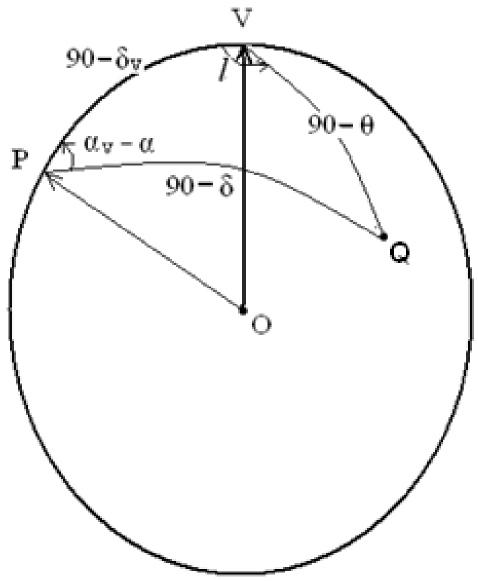

Figure 1. The apical latitude θ and the apical longitude l of a quasar in the apical system.

The apical coordinates of a quasar can be related to its astronomical coordinates the *right ascension α* and the *declination δ* (that are known from the ICRF2 catalogue) by means of some equations derived from spherical astronomy[31,32].

$$\sin\theta = \sin\delta_v \sin\delta + \cos\delta_v \cos\delta \cos(\alpha_v - \alpha) \tag{1}$$

$$\cos\theta = \frac{\cos\delta \sin(\alpha_v - \alpha)}{\sin l} \tag{2}$$

$$\cos l = \frac{\sin\delta \cos\delta_v - \cos\delta \cos\delta_v \cos(\alpha_v - \alpha)}{\cos\theta} \tag{3}$$

where α_v and δ_v are the stationary coordinates of the vector velocity of the spacecraft that have to be recomputed during the voyage.

The spaceship should determine its position by means of the apical coordinates of a number of quasars. It was shown that spacecraft's position and velocity can be determined using only three quasars by means of automatic measurements on-board the spacecraft of the angular distances ψ_i between the quasars[7].

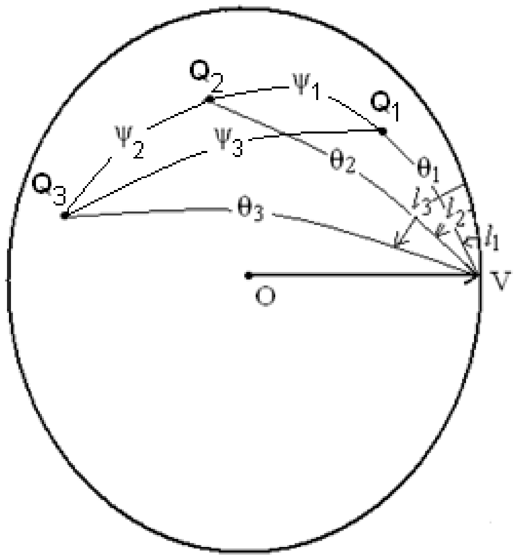

Figure 2. The apical coordinates θ_i and l_i (i=1,2,3) of three quasars in the apical system (where the spacecraft is at rest).

Indeed, the angular distances ψ_1, ψ_2, ψ_3 between three quasars named Q_1, Q_2, Q_3, pointed out in Figure 2, can be related to their apical coordinates applying the II Gauss formula to the spherical triangles represented in Figure 2:

$$\cos\psi_1 = \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2\cos(l_2 - l_1) \tag{4}$$

$$\cos\psi_2 = \cos\theta_3\cos\theta_2 + \sin\theta_3\sin\theta_2\cos(l_3 - l_2) \tag{5}$$

$$\cos\psi_3 = \cos\theta_3\cos\theta_1 + \sin\theta_3\sin\theta_1\cos(l_3 - l_1) \tag{6}$$

Measurements on-board the spaceship of the angular distances ψ_1, ψ_2, ψ_3 could be used to obtain the apical coordinates, determining the position and velocity of a spacecraft.

THE RELATIVISTIC ABERRATION AND THE ABERRATED APICAL COORDINATES

The most relevant effect to be taken into account at relativistic velocities is represented by the change in direction of a stellar object because the point of view of an object from a moving observer depends on its velocity and this change is not negligible if the velocity is comparable to the velocity of light. Indeed, according to the relativistic aberration, during the motion of an object its apical coordinates θ_i change into θ_i' as follows[31,32]:

$$\cos\theta' = \frac{\cos\theta + \beta'}{1 + \beta'\cos\theta} \tag{7}$$

$$\sin\theta' = \frac{\sin\theta}{\gamma\,(1 + \beta'\cos\theta)} \tag{8}$$

where $\beta' = {}^{v}/_{c}\,,\ \gamma = {}^{1}\big/_{\sqrt{1-\beta'^2}}$ (9)

(v and c are the velocities of the spacecraft and of light, respectively).

As the velocity of a spacecraft during an interstellar space mission should range from 0.1c to 0.3c, the apical coordinates θ_i and l_i of the spacecraft (i=1,2,3) and the angular distances ψ_i between the quasars should change in the "aberrated coordinates" θ_i' , l_i', ψ_i', respectively. However, the aberrated coordinates of the spaceship can be related to its apical coordinates applying some relations analogue to Equations (4), (5), (6).

In particular, the spacecraft's velocity value V_{j+1} at the time (j+1) and the related aberrated coordinates can be obtained from the spacecraft's velocity V_j and the aberrated coordinates at the time (j) by means of measurements of three quasars at least and some expressions derived from spherical astronomy[7]. The apical and aberrated coordinates of the velocities V_1 (j=1) and V_2 are pointed out in Figure 3. The vectors V_1 and V_2 may be represented in the apical system by their apical coordinates [($\theta_{A,1}$, $l_{A,1}$), ($\theta_{B,1}$, $l_{B,1}$), ($\theta_{C,1}$, $l_{C,1}$)] and [($\theta_{A,2}$, $l_{A,2}$), ($\theta_{B,2}$, $l_{B,2}$), ($\theta_{C,2}$, $l_{C,2}$)], respectively.

Nevertheless, only measurements of the aberrated angular distances ψ_i'' (i = 1,2,3) between the quasars can be carried out aboard the spacecraft. Applying the II Gauss' formulae to the spherical triangles ABV_2 and $A''B''V_2$, BCV_2 and $B''C''V_2$, CAV_2 and $C''A''V_2$, the following relations can be obtained:

$$\cos\psi_1'' = \cos\theta_{A_2}''\cos\theta_{B_2}'' + \sin\theta_{A_2}''\sin\theta_{B_2}''\cos E_{2,1}'' \tag{10}$$

$$\cos\psi_2'' = \cos\theta_{B_2}''\cos\theta_{C_2}'' + \sin\theta_{B_2}''\sin\theta_{C_2}''\cos E_{2,2}'' \tag{11}$$

$$\cos\psi_3'' = \cos\theta_{A_2}''\cos\theta_{C_2}'' + \sin\theta_{A_2}''\sin\theta_{C_2}''\cos E_{2,3}'' \tag{12}$$

Assuming that the dihedral angles $E_{i,j}$ (i, j = 1, 2) between two quasars do not change because their large distances from the observer (i.e., $\cos E_{2,i} = \cos E_{2,i}''$, i=1,2, 3) we can relate Equations (10), (11) and (12) to analogue expressions where the not-aberrated coordinates appear:

$$\frac{\cos\psi_1'' - \cos\theta_{A_2}''\cos\theta_{B_2}''}{\sin\theta_{A_2}''\sin\theta_{B_2}''} = \frac{\cos\psi_1 - \cos\theta_{A_2}\cos\theta_{B_2}}{\sin\theta_{A_2}\sin\theta_{B_2}} \tag{13}$$

$$\frac{\cos\psi_2'' - \cos\theta_{B_2}''\cos\theta_{C_2}''}{\sin\theta_{B_2}''\sin\theta_{C_2}''} = \frac{\cos\psi_2 - \cos\theta_{B_2}\cos\theta_{C_2}}{\sin\theta_{B_2}\sin\theta_{C_2}} \tag{14}$$

$$\frac{\cos\psi_3'' - \cos\theta_{A_2}'' \cos\theta_{C_2}''}{\sin\theta_{A_2}'' \sin\theta_{C_2}''} = \frac{\cos\psi_3 - \cos\theta_{C_2} \cos\theta_{A_2}}{\sin\theta_{C_2} \sin\theta_{A_2}} \tag{15}$$

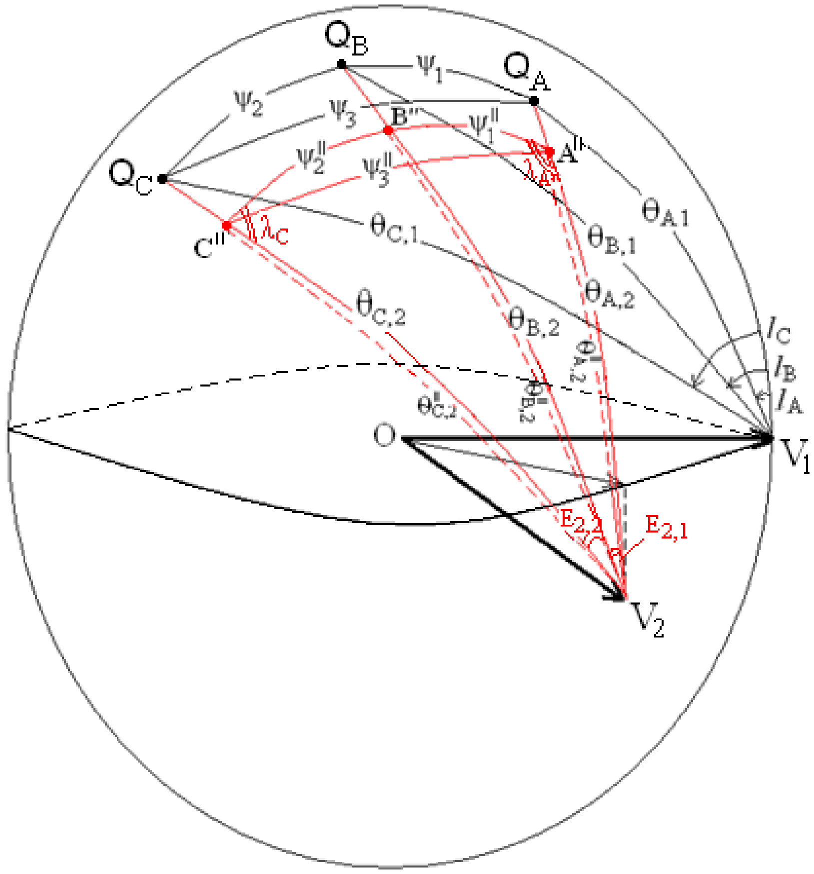

Figure 3. The apical coordinates of a spacecraft at two velocities V_1 ($\theta_{i,1}$; i = A, B, C) and V_2 ($\theta_{i,2}$; i = A, B, C). The aberrated angular distances ψ_1'', ψ_2'', ψ_3'' between three quasars (Q_A, Q_B, Q_C) and the aberrated coordinates $\theta_{A,2}''$, $\theta_{B,2}''$, $\theta_{C,2}''$ referred to the velocity V_2 are pointed out.

Applying the formulation of the relativistic aberration to the aberrated coordinates θ_{A2}'', θ_{B2}'', θ_{C2}'', the apical coordinates θ_{A2}, θ_{B2}, θ_{C2}, can be obtained, respectively, by means of the following equations, derived from Equations (7) and (8):

$$\sin\theta_{A_2}'' = \frac{\sin\theta_{A_2}}{\gamma\,(1+\beta''\cos\theta_{A_2})} \qquad \cos\theta_{A_2}'' = \frac{\cos\theta_{A_2}+\beta''}{1+\beta''\cos\theta_{A_2}} \tag{16}$$

$$\sin\theta''_{B_2} = \frac{\sin\theta_{B_2}}{\gamma\,(1+\beta''\cos\theta_{B_2})} \qquad \cos\theta''_{B_2} = \frac{cos\theta_{B_2} + \beta''}{1+\beta''\cos\theta_{B_2}} \tag{17}$$

$$\sin\theta''_{C_2} = \frac{\sin\theta_{C_2}}{\gamma\,(1+\beta''\cos\theta_{C_2})} \qquad \cos\theta''_{C_2} = \frac{cos\theta_{C_2} + \beta''}{1+\beta''\cos\theta_{C_2}} \tag{18}$$

The set of Equations (16), (17) and (18) allows to express the set of Equations (13), (14) and (15) as a function of the apical coordinates of the spaceship θ_{A2}, θ_{B2}, θ_{C2}, and of the aberrated modulus of the velocity β" (in unit of c). Applying again the II Gauss' equation and the sinus theorem to the spherical triangles B"C"V_2 and A"B"V_2 we obtain:

$$\cos\theta''_{B_2} = \cos\psi''_2\cos\theta''_{C_2} + \sin\psi''_2\sin\theta''_{C_2}\cos\lambda_C \tag{19}$$

$$\cos\theta''_{B_2} = cos\,\psi''_1\cos\theta''_{A_2} + \sin\psi''_1\sin\theta''_{A_2}\cos\lambda_A \tag{20}$$

$$\sin\lambda_A = \frac{\sin E_{2,1}\sin\theta''_{B_2}}{\sin\psi''_1} \qquad \sin\lambda_C = \frac{\sin E_{2,2}\sin\theta''_{B_2}}{\sin\psi''_2} \tag{21}$$

The set of equations above reported allows to obtain the apical coordinates θ_{A2}, θ_{B2}, θ_{C2} (at time j=2) as a function of the apical coordinates θ_{A1}, θ_{B1}, θ_{C1} (at time j=1) and of the measured quasars' angular distances ψ_1", ψ_2", ψ_3". Hence, this algorithm can provide the direction and modulus of velocity that have to be used to rectify the trajectory of the spacecraft towards its target.

OPTIMIZING THE ACCURACY OF DETERMINING THE SPACECRAFT'S ABERRATED VELOCITY AND ABERRATED APICAL COORDINATES

The solution of the set of equations above reported can provide the exact values of the aberrated coordinates of a spaceship that are required for navigation and guidance during an interstellar space mission. Nevertheless, previous results showed that the accuracy of determining the aberrated coordinates depends on the apical coordinates of the quasars that are used. Indeed, it was shown that applying the algorithm to typical apical latitudes around 90° of three quasars, the best accuracy in the determination of apical coordinates can be obtained using quasars with apical longitudinal angular distances around 90° and 180°[7].

In this simulation study, instead of simulating a variation of quasars' apical longitudes, quasars' apical latitudes ranging from 5° to 150° have been used, using also the value of the spaceship's aberrated velocity modulus β = 0.1 (in unit of c) and the values θ = *l* = 0.5° that represent a change in direction of the motion of the spacecraft. The results of this simulation study have been reported in the Tables 1-16 (in the Appendix section) where quasars' aberrated angular distances, spacecraft's aberrated apical latitudes and aberrated velocity modulus values have been reported as a function of typical quasars' apical latitudes and longitudes. Looking at the results reported in the Tables 1-16, it appears that the aberrated velocity modulus values obtained from the input value β" = 0.1 are all very close to this value, confirming the reliability of the algorithm. In addition, the variations of the aberrated velocity modulus resulted to decrease with decreasing quasars' apical latitude, getting the best accuracy using quasars' aberrated apical latitudes lower than 50°. The average of the values of spacecraft's aberrated velocity modulus obtained at typical apical longitudes was plotted as a function of quasars' aberrated apical latitude and a sigmoidal fit was used with upper and lower asymptotes equal to 7.08x10^{-7} and 4.88x10^{-8}, respectively (see Figure 4). A statistical analysis was carried out applying Student's t-test for comparison between two groups, the group of quasars whose aberrated apical latitude are $\theta \leq 50°$ and the group of quasars with $\theta > 90°$, with $p < 0.05$ considered significant. The t-test provided the result that the group of quasars with $\theta \leq 50°$ is significantly different in comparison to the other group ($p < 0.01$), showing that the accuracy of determining the spacecraft's aberrated velocity β'' increases using quasars' aberrated apical latitude $\theta \leq 50°$. Hence, the result of this simulation study has confirmed that a celestial reference frame consisting of three quasars can be successful used for interstellar navigation regardless of their apical coordinates, but the best accuracy in the determination of spacecraft's apical coordinates can be obtained using quasars whose aberrated apical latitudes are lower than 50°.

Furthermore, as the aberrated coordinates of an interstellar spaceship are related to its aberrated velocity by means of Equation (16), (17) and (18), the minimization of the uncertainty in the determination of the spacecraft's aberrated velocity provides an increase in accuracy of determining the aberrated apical coordinates using quasars in that range of aberrated apical latitudes.

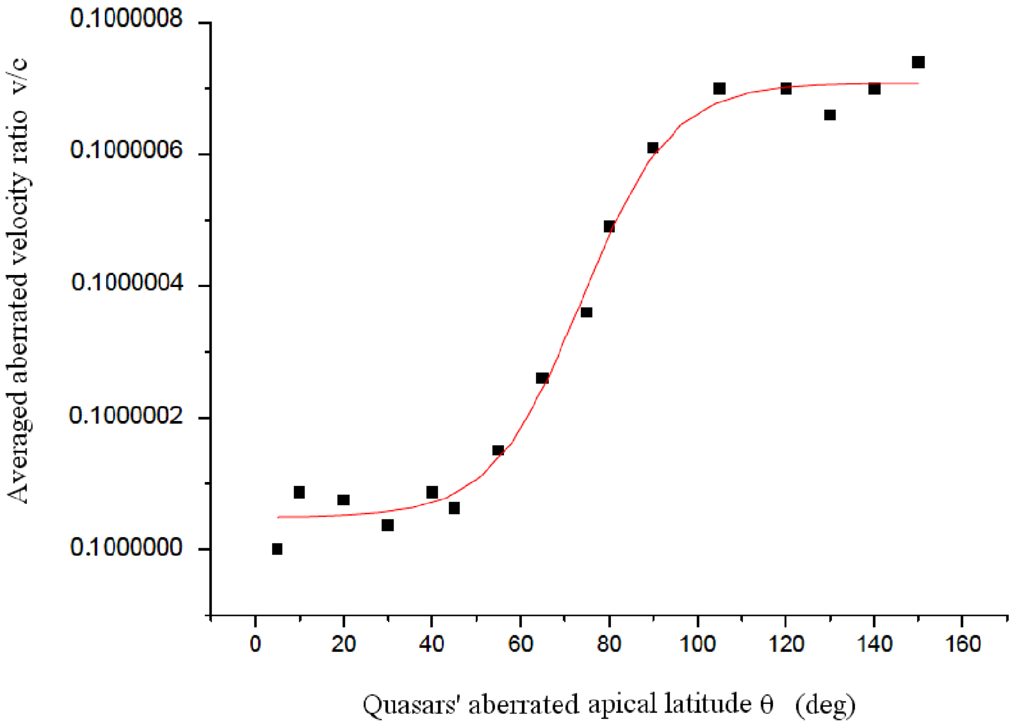

Figure 4. The averaged aberrated velocity β'' (in unit of c) of a relativistic interstellar spacecraft (with $\beta = 0.1$) as a function of quasars' aberrated apical latitude.

This result suggests that one or more normal-sized telescopes aboard the spacecraft can carry out feasible maneuvers along the direction of motion of the spaceship for the automatic measurements of quasars' angular distances because quasars to be used are within a cone with the axis in the direction of motion of the spaceship and an angular aperture of 50° (see Fig. 5). The large number of quasars whose coordinates have been measured in radio and optical domains[18,19,20,24,25] and quasars' uniform distribution over the sky[33,34], can ensure the feasibility of this design.

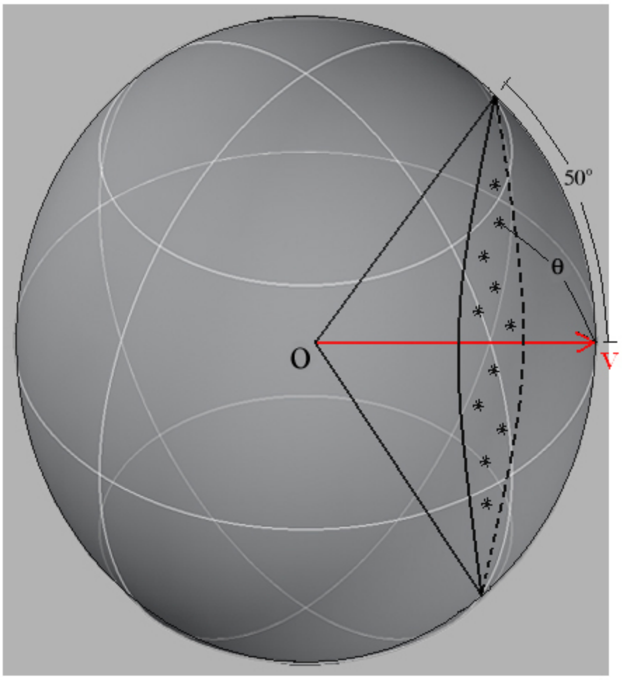

Figure 5. The accuracy of determining the spacecraft's aberrated velocity and apical coordinates increases using quasars' aberrated apical latitude within a cone with the axis in the direction of motion of the spaceship and an angular aperture of 50°.

Furthermore, the limit of accuracy of determining the aberrated coordinates and velocity of an interstellar spacecraft depends on the technique which can be used aboard the spaceship for measuring angular distances between quasars. As described in the previous sections, a positional precision close to 50 µAs for quasars with magnitude lesser than 18 should be reached by means of Gaia space mission which will define a new celestial reference frame in the optical domain, the LQRF. We have performed a simulation study assuming that angular measurements between quasars can be carried out on-board the spaceship with errors within 1 mAs. It may be considered a reasonable estimate of accuracy of automatic angular measurements aboard an interstellar spacecraft, because it represents a value conservatively much smaller than that will be reached in the future astrometry space missions above mentioned. In addition, coordinates evolution of "stable" quasars is assumed to be around 0.2 mAs[35], so that this uncertainty cannot influence measurements of quasars' angular distance aboard the spacecraft within the assumed accuracy of 1mAs.

Hence, the uncertainty of spacecraft's aberrated apical latitude can be related to the errors of angular measurements carried out on-board the spaceship by means of the following equation derived from Equation (21):

$$\Delta\theta'' = \left| \frac{(\cos\lambda \sin\psi\, \Delta\lambda + \sin\lambda \cos\psi\, \Delta\psi) \sin E - \cos E \sin\lambda \sin\psi\, \Delta E}{\cos\theta'' \sin E^2} \right| \quad (22)$$

The uncertainty values Δθ" of the aberrated apical latitude have been computed using the values obtained from this simulation study, assuming that a reasonable estimate of uncertainty of measurements on-board the spacecraft is $\Delta\psi = \Delta\lambda = \Delta E = 1$ mAs. The results of the computation have been reported in the last columns of the Tables 1-16.

Looking at the values Δθ" reported in these columns, it appears that the relative error of the aberrated apical latitude $\frac{\Delta\theta''}{\theta''}$ decreases with decreasing the aberrated apical latitude, providing the lowest-order relative error values ranging from 10^{-7} to 10^{-9} using aberrated apical latitudes lesser than 50°. This result is in agreement with the previous result regarding the increase in accuracy of determining the spacecraft's aberrated velocity β" which has been obtained using quasars' aberrated apical latitude $\theta \leq 50°$.

CONCLUSIONS

In this study it has been confirmed that three quasars, at least, can represent an inertial reference frame for interstellar space missions. In particular, an algorithm has been used to carry out a simulation study aimed at increasing the accuracy in the determination of the aberrated coordinates of a relativistic spacecraft during an interstellar space mission. It has been shown that the uncertainties of measurements in navigation control can be minimized selecting the set of quasars. The accuracy of determining the aberrated velocity and the aberrated apical coordinates of a spacecraft resulted increased significantly ($p < 0.01$) using an inertial reference frame formed by quasars with aberrated apical latitudes lower than 50°. This result suggests feasible design techniques for measurements of quasars' aberrated angular distances aboard the spaceship within a cone with the axis in the direction of motion of the spaceship and an angular aperture of 50°.

A further simulation has been performed assuming that measurements of quasars' angular distances can be carried out on-board the spacecraft with accuracy within 1 mAs. The related uncertainties of the aberrated apical latitudes of the spacecraft have been obtained providing small relative errors ranging from 10^{-7} to 10^{-9} using quasars' apical latitudes less than 50°, in agreement with the previous result. However, further corrections can be carried out taking into account corrections for secular aberration drift, for Doppler shift and for the expansion of the Universe.

REFERENCES

[1] F. Dyson, "Project Orion the atomic spaceship 1957-1965, Penguin Books, USA, 2002.

[2] A. Bond et al., "Project Daedalus - the final report on the BIS starship study, JBIS, 1978.

[3] K.F. Long, R.K. Obousy, A.C. Tziolas, A. Mann, R. Osborne, A. Presby, M. Fogg, "PROJECT ICARUS: Son of Daedalus- Flying Closer to Another Star, presented at the British Interplanetary Society Daedalus after 30 years symposium (30th September 2009) – arXiv:1005.3833v1.

[4] R.H. Frisbee, "Advanced Space Propulsion for the 21st Century." *Journal of Propulsion and Power*. Vol. 19, No. 6, 2003.

[5] G. Piefer, Advanced Fusion for Space Application, prepared for Neep 602, 2000.

[6] D.G. Andrews, "Interstellar Transportation using Today's Physics", Joint Propulsion Conference & Exhibit, Huntsville, Alabama, 2003.

[7] E. Calabrò, "Relativistic aberrational interstellar navigation". *Acta Astronautica*. Vol. 69, 2011, pp. 360-364.

[8] C. Ma, M. Feissel, "Definition and Realization of the International Celestial Reference System by VLBI Astrometry of Extragalactic Objects". *IERS Technical Note*. Vol. 23, International Earth Rotation and Reference System Service, 1997, Observatoire de Paris.

[9] D.S. Downs, "Interplanetary Navigation using Pulsating Radio Sources". *NASA Technical Reports* TR N74-34150, October 1974, pp.1-12.

[10] P.S. Ray, K.S. Wood, B.F. Phlips, "Spacecraft Navigation using X-ray Pulsars". *Naval Research Laboratory Review Featured Research*, 2006, pp.95-102.

[11] S.I. Sheikh, A.R. Golshan, D.J. Pines, "Absolute and relative position determination using variable celestial X-ray sources", in 30th Annual AAS Guidance and Control Conference, 2007, pp.855-874.

[12] K. Wallace, "Radio stars, what they are and the prospects for their use in navigational systems". *Journal of Navigation*. Vol.41, 1988, pp. 358-374.

[13] A. Shearer, A. Golden, "Implications of the optical observation of isolated neutron stars". *The Astrophysical Journal*. Vol. 547, 2001, p. 967.

[14] S.I. Sheikh, D.J. Pines, P.S. Ray, K.S. Wood, M.N. Lovellette, M.T. Wolff, "Spacecraft navigation using X-ray pulsars". *Journal of Guidance, Control, and Dynamics*. Vol. 29, 2006, pp.49-63.

[15] G. Hobbs, A. Lyne, M. Kramer, "An analysis of the timing irregularities for 366 pulsars". *Monthly Notices of the Royal Astronomical Society*. Vol. 402, 2010, pp. 1027-1048.

[16] X.P. Deng, G. Hobbs, X.P. You, M.T. Li, M.J. Keith, R.M. Shannon, W. Coles, R.N. Manchester, J.H. Zheng, X.Z. Yu, D. Gao, X. Wu, D. Chen, "Interplanetary spacecraft navigation using pulsars". Advances in Space Research, 2013, preprint arXiv:1307.5375v1 [astro-ph.IM].

[17] N. Ashby, D.A. Howe, "Relativity and Timing in X-ray Pulsar Navigation", Contribution of the U.S. government, 1-4244-0074-0/06, 2006 IEEE.

[18] C. Ma et al., "The International Celestial Reference Frame as Realized by Very Long Baseline Interferometry". The Astronomical Journal. Vol. 116, no.1, 1998, pp.516-546.

[19] Fey, A. L., Ma, C., Arias, E. F., Charlot, P., Feissel-Vernier, M., Gontier, A. M., Jacobs, C. S., Li, J., & MacMillan, D. S., "The Second Extension of the International Celestial Reference Frame: ICRF-EXT.1", *Astronomical Journal*. Vol. 127, 2004, pp.3587-3608.

[20] Ma, C., Arias, E. F., Bianco, G., Boboltz, D. A., Bolotin, S. L., Charlot, P., Engelhardt, G., Fey, A. L., Gaume, R. A., Gontier, A.-M., Heinkelmann, R., Jacobs, C. S., Kurdubov, S., Lambert, S. B., Malkin, Z. M., Nothnagel, A., Petrov, L., Skurikhina, E., Sokolova, J. R., Souchay, J., Sovers, O. J., Tesmer, V., Titov, O. A., Wang, G., Zharov, V. E., Barache, C., Boeckmann, S., Collioud, A., Gipson, J. M., Gordon, D., Lytvyn, S. O., MacMillan, D. S., & Ojha, R., "*The Second Realization of the International Celestial Reference Frame by Very Long Baseline Interferometry*", IERS Technical Note No. 35, 2009, International Earth Rotation and Reference System Service (Frankfurt am Main: Verlag des Bundsesamts fur Kartographie und Geodasie).

[21] A.L. Fey, T.M. Eubanks, K.A. Kingham, A.J. Vol. 114, 1997, p. 2284.

[22] O. Titov, S.B. Lambert, A.-M. Gontier, "VLBI measurement of the secular aberration drift", *Astronomy and Astrophysics*, manuscript no. 15718, April 2011.

[23] O. Titov, "Secular aberration drift and IAU definition of International Celestial Reference System", MNRAS Lett. Vol. 407, 2010, pp. L46-L48.

[24] M.A.C. Perryman, "The Hipparcos and Tycho Catalogues", ESA SP-1200, 1997.

[25] J. Kovalevsky, L. Lindegren, M.A.C. Perryman, P.D. Hemenway, K.J. Johnston, V.S. Kislyuk, J.-F. Lestrade, L.V. Morrison, I. Platais, S. Röser et al., "The Hipparcos Catalogue as a realisation of the extragalactic reference system", A&A. Vol. 323, 1997, pp. 620-633.

[26] L. Lindegren, C. Babusiaux, C. Bailer-Jones, U. Bastian, A.G.A. Brown, M. Cropper et al., “The Gaia mission: science, organization and present status”, Proceedings IAU Symposium No. 248, 2007.

[27] A.H. Andrei, J. Souchay, N. Zacharias, R.L. Smart, R. Vieira Martins, D.N. da Silva Neto et al., “The large quasar reference frame (LQRF)”, A & A. Vol. 505, 2009, pp. 385-404.

[28] G. Bourda, P. Charlot, J.-F. Le Campion, “Astrometric suitability of optically-bright ICRF sources for the alignment with the future Gaia celestial reference frame”, A & A. Vol. 490, 2008, pp. 403-408.

[29] G. Bourda, P. Charlot, “Celestial Reference Frames in the Gaia Era”, Société Francaise d’Astronomie et d’Astrophysique (SF2A), 2009, M. Heydari-Malayeri, C. Reylé and R. Samadi (eds).

[30] R. Goulliod, J.H. Catanzarite, F.G. Dekens, M. Shao, J.C. Marr IV, “Overview of the SIM PlanetQuest Light (SIM-Lite) mission concept”, Jet Propulsion Laboratory, California Institute of Technology, 4800 Oak Grove Dr, Pasadena, USA.

[31] R.M. Green, Spherical Astronomy, Cambridge University Press, 1985.

[32] M. Harwit, Astrophysical Concepts, Springer-Verlag, 1988.

[33] A. Webster, “The clustering of radio sources – III. The Parkes 2700-MHz and Bologna B2 surveys”, Mon. Not. R. Astr. Soc. Vol. 179, 1977, pp. 511-515.

[34] C. Fanti, C. Lari, M.C. Olori, “Isotropy of radio sources in the B2 catalogue”, A. & A. Vol. 67, 1978, pp. 175-184.

[35] S. Lambert, “Time stability of the ICRF2 axes”, A. & A. Vol. 553, 2007, A122, 7 pages.

APPENDIX: SPACECRAFT'S ABERRATED VELOCITY AND APICAL LATITUDES AS A FUNCTION OF QUASARS' ABERRATED APICAL LATITUDE

Table1. Quasars' aberrated angular distances, spacecraft's aberrated apical latitudes, uncertainties and aberrated velocity as a function of quasars' apical latitudes $\theta_A = \theta_B = \theta_C = 5°$.

Apical latitudes (deg) $\theta_A = 5$; $\theta_B = 5$; $\theta_C = 5$				
Apical longitude values (deg) l_A l_B l_C	Quasars' aberrated angular distances (rad) Ψ_1'' Ψ_2'' Ψ_3''	Aberrated apical latitudes (rad) θ_{A2}'' θ_{B2}'' θ_{C2}''	Aberrated velocity $\beta'' = v''/c$	Uncertainty of aberrated apical latitudes $\Delta\theta''$ (rad)
0	6.036476×10^{-2}	7.104854×10^{-2}		
45	6.036476×10^{-2}	7.352113×10^{-2}	.1	1.58638×10^{-9}
90	.1115786	7.104854×10^{-2}		
0	.1366982	7.104854×10^{-2}		
120	.1366982	8.311642×10^{-2}	.1	9.794508×10^{-9}
240	.1366908	7.104854×10^{-2}		
0	.1572874	7.104854×10^{-2}		
170	.1400199	8.672082×10^{-2}	.1	7.521397×10^{-8}
45	6.036476×10^{-2}	7.351951×10^{-2}		
45	.157287	7.352113×10^{-2}		
215	.1400161	8.377577×10^{-2}	.1	5.19094×10^{-7}
90	6.036476×10^{-2}	7.104854×10^{-2}		
90	6.03667×10^{-2}	7.926836×10^{-2}		
45	.1115813	7.352033×10^{-2}	.1	1.489584×10^{-9}
135	6.03667×10^{-2}	7.362574×10^{-2}		
135	.1115866	8.466642×10^{-2}		
45	.1458454	7.352113×10^{-2}	.1	4.037413×10^{-9}
180	.0603697	7.940599×10^{-2}		
225	.1458474	8.475729×10^{-2}		
90	.1115829	7.104854×10^{-2}	.1	1.405048×10^{-8}
0	.1458528	7.927288×10^{-2}		
180	.1458503	8.684048×10^{-2}		
45	6.036476×10^{-2}	7.352113×10^{-2}	.1	1.168942×10^{-8}
90	.1115866	7.104854×10^{-2}		

Table 2. Quasars' aberrated angular distances, spacecraft's aberrated apical latitudes, uncertainties and aberrated velocity as a function of quasars' apical latitudes $\theta_A = \theta_B = \theta_C = 10°$.

Apical latitudes (deg) $\theta_A = 10$; $\theta_B = 10$; $\theta_C = 10$				
Apical longitude values (deg) l_A l_B l_C	**Quasars' aberrated angular distances (rad)** Ψ_1'' Ψ_2'' Ψ_3''	**Aberrated apical latitudes (rad)** θ_{A2}'' θ_{B2}'' θ_{C2}''	**Aberrated velocity** $\beta'' = v''/c$	**Uncertainty of aberrated apical latitudes** $\Delta\theta''$ **(rad)**
0 45 90	.1204418 .1204418 .2228695	.1500399 .1524055 .1500403	.1	3.873324×10^{-9}
0 120 240	.2732715 .2732715 .2732428	.1500399 .1619809 .1500399	.1	1.859782×10^{-8}
0 170 45	.3146759 .2799516 .1204418	.1500399 .1657227 .1524067	.1	1.405435×10^{-7}
45 215 90	.3146734 .279938 .1204418	.1524059 .1626599 .1500399	.1	2.326587×10^{-7}
90 45 135	.1204502 .2228789 .1204502	.1580701 .1524059 .1525068	.1000007	3.756036×10^{-9}
135 45 180	.2229005 .2916495 .1204621	.1635804 .1524059 .1582079	.1	7.86745×10^{-9}
225 90 0	.2916580 .2228851 .2916783	.1636748 .1500399 .1580879	.1	2.711515×10^{-8}
180 45 90	.2916696 .1204418 .2229005	.1658484 .1524059 .1500399	.1	2.471196×10^{-8}

Table 3. Quasars' aberrated angular distances, spacecraft's aberrated apical latitudes, uncertainties and aberrated velocity as a function of quasars' apical latitudes $\theta_A = \theta_B = \theta_C = 20°$.

Apical latitudes (deg) $\theta_A = 20$; $\theta_B = 20$; $\theta_C = 20$				
Apical longitude values (deg) l_A l_B l_C	**Quasars' aberrated angular distances** (rad) Ψ_1'' Ψ_2'' Ψ_3''	**Aberrated apical latitudes** (rad) θ_{A2}'' θ_{B2}'' θ_{C2}''	**Aberrated velocity** $\beta'' = v''/c$	**Uncertainty of aberrated apical latitudes** $\Delta\theta''$ (rad)
0 45 90	.2385968 .2385968 .4434312	.3083875 .3107102 .3083873	.1	8.906437×10^{-9}
0 120 240	.5454995 .5454995 .5453854	.3083875 .3203031 .3083873	.1	3.621693×10^{-8}
0 170 45	.6301567 .5591298 .2385968	.3083875 .3241316 .3107106	.1	2.721964×10^{-7}
45 215 90	.6301463 .5590756 .2385968	.3107102 .3209947 .3083875	.1	3.33884×10^{-7}
90 45 135	.2386291 .4434671 .2386296	.3163490 .3107102 .3108094	.1000001	8.764387×10^{-9}
135 45 180	.4435533 .5829566 .2386760	.3219346 .3107102 .3164873	.1	1.532988×10^{-8}
225 90 0	.5829908 .4434926 .5830727	.3220308 .3083875 .3164183	.1	5.315188×10^{-8}
180 45 90	.5830376 .2385965 .4435540	.3242611 .3107100 .3083875	.1000005	5.07998×10^{-8}

Table 4. Quasars' aberrated angular distances, spacecraft's aberrated apical latitudes, uncertainties and aberrated velocity as a function of quasars' apical latitudes $\theta_A = \theta_B = \theta_C = 30°$.

Apical latitudes (deg) $\theta_A = 30$; $\theta_B = 30$; $\theta_C = 30$				
Apical longitude values (deg) l_A l_B l_C	Quasars' aberrated angular distances (rad) Ψ_1'' Ψ_2'' Ψ_3''	Aberrated apical latitudes (rad) θ_{A2}'' θ_{B2}'' θ_{C2}''	Aberrated velocity $\beta'' = v''/c$	Uncertainty of aberrated apical latitudes $\Delta\theta''$ (rad)
0 45 90	.3521147 .3521147 .6590953	.4675786 .4698990 .4675784	.1000003	1.502422×10^{-8}
0 120 240	.8153257 .8153257 .8150669	.4675787 .4795486 .4675787	.1	5.38223×10^{-8}
0 170 45	.9472179 .8364417 .3521149	.4675787 .4834285 .4699020	.1	4.057065×10^{-7}
45 215 90	.9471941 .8363186 .3521149	.4698993 .4802482 .4675791	.1	4.579924×10^{-7}
90 45 135	.3521857 .6591761 .3521866	.4755584 .4698993 .4699984	.1	1.484936×10^{-8}
135 45 180	.6593686 .8733605 .3522882	.4811999 .4698993 .4756976	.1	2.243538×10^{-8}
225 90 0	.8734387 .6592328 .8736262	.4812975 .4675787 .4756282	.1	7.921381×10^{-8}
180 45 90	.8735464 .3521149 .6593703	.4835601 .4698993 .4675786	.1	7.698898×10^{-8}

Table 5. Quasars' aberrated angular distances, spacecraft's aberrated apical latitudes, uncertainties and aberrated velocity as a function of quasars' apical latitudes $\theta_A = \theta_B = \theta_C = 40°$.

Apical latitudes (deg) $\theta_A = 40$; $\theta_B = 40$; $\theta_C = 40$				
Apical longitude values (deg) l_A l_B l_C	**Quasars' aberrated angular distances (rad)** Ψ_1'' Ψ_2'' Ψ_3''	**Aberrated apical latitudes (rad)** θ_{A2}'' θ_{B2}'' θ_{C2}''	**Aberrated velocity** $\beta'' = v''/c$	**Uncertainty of aberrated apical latitudes** $\Delta\theta''$ **(rad)**
0 45 90	.4584757 .4584757 .8665489	.6280316 .6303636 .6280315	.1000001	2.306603×10^{-8}
0 120 240	1.080622 1.080622 1.080156	.6280316 .6400977 .6280323	.1000001	7.157303×10^{-8}
0 170 45	1.266572 1.110074 .4584759	.6280316 .6440277 .6303676	.1	5.437029×10^{-7}
45 215 90	1.266527 1.109851 .4584759	.6303637 .640806 .6280328	.1	5.889684×10^{-7}
90 45 135	.4585961 .8666899 .4585977	.6360657 .6303634 .6304636	.1000002	2.284043×10^{-8}
135 45 180	.867028 1.161710 .4587704	.6417693 .6303636 .6362065	.1	2.902025×10^{-8}
225 90 0	1.161853 .8667899 1.162195	.6418682 .6280316 .6361363	.1	1.05652×10^{-7}
180 45 90	1.162049 .4584756 .8670309	.6441610 .6303634 .6280313	.1000003	1.036194×10^{-7}

Table 6. Quasars' aberrated angular distances, spacecraft's aberrated apical latitudes, uncertainties and aberrated velocity as a function of quasars' apical latitudes $\theta_A = \theta_B = \theta_C = 45°$.

Apical latitudes (deg) $\theta_A = 45$; $\theta_B = 45$; $\theta_C = 45$				
Apical longitude values (deg) l_A l_B l_C	**Quasars' aberrated angular distances (rad)** Ψ_1'' Ψ_2'' Ψ_3''	**Aberrated apical latitudes (rad)** θ_{A2}'' θ_{B2}'' θ_{C2}''	**Aberrated velocity** $\beta'' = v''/c$	**Uncertainty of aberrated apical latitudes** $\Delta\theta''$ **(rad)**
0 45 90	.5081233 .5081233 .9658515	.7088562 .7111977 .7088549	.1000001	2.821275×10^{-8}
0 120 240	1.210536 1.210536 1.209941	.7088562 .7209837 .7088549	.1	8.064029×10^{-8}
0 170 45	1.427298 1.244587 .5081232	.7088562 .7249402 .7111935	.1000001	6.160802×10^{-7}
45 215 90	1.427240 1.244302 .5081233	.7111977 .7216963 .7088572	.1	6.578899×10^{-7}
90 45 135	.5082715 .9660284 .5082732	.7169275 .7111976 .7112963	.1000002	2.795166×10^{-8}
135 45 180	.9664523 1.304455 .5084859	.7226662 .7111977 .7170680	.1	3.206421×10^{-8}
225 90 0	1.304639 .9661539 1.305079	.7227656 .7088564 .7169986	.1	1.192555×10^{-7}
180 45 90	1.304891 .5081232 .9664560	.7250745 .7111977 .7088549	.1000001	1.173458×10^{-7}

Table 7. Quasars' aberrated angular distances, spacecraft's aberrated apical latitudes, uncertainties and aberrated velocity as a function of quasars' apical latitudes $\theta_A = \theta_B = \theta_C = 55°$.

Apical latitudes (deg) $\theta_A = 55$; $\theta_B = 55$; $\theta_C = 55$				
Apical longitude values (deg) l_A l_B l_C	**Quasars' aberrated angular distances (rad)** Ψ_1'' Ψ_2'' Ψ_3''	**Aberrated apical latitudes (rad)** θ_{A2}'' θ_{B2}'' θ_{C2}''	**Aberrated velocity** $\beta'' = v''/c$	**Uncertainty of aberrated apical latitudes** $\Delta\theta''$ **(rad)**
0 45 90	.5985235 .5985235 1.152048	.8719386 .8743041 .8719379	.1	4.217321×10^{-8}
0 120 240	1.461552 1.461552 1.460648	.8719386 .8842108 .871938	.1000001	9.980699×10^{-8}
0 170 45	1.751104 1.505919 .5985236	.8719386 .8882255 .8743089	.1	7.747448×10^{-7}
45 215 90	1.751009 1.505481 .5985233	.8743038 .8849332 .8719363	.1000006	8.090532×10^{-7}
90 45 135	.5987303 1.152306 .5987327	.8801010 .8743043 .8744049	.1	4.180844×10^{-8}
135 45 180	1.152921 1.584652 .5990289	.8859171 .8743041 .8802433	.1000003	3.75483×10^{-8}
225 90 0	1.584939 1.152488 1.585625	.8860181 .8719386 .8801725	.1000002	1.483053×10^{-7}
180 45 90	1.585333 .5985235 1.152927	.8883619 .8743042 .871938	.1	1.467092×10^{-7}

Table 8. Quasars' aberrated angular distances, spacecraft's aberrated apical latitudes, uncertainties and aberrated velocity as a function of quasars' apical latitudes $\theta_A = \theta_B = \theta_C = 65°$.

Apical latitudes (deg) $\theta_A = 65$; $\theta_B = 65$; $\theta_C = 65$				
Apical longitude values (deg) l_A l_B l_C	**Quasars' aberrated angular distances (rad)** Ψ_1'' Ψ_2'' Ψ_3''	**Aberrated apical latitudes (rad)** θ_{A2}'' θ_{B2}'' θ_{C2}''	**Aberrated velocity** $\beta'' = v''/c$	**Uncertainty of aberrated apical latitudes** $\Delta\theta''$ **(rad)**
0 45 90	.6744785 .6744785 1.31547	1.037221 1.039617 1.037223	.1000003	$6.49072\text{x}10^{-8}$
0 120 240	1.693532 1.693532 1.692253	1.037222 1.049665 1.037221	.1	$1.224615\text{x}10^{-7}$
0 170 45	2.078023 1.74993 .674478	1.037221 1.053743 1.039607	.1000006	$9.741514\text{x}10^{-7}$
45 215 90	2.077874 1.749306 .6744784	1.039617 1.050399 1.037222	.1000002	$9.992526\text{x}10^{-7}$
90 45 135	.6747422 1.315813 .6747456	1.045493 1.039617 1.039722	.1000002	$6.436005\text{x}10^{-8}$
135 45 180	1.316635 1.851687 .6751235	1.051398 1.039617 1.045640	.1000002	$4.208667\text{x}10^{-8}$
225 90 0	1.852109 1.316056 1.853118	1.051501 1.037221 1.045566	.1000004	$1.831758\text{x}10^{-7}$
180 45 90	1.852688 .6744785 1.316643	1.053883 1.039617 1.037223	.1000002	$1.820385\text{x}10^{-7}$

Table 9. Quasars' aberrated angular distances, spacecraft's aberrated apical latitudes, uncertainties and aberrated velocity as a function of quasars' apical latitudes $\theta_A = \theta_B = \theta_C = 75°$.

Apical latitudes (deg) $\theta_A = 75$; $\theta_B = 75$; $\theta_C = 75$				
Apical longitude values (deg) l_A l_B l_C	**Quasars' aberrated angular distances** (rad) Ψ_1'' Ψ_2'' Ψ_3''	**Aberrated apical latitudes** (rad) θ_{A2}'' θ_{B2}'' θ_{C2}''	**Aberrated velocity** $\beta'' = v''/c$	**Uncertainty of aberrated apical latitudes** $\Delta\theta''$ (rad)
0 45 90	.7328074 .7328074 1.446703	1.205014 1.207440 1.205018	.1	1.086075×10^{-7}
0 120 240	1.892254 1.892254 1.890555	1.205014 1.217651 1.205017	.1	1.559243×10^{-7}
0 170 45	2.406806 1.962084 .7328069	1.205014 1.221801 1.207434	.1000002	1.289855×10^{-6}
45 215 90	2.406558 1.961239 .7328073	1.207444 1.218397 1.205012	.1000001	1.301131×10^{-6}
90 45 135	.7331199 1.447127 .7331235	1.213411 1.207444 1.207545	.1000007	1.076865×10^{-7}
135 45 180	1.448146 2.091397 .7335716	1.219414 1.207444 1.213555	.1000007	4.551616×10^{-8}
225 90 0	2.091990 1.447428 2.093409	1.219517 1.205014 1.213484	.1000006	2.351534×10^{-7}
180 45 90	2.092804 .7328073 1.448155	1.221941 1.207444 1.205017	.1000006	2.348527×10^{-7}

Table 10. Quasars' aberrated angular distances, spacecraft's aberrated apical latitudes, uncertainties and aberrated velocity as a function of quasars' apical latitudes $\theta_A = \theta_B = \theta_C = 80°$.

Apical latitudes (deg) $\theta_A = 80$; $\theta_B = 80$; $\theta_C = 80$				
Apical longitude values (deg) l_A l_B l_C	Quasars' aberrated angular distances (rad) Ψ_1'' Ψ_2'' Ψ_3''	Aberrated apical latitudes (rad) θ_{A2}'' θ_{B2}'' θ_{C2}''	Aberrated velocity $\beta'' = v''/c$	Uncertainty of aberrated apical latitudes $\Delta\theta''$ (rad)
0 45 90	.7543896 .7543896 1.496816	1.289934 1.292382 1.289936	.1000005	1.50339×10^{-7}
0 120 240	1.972387 1.972387 1.970484	1.289934 1.302674 1.289936	.1000007	1.84351×10^{-7}
0 170 45	2.570271 2.048865 .7543894	1.289934 1.306861 1.292377	.1000002	1.570314×10^{-6}
45 215 90	2.569936 2.047908 .7543896	1.292383 1.303427 1.289933	.1000002	1.569307×10^{-6}
90 45 135	.7547213 1.497276 .7547255	1.298397 1.292383 1.292491	.1000007	1.490476×10^{-7}
135 45 180	1.498376 2.192794 .755201	1.304452 1.292383 1.298548	.1000002	4.676693×10^{-8}
225 90 0	2.193482 1.49760 2.195129	1.304557 1.289934 1.298471	.1000007	2.789272×10^{-7}
180 45 90	2.194427 .7543898 1.498386	1.307003 1.292383 1.289937	.1000007	2.794903×10^{-7}

Table 11. Quasars' aberrated angular distances, spacecraft's aberrated apical latitudes, uncertainties and aberrated velocity as a function of quasars' apical latitudes $\theta_A = \theta_B = \theta_C = 90°$.

Apical latitudes (deg) $\theta_A = 90$; $\theta_B = 90$; $\theta_C = 90$				
Apical longitudes values (deg) l_A l_B l_C	**Quasars' aberrated angular distances (rad)** Ψ_1'' Ψ_2'' Ψ_3''	**Aberrated apical latitudes (rad)** θ_{A2}'' θ_{B2}'' θ_{C2}''	**Aberrated velocity** $\beta'' = v''/c$	**Uncertainty of aberrated apical latitudes** $\Delta\theta''$ **(rad)**
0 45 90	.7806323 .7806323 1.559071	1.46195 1.464438 1.461949	.1000006	4.35259×10^{-7}
0 120 240	2.076409 2.076409 2.074203	1.461950 1.474906 1.461950	.1000007	3.782526×10^{-7}
0 170 45	2.875982 2.163023 .7806325	1.461950 1.479170 1.464444	.1000005	3.594844×10^{-6}
45 215 90	2.875224 2.161893 .7806324	1.464438 1.475672 1.461953	.1000002	3.465266×10^{-6}
90 45 135	.7809882 1.559575 .7809924	1.470553 1.464438 1.464544	.1000007	4.313473×10^{-7}
135 45 180	1.560786 2.330901 .7815026	1.476717 1.464438 1.470704	.1000008	4.824766×10^{-8}
225 90 0	2.331752 1.559933 2.333790	1.476824 1.461950 1.470629	.1000007	5.535144×10^{-7}
180 45 90	2.332921 .78063222 1.560797	1.479315 1.464438 1.461949	.1000007	5.63842×10^{-7}

Table 12. Quasars' aberrated angular distances, spacecraft's aberrated apical latitudes, uncertainties and aberrated velocity as a function of quasars' apical latitudes $\theta_A = \theta_B = \theta_C = 105°$.

Apical latitudes (deg) $\theta_A = 105$; $\theta_B = 105$; $\theta_C = 105$				
Apical longitudes values (deg) l_A l_B l_C	Quasars' aberrated angular distances (rad) Ψ_1'' Ψ_2'' Ψ_3''	Aberrated apical latitudes (rad) θ_{A2}'' θ_{B2}'' θ_{C2}''	Aberrated velocity $\beta'' = v''/c$	Uncertainty of aberrated apical latitudes $\Delta\theta''$ (rad)
0 45 90	.7737214 .7737214 1.542527	-1.415936 -1.413387 -1.415934	.1000007	3.270342×10^{-7}
0 120 240	2.048211 2.048211 2.046092	-1.415936 -1.402644 -1.415936	.1000008	8.354338×10^{-8}
0 170 45	2.770738 2.131879 .7737212	-1.415936 -1.398259 -1.413391	.1000007	1.055704×10^{-6}
45 215 90	2.770202 2.130802 .7737214	-1.413387 -1.401856 -1.415935	.1000008	1.061392×10^{-6}
90 45 135	.7740707 1.543019 .7740752	-1.407115 -1.413387 -1.413275	.1000007	3.237373×10^{-7}
135 45 180	1.544199 2.292566 .7745759	-1.400782 -1.413387 -1.406957	.1000006	4.780046×10^{-8}
225 90 0	2.293366 1.543368 2.295284	-1.400672 -1.415936 -1.407037	.1000007	1.896711×10^{-7}
180 45 90	2.294466 .7737215 1.544210	-1.398109 -1.413387 -1.415934	.1000007	1.861201×10^{-7}

Table 13. Quasars' aberrated angular distances, spacecraft's aberrated apical latitudes, uncertainties and aberrated velocity as a function of quasars' apical latitudes $\theta_A = \theta_B = \theta_C = 120°$.

Apical latitudes (deg) $\theta_A = 120$; $\theta_B = 120$; $\theta_C = 120$				
Apical longitudes values (deg) l_A l_B l_C	**Quasars' aberrated angular distances** (rad) Ψ_1'' Ψ_2'' Ψ_3''	**Aberrated apical latitudes** (rad) θ_{A2}'' θ_{B2}'' θ_{C2}''	**Aberrated velocity** $\beta'' = v''/c$	**Uncertainty of aberrated apical latitudes** $\Delta\theta''$ (rad)
0 45 90	.7084652 .7084652 1.391237	-1.145326 -1.142719 -1.145329	.1000007	1.25511×10^{-7}
0 120 240	1.806544 1.806544 1.805039	-1.145326 -1.131707 -1.145324	.1000007	1.622084×10^{-8}
0 170 45	2.256094 1.870123 .7084649	-1.145326 -1.127201 -1.142729	.1000007	8.497113×10^{-8}
45 215 90	2.255899 1.869381 .7084652	-1.142719 -1.130897 -1.145327	.1000008	8.359305×10^{-8}
90 45 135	.7087568 1.391625 .7087601	-1.136293 -1.142719 -1.142611	.1000006	1.239996×10^{-7}
135 45 180	1.392558 1.986321 .7091784	-1.129794 -1.142719 -1.136131	.1000007	4.385023×10^{-8}
225 90 0	1.986831 1.391901 1.988053	-1.129681 -1.145327 -1.136214	.1000007	9.03231×10^{-10}
180 45 90	1.987532 .7084652 1.392566	-1.127048 -1.142719 -1.145324	.1000007	1.083175×10^{-9}

Table 14. Quasars' aberrated angular distances, spacecraft's aberrated apical latitudes, uncertainties and aberrated velocity as a function of quasars' apical latitudes $\theta_A = \theta_B = \theta_C = 130°$.

Apical latitudes (deg) $\theta_A = 130$; $\theta_B = 130$; $\theta_C = 130$				
Apical longitudes values (deg) l_A l_B l_C	**Quasars' aberrated angular distances (rad)** Ψ_1'' Ψ_2'' Ψ_3''	**Aberrated apical latitudes (rad)** θ_{A2}'' θ_{B2}'' θ_{C2}''	**Aberrated velocity** $\beta'' = v''/c$	**Uncertainty of aberrated apical latitudes** $\Delta\theta''$ **(rad)**
0 45 90	.6336025 .6336025 1.226604	-.9612522 -.9586096 -.9612507	.1000007	$8.632905x10^{-8}$
0 120 240	1.565702 1.565702 1.564642	-.9612522 -.9474325 -.9612541	.1000002	$2.933457x10^{-8}$
0 170 45	1.893283 1.615097 .6336023	-.9612522 -.9428508 -.9586159	.1000008	$6.650787x10^{-8}$
45 215 90	1.893168 1.614583 .6336024	-.9586101 -.9466096 -.9612538	.1000007	$6.251612x10^{-8}$
90 45 135	.6338347 1.226899 .6338376	-.9520916 -.9586099 -.9584945	.1000007	$8.510795x10^{-8}$
135 45 180	1.227604 1.703298 .6341703	-.9454886 -.9586101 -.9519283	.1000008	$3.93128x10^{-8}$
225 90 0	1.703639 1.227107 1.704455	-.9453734 -.9612522 -.9520106	.1000007	$2.681128x10^{-8}$
180 45 90	1.704107 .6336024 1.227610	-.9426943 -.9586099 -9612509	.1000007	$2.699725x10^{-8}$

Table 15. Quasars' aberrated angular distances, spacecraft's aberrated apical latitudes, uncertainties and aberrated velocity as a function of quasars' apical latitudes $\theta_A = \theta_B = \theta_C = 140°$.

Apical latitudes (deg) $\theta_A = 140$; $\theta_B = 140$; $\theta_C = 140$				
Apical longitudes values (deg) l_A l_B l_C	Quasars' aberrated angular distances (rad) Ψ_1'' Ψ_2'' Ψ_3''	Aberrated apical latitudes (rad) θ_{A2}'' θ_{B2}'' θ_{C2}''	Aberrated velocity $\beta'' = v''/c$	Uncertainty of aberrated apical latitudes $\Delta\theta''$ (rad)
0 45 90	.5362356 .5362356 1.022941	-.7745201 -.7718490 -.7745184	.1000007	6.196224×10^{-8}
0 120 240	1.286341 1.286341 1.285662	-.7745201 -.7605262 -.7745192	.1000007	3.151045×10^{-8}
0 170 45	1.522930 1.323285 .5362357	-.7745201 -.7558734 -.7718436	.1000007	1.199885×10^{-7}
45 215 90	1.522862 1.322958 .5362356	-.7718493 -.7596914 -.7745194	.1000008	1.110272×10^{-7}
90 45 135	.5364031 1.023141 .5364031	-.7652512 -.7718493 -.7717324	.1000007	6.08850×10^{-8}
135 45 180	1.023619 1.388377 .536640	-.7585528 -.7718493 -.7650857	.1000008	3.335228×10^{-8}
225 90 0	1.388588 1.023282 1.389094	-.7584358 -.7745202 -.7651689	.1000007	3.400285×10^{-8}
180 45 90	1.388878 .5362356 1.023623	-.7557144 -.7718493 -.7745187	.1000007	3.433049×10^{-8}

Table 16. Quasars' aberrated angular distances, spacecraft's aberrated apical latitudes, uncertainties and aberrated velocity as a function of quasars' apical latitudes $\theta_A = \theta_B = \theta_C = 150°$.

Apical latitudes (deg) $\theta_A = 150$; $\theta_B = 150$; $\theta_C = 150$				
Apical longitudes values (deg) l_A l_B l_C	**Quasars' aberrated angular distances (rad)** Ψ_1'' Ψ_2'' Ψ_3''	**Aberrated apical latitudes (rad)** θ_{A2}'' θ_{B2}'' θ_{C2}''	**Aberrated velocity** $\beta'' = v''/c$	**Uncertainty of aberrated apical latitudes** $\Delta\theta''$ **(rad)**
0 45 90	.4197701 .4197701 .7903064	-.5854905 -.5828009 -.5854905	.1000007	4.373279×10^{-8}
0 120 240	.9822838 .9822838 .9819026	-.5854905 -.5713606 -.5854912	.1000007	2.77957×10^{-8}
0 170 45	1.147026 1.008501 .4197701	-.5854905 -.5666404 -.5827973	.1000008	1.231185×10^{-7}
45 215 90	1.146990 1.008319 .4197701	-.5828009 -.5705143 -.5854912	.1000007	1.103616×10^{-7}
90 45 135	.4198708 .7904230 .4198721	-.5761420 -.5828009 -.5826859	.1000007	4.273432×10^{-8}
135 45 180	.7907025 1.054402 .4200169	-.5693601 -.5828009 -.5759767	.1000007	2.613807×10^{-8}
225 90 0	1.054518 .7905056 1.054796	-.5692415 -.5854905 -.5760588	.1000008	3.16066×10^{-8}
180 45 90	1.054678 .4197701 .7907050	-.5664790 -.5828009 -.5854905	.1000008	3.208496×10^{-8}