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THRUSTER FAILURE RECOVERY STRATEGIES FOR LIBRATION POINT MISSIONS

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Recovery strategies in case of possible thruster failure are proposed for collinear libration point missions. It is implied that, instead of a failed main thruster, a redundant set of thrusters is used to transfer a spacecraft to periodic orbits close to the nominal one or to associated stable manifolds. The basic idea behind this approach consists in the fact that making a spacecraft to follow a reference periodic orbit is often not critical while it is much more important to save enough fuel for station-keeping during the planned mission term. In the paper, a family of planar Lyapunov orbits around the Sun-Earth L2 point is considered though similar results can be obtained in the same way for halo orbits. Transfers to the "cheapest-to-get" periodic orbit and to the reference one are compared for different correction maneuver delay values.

INTRODUCTION

Nowadays, there is a growing interest in missions to collinear libration points and associated periodic orbits since they provide a useful platform for investigation of both the solar system and the universe. Based on experience acquired in many successfully accomplished missions (ISEE-3, WIND, SOHO, ACE, Genesis, Gaia), a number of promising near-future projects have been proposed by leading space agencies: Deep Space Climate Observatory (NASA), LISA Pathfinder (ESA/NASA), Spektr-RG (Roscosmos/ESA). The majority of them exploit the L1/L2 libration points of the Sun-Earth system.

One of the important features related to (quasi)periodic trajectories around collinear libration points is high instability of motion, which requires the application of station-keeping techniques. For that purpose, accurate trajectory determination and regular control-law updates are essential. The optimal placement of two statistical control maneuvers for keeping a spacecraft near collinear libration point is studied in (Reference 1). Using the linear theory, explicit formulas for spacecraft control as well as for the mean ΔV and standard deviation are found. The optimal spacing of correction maneuvers is shown to be related to the characteristic time of instability. For a continuous thrust, the optimal control of a system linearized relative to a nominal periodic orbit is obtained in (Reference 2). If the total level of orbit determination uncertainty is fixed, there is an optimal way to distribute uncertainty between the position and velocity parts of a state vector. These ideas and results were applied to spacecraft control in the vicinity of a halo orbit in the Hill problem.

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Since station-keeping operations imply using thrusters, any possible thruster failure threatens a mission: it prevents correction maneuvers from being performed and thus can lead to a significant deviation of the spacecraft from the nominal periodic orbit. This problem is of high importance as the largest percentage of all fail occurrences relating to the control system falls on thruster failure (Reference 3)[*]. Generally, if a thruster fails, the control is allocated to a redundant set of thrusters (attitude control thrusters or a backup orbital thruster). It is worth noting that most of publications devoted to the thruster failure issue are related only to collision avoidance during rendezvous and docking (see, for example, References 4 and 5). To the authors' knowledge, the problem of libration point mission recovery has not been deeply studied yet.

In the present paper, a family of horizontal Lyapunov periodic orbits around the Sun-Earth L2 point is considered. We suppose that the main orbit control thruster failed and produces no thrust. As a result, the planned correction maneuver is not performed on time. With some delay, a redundant set of thrusters is used. Since it usually has less fuel available (moreover, the orbit decay can already be considerable), a transfer to the nominal periodic orbit may appear to be too expensive, which means that not enough fuel is left to perform station-keeping maneuvers during the planned mission lifetime. So, an investigation of transfers to other periodic orbits close to the nominal one and acceptable from the viewpoint of the mission goal is proposed. Two strategies are considered: to transfer a spacecraft directly to some periodic orbit or to an associated stable manifold. In both cases, the aim is the same – to find the "cheapest-to-get" periodic orbit for different values of *correction maneuver delay* (the time passed since the moment of unsuccessful correction maneuver).

COLLINEAR LIBRATION POINTS AND PERIODIC ORBITS

In this paper, the planar circular restricted three-body problem (CR3BP) is studied, though the same approach can be applied to the spatial case. According to the CR3BP model, a spacecraft of negligible mass moves under the gravitational influence of two masses m_1 and m_2, referred to as the *primaries*, such that m_1 and m_2 move in circular orbits about their center of mass C. As was said, the spacecraft is supposed to move in the orbital plane of the primaries. To avoid ambiguity, let m_1 be greater than m_2. In particular, m_1 can represent the Sun while m_2 represents the Earth.

It is convenient to write the equations of spacecraft motion in the standard for the CR3BP non-dimensional rotating (sometimes referred to as *synodic*) coordinate frame. The masses m_1 and m_2 are normalized so that $m_1 = 1 - \mu$ and $m_2 = \mu$, where $\mu = m_2 / (m_1 + m_2)$ stands for the mass parameter of the system. The origin of the frame is chosen at C. If we normalize to one the angular velocity of the rotating frame and the distance between the primaries, they would be at fixed positions along the *x*-axis at points $(-\mu, 0)$ and $(1-\mu, 0)$, respectively. Note that the value of m_2 for the Sun-Earth system includes the mass of the Moon and therefore $\mu = 3.03939 \cdot 10^{-6}$. The planar equations of spacecraft motion can be expressed as follows:

$$\ddot{x} - 2\dot{y} = -U_x, \tag{1}$$

$$\ddot{y} + 2\dot{x} = -U_y, \tag{2}$$

[*] Apart from a thruster failure, a communication failure can also prevent the orbit correction: in 1998, flight controllers at NASA lost the contact with the SOHO spacecraft; the recovery operation required about 7 m/s of additional delta-v.

where

$$U(x,y) = -\frac{1}{2}\left(x^2+y^2\right) - \frac{1-\mu}{r_1} - \frac{\mu}{r_2} - \frac{1}{2}\mu(1-\mu), \tag{3}$$

is the so called *effective potential*; U_x and U_y are the partial derivatives of U with respect to the position variables. The distances between the spacecraft and the primaries are given by equalities $r_1^2 = (x+\mu)^2 + y^2$, $r_2^2 = (x-1+\mu)^2 + y^2$.

The system has five relative equilibrium points; three of them lie at the *x*-axis and are referred to as *collinear libration points*. Usually denoted by L_1, L_2, and L_3, these points are proved to be unstable. Their *x*-coordinates for the Sun-Earth system are respectively equal to $x_{L1} = 0.989987$, $x_{L2} = 1.0100740$, and $x_{L3} = -1.000001$. In further text, L stands for either L_1 or L_2.

It is known that there exists a family of planar Lyapunov periodic orbits around collinear libration points. For their computation, one can use the differential correction technique and continuation methods (Reference 6). Several analytical approximations have also been developed. Below, Richardson's third-order approximation is exploited (see Reference 7). It involves the parameterization of all the periodic orbits in terms of shifted normalized variables $\bar{x} = (x - x_L)/\gamma$, $\bar{y} = y/\gamma$, $\bar{v}_x = \dot{x}/\gamma$, $\bar{v}_y = \dot{y}/\gamma$. Here $\gamma = |x_L - 1 + \mu|$ denotes the distance between L and m_2. According to this approximation, any Lyapunov periodic orbit can be parameterized in phase space as follows:

$$\bar{x} = a_{21}A_x^2 - A_x\cos\tau_1 + a_{23}A_x^2\cos 2\tau_1 + a_{31}A_x^3\cos 3\tau_1, \tag{4}$$

$$\bar{y} = kA_x\sin\tau_1 + b_{21}A_x^2\sin 2\tau_1 + b_{31}A_x^3\sin 3\tau_1, \tag{5}$$

$$\bar{v}_x/(\omega_p\nu) = A_x\sin\tau_1 - 2a_{23}A_x^2\sin 2\tau_1 - 3a_{31}A_x^3\sin 3\tau_1, \tag{6}$$

$$\bar{v}_y/(\omega_p\nu) = kA_x\cos\tau_1 + 2b_{21}A_x^2\cos 2\tau_1 + 3b_{31}A_x^3\cos 3\tau_1, \tag{7}$$

where

$$k = \left(\omega_p^2 + 2c_2 + 1\right)/(2\omega_p) > 0, \tag{8}$$

$$\omega_p = \sqrt{\left(2 - c_2 + \sqrt{9c_2^2 - 8c_2}\right)/2}, \tag{9}$$

$$c_2 = \mu|x_L - 1 + \mu|^{-3} + (1-\mu)|x_L + \mu|^{-3}. \tag{10}$$

Here $\tau_1 = \omega_p\nu t + \varphi$, $\nu = 1 + s_1A_x^2$, the phase φ parameterizes a point in orbit, A_x is the third-order approximation of the *x*-axis amplitude of the periodic orbit. $a_{21}, a_{23}, a_{31}, b_{21}, b_{31}, s_1$ are some constant factors for which we omit the expressions due to their complexity (for details, see Reference 7). For the Sun-Earth L_2 point, $c_2 = 3.9405$, $\omega_p = 2.0570$, $k = 3.1872$. The formulas (4)-(7) can also be used to approximate the stable and unstable manifolds of the corresponding periodic orbit. In this case, an additional exponential term $A_{\exp}\exp(-\lambda t)u_{\text{st}}$ or $A_{\exp}\exp(\lambda t)u_{\text{unst}}$ must be included where u_{st} and u_{unst} are the directions in phase space along the stable and unstable asymptotic trajectories, respectively:

$$u_{\text{st}} = (1, \sigma, -\lambda, -\lambda\sigma), \tag{11}$$

$$u_{\text{unst}} = (1, -\sigma, \lambda, -\lambda\sigma). \tag{12}$$

The notation

$$\lambda = \sqrt{\left(c_2 - 2 + \sqrt{9c_2^2 - 8c_2}\right)\Big/2}, \tag{13}$$

$$\sigma = 2\lambda \Big/ \left(\lambda^2 + c_2 - 1\right), \tag{14}$$

is used here. For the L_2 point of the Sun-Earth system, $\lambda = 2.4843$, $\sigma = 0.5453$.

PROBLEM STATEMENT

Consider a planar periodic orbit with an *x*-amplitude of A_x around the Sun-Earth L_2 point. Let $\hat{x} = (\hat{\mathbf{r}}, \hat{\mathbf{v}})$ be a state corresponding to some point of orbit. At the initial moment of time $t_0 = 0$, it is supposed that the main orbital thruster failed, and so the planned correction maneuver was unsuccessful. To investigate the worst possible case of orbital decay, we assume that the phase state of the spacecraft $x_0 = (\mathbf{r}_0, \mathbf{v}_0)$ is shifted along the unstable manifold of the reference orbit:

$$\mathbf{r}_0 = \hat{\mathbf{r}} + \sigma_r \mathbf{u}_{r,\text{unst}}, \tag{15}$$

$$\mathbf{v}_0 = \hat{\mathbf{v}} + \sigma_v \mathbf{u}_{v,\text{unst}}, \tag{16}$$

where σ_r and σ_v are some deviations in position and velocity at $t_0 = 0$; $u_{\text{unst}} = \left(\mathbf{u}_{r,\text{unst}}, \mathbf{u}_{v,\text{unst}}\right)$ is the direction along the unstable manifold of the reference orbit at point $\hat{x}$.

Let t_1 be the duration of spacecraft motion without any control since the moment $t_0 = 0$, i.e. a delay since the moment of unsuccessful correction maneuver. At the moment t_1 the possibility of controlling the spacecraft is supposed to come back. Let x_1 be the phase state of the spacecraft at $t = t_1$. Our aim is to obtain a transfer to the *backup* periodic orbit that is close to the reference one and, at the same time, is the best in terms of delta-v spent by the redundant set of thrusters. To do so, a few optimization procedures are implemented. First, we optimize a transfer to the reference periodic orbit. The obtained solution is compared with the optimal transfer to the best backup periodic orbit (periodic orbit targeting strategy). Then the same procedure is done with the solutions to the problem of optimal transfer to the stable manifolds associated with the reference periodic orbit and with the best backup periodic orbit (stable manifold targeting strategy). In both cases the optimization problem has a form

$$J(y) \to \min, \tag{17}$$

where $J = J(y)$ is the objective function and y is the vector of optimized parameters. Different problems are characterized by different parameters y and the objective function J.

Transfers to Periodic Orbits (Periodic Orbit Targeting)

For a given delay since last unsuccessful correction t_1, the phase state x_1 at t_1 is obtained. After that, a phase state $x_2 = \left(\mathbf{r}_2, \mathbf{v}_2^-\right)$ of the spacecraft at the time $t_1 + T_1$ is obtained. For a given φ

that parameterizes the periodic orbit with amplitude A_x, an appropriate phase state $x_3 = \left(\mathbf{r}_3, \mathbf{v}_3^+\right)$ is obtained (see Eq. (4)–(7)). Choosing a time of flight T_2, the given values $\mathbf{r}_2$, $\mathbf{r}_3$ and T_2 allows one to solve a two-boundary problem and obtain velocities $\mathbf{v}_2^+$ and $\mathbf{v}_3^-$ at $\mathbf{r}_2$ and $\mathbf{r}_3$, respectively, required for performing the transfer. The cost of the transfer is the sum of $\Delta v_1 = \left|\mathbf{v}_2^+ - \mathbf{v}_2^-\right|$ and $\Delta v_2 = \left|\mathbf{v}_3^+ - \mathbf{v}_3^-\right|$, so the objective functional is $J = \Delta v_1 + \Delta v_2$. In general, the vector of optimized parameters is $y = \left(A_x, T_1, T_2, \varphi\right)$. If a transfer to the reference periodic orbit is considered, then the amplitude A_x is fixed and $y = \left(T_1, T_2, \varphi\right)$.

Transfers to Stable Manifolds (Stable Manifold Targeting)

Transfers to manifolds obtained in the same way as transfers to periodic orbits but with one difference: a point with the parameters t, φ belonging to the manifold associated with a periodic orbit of amplitude A_x is chosen. Thus, the vector of optimized parameters is $y = \left(A_x, T_1, T_2, \varphi, t\right)$. If we consider a transfer to the manifold associated with the reference periodic orbit, then the amplitude A_x is fixed and $y = \left(T_1, T_2, \varphi, t\right)$. If fixing $t = 0$, transfer optimization to manifolds turns into transfer optimization to periodic orbits.

RESULTS

The results are obtained for a number of reference orbits with amplitudes of $A_x = 0.5 \times 10^5$ km, 1.0×10^5 km, 1.5×10^5 and 2.0×10^5 km. The deviation in position and velocity are chosen equal to $\sigma_r = 300$ km and $\sigma_v = 0.3$ m/s, respectively. The time delay t_1 is varied from $0.05 \cdot T$ to $0.7 \cdot T$ where T is the reference orbit period.

Dependencies $\Delta v_{\text{ref}}\left(t_1\right)$, $\Delta v_{\text{ref}}\left(t_1\right) - \Delta v_{\text{best}}\left(t_1\right)$ and $A_{x,\text{best}}\left(t_1\right) - A_{x,\text{ref}}\left(t_1\right)$ are shown in Figures 1–3 (for transfers to periodic orbits) and Figures 4–6 (for transfers to stable manifolds).

In the first case, Δv_{ref} stands for the cost of a transfer to the reference periodic orbit, Δv_{best} stands for the cost of a transfer to the best found backup periodic orbit, $A_{x,\text{best}}$ denotes the amplitude of the best found backup periodic orbit, and $A_{x,\text{ref}}$ is the amplitude of the reference periodic orbit. In the second case, Δv_{ref} and Δv_{best} are related to transfers to stable manifolds associated respectively with the reference periodic orbit of amplitude $A_{x,\text{ref}}$ and with the best found backup periodic orbit of amplitude $A_{x,\text{best}}$.

Figures 2 and 5 show that in the case when the spacecraft is moving far from the reference periodic orbit ($t_1 \approx 0.3\text{-}0.4$), the gain in delta-v considerably contributes to the budget of subsequent station-keeping. Another observation is that the stable manifold targeting strategy, although being the generalization of the periodic orbit targeting strategy, has almost no advantage compared with the latter.

CONCLUSION

Two recovery strategies in case of possible thruster failure – periodic orbit targeting and stable manifold targeting – are proposed for collinear libration point missions. The strategies imply performing several optimization procedures. First, an optimal two-impulse transfer to the reference periodic orbit is considered. The solution is compared with a transfer to the "cheapest-to-get" periodic orbit. Then the same approach is applied to the problem of transfers to the stable manifold associated with the reference orbit and to the "cheapest-to-get" stable manifold of some other periodic orbit. The results of applying both strategies for different correction maneuver delay values are presented. They show that the corresponding gain in delta-v is often significant for subsequent station-keeping. Taking advantage of proposed strategies can extend the spacecraft lifetime up to several years.

Some further work is needed if periodic orbits with larger amplitudes (including vertical Lyapunov and halo orbits) are considered. In this case, using higher-order approximation is probably required. The study of recovery strategies for a Lissajous or quasi-halo reference orbit is also of interest.

ACKNOWLEDGMENTS

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APPENDIX: FIGURES

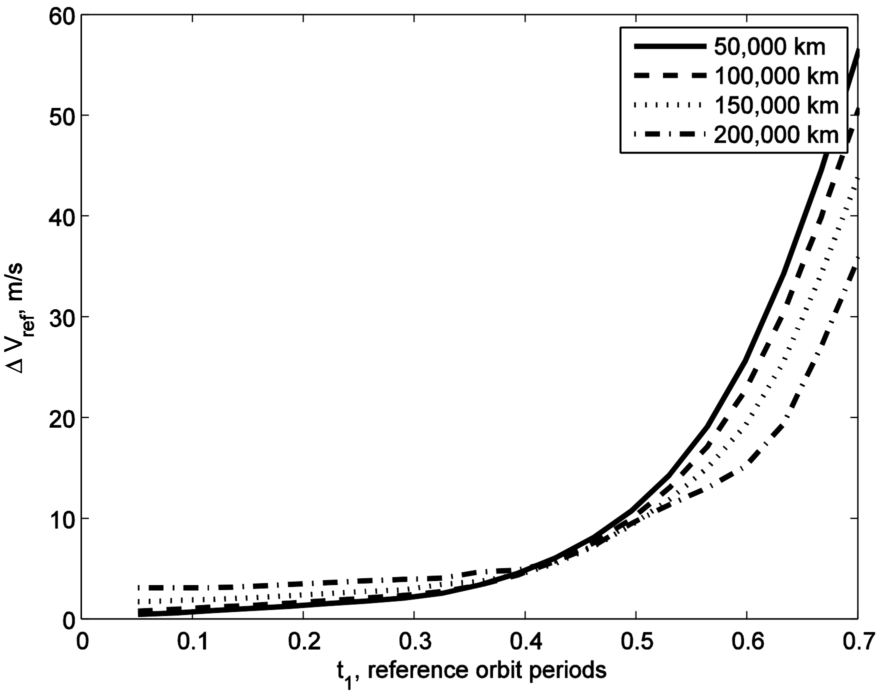

Figure 1. Characteristic Velocity for a Transfer to the Reference Periodic Orbit

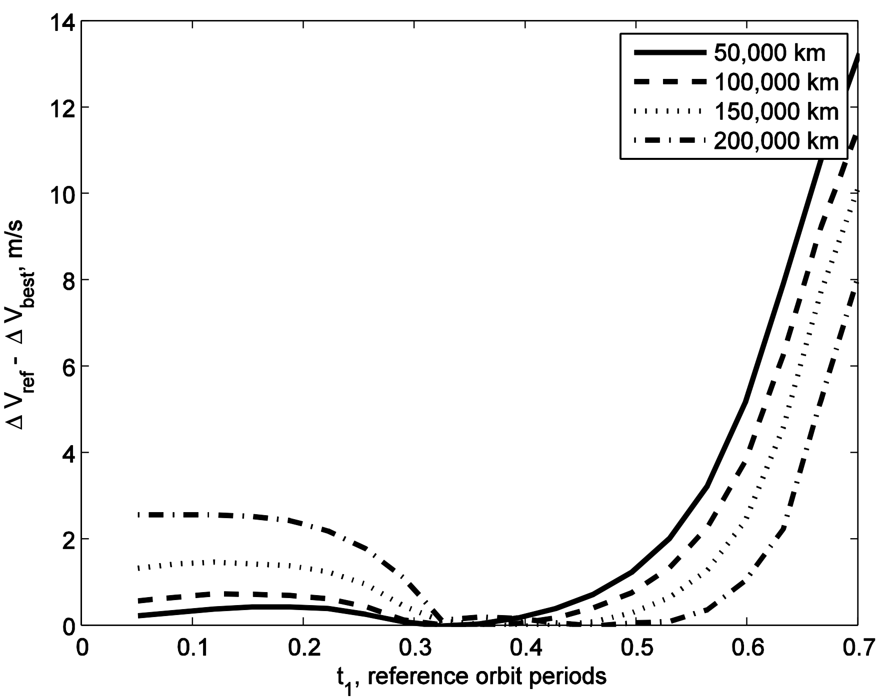

Figure 2. Difference between the Characteristic Velocities for Transfers to the Reference Periodic Orbit and to the Best Backup Periodic Orbit

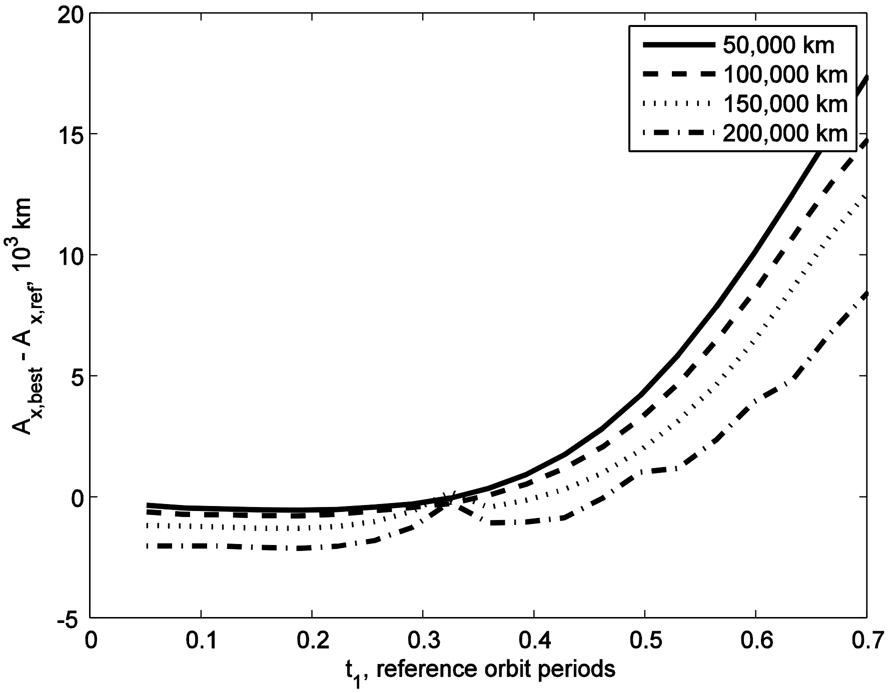

Figure 3. Difference between the Amplitudes of the Best Backup Periodic Orbit and of the Reference Periodic Orbit (Periodic Orbit Targeting Strategy)

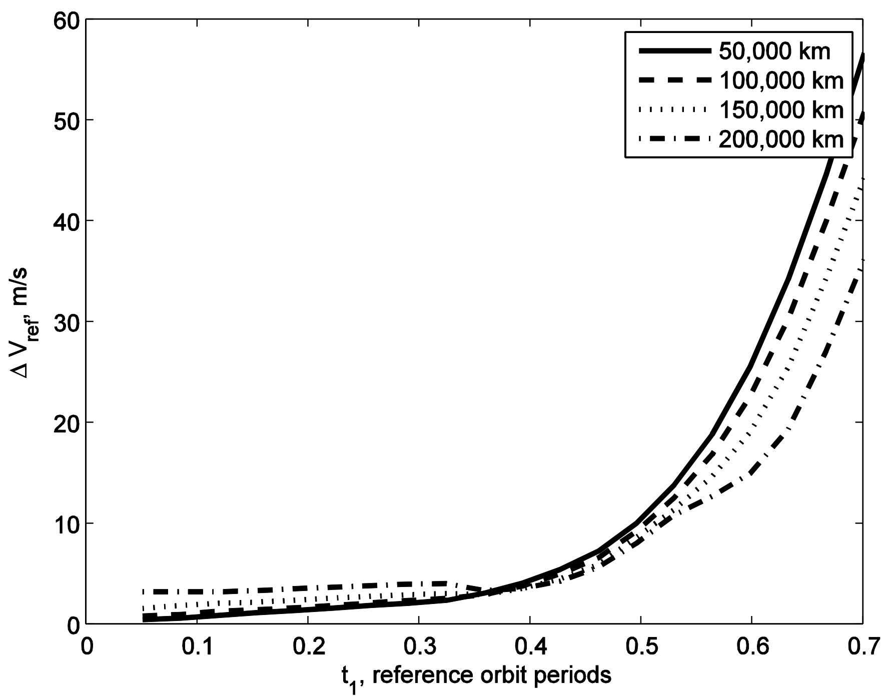

Figure 4. Characteristic Velocity for a Transfer to the Stable Manifold of the Reference Periodic Orbit

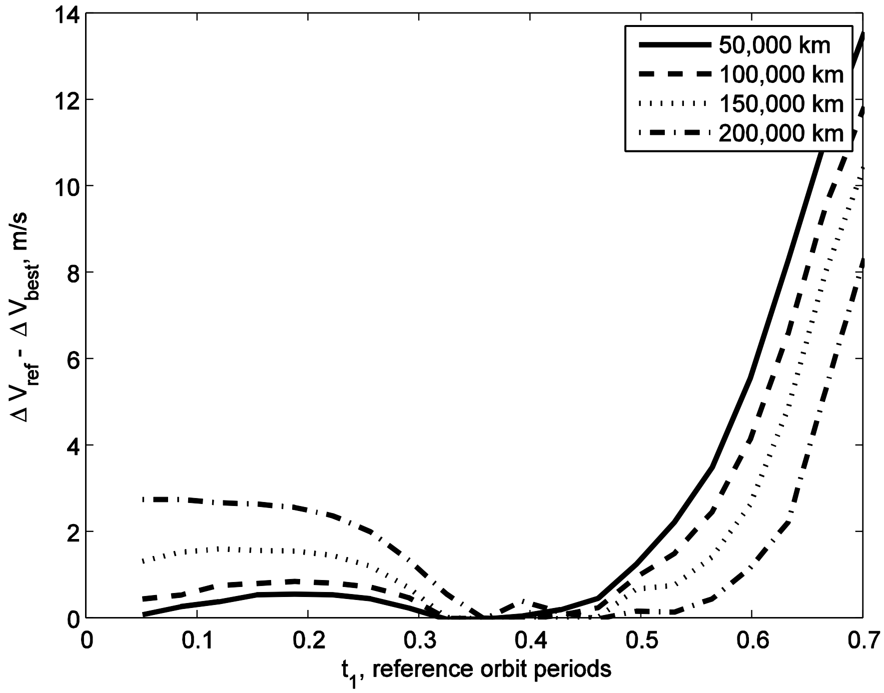

Figure 5. Difference between the Characteristic Velocities for Transfers to the Stable Manifolds of the Reference Periodic Orbit and of the Best Backup Periodic Orbit

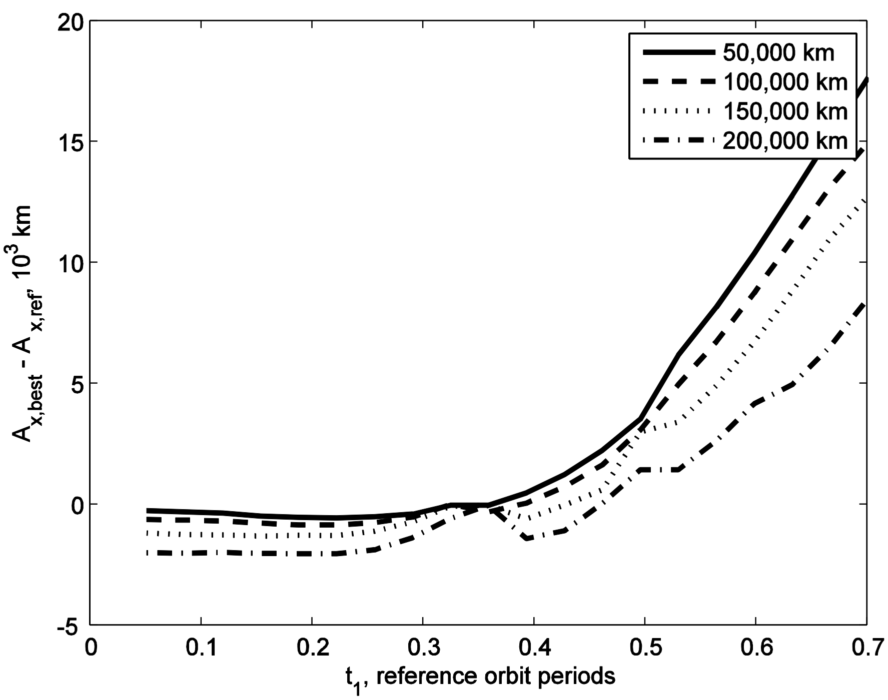

Figure 6. Difference between the Amplitudes of the Best Backup Periodic Orbit and of the Reference Periodic Orbit (Stable Manifold Targeting Strategy)