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POWER CONSUMPTION BASED ON A FOUR REACTION WHEELS IN A PYRAMIDAL CONFIGURATION

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A cluster of four reaction wheels, used for attitude control in a 3U CubeSat, has been chosen for a power consumption analysis. The reaction wheels cluster is arranged in a pyramidal configuration. The reaction wheels were modeled as a dc motor. It's shown how the power consumption for this configuration is affected according to the pyramid angle. The power consumption was evaluated while the orientation process occurred. The simulation was made for a particular desired attitude, varying the pyramid angle each time. The results show the power consumption considering the voltage and current in each wheel and the entire cluster.

INTRODUCTION

Satellital mission *Libertad 2*, a 3U CubeSat, is a Sergio Arboleda University initiative. This initiative started with the Libertad 1 mission in 2007.[1] At difference with the Libertad 1, which has a passive control, *Libertad 2* will be equipped with a three axis control subsystem, in order to achieve its principal aim, Earth observation. This kind of objectives require control pointing. The satellital subsystem in charge to do that is the Attitude Determination and Control System (ADCS). The research in the ADCS field, by the Sergio Arboleda University ADCS team, is focused in the study of the dynamic, kinematic, sensors, actuators and the control algorithms. All this with the purpose to include in future missions an ADCS developed in the University.

One of the control actuators studied is the reaction wheel. This kind of actuator is a good start point, has been used for a lot of satellital missions and in several simulation proposals. Using from one reaction wheel, for axis control (*e.g*, pitch control), passing through three reaction wheels aligned with satellite principal axes, until four reaction wheels. In this paper a cluster of four reaction wheels in a pyramidal configuration have been used. Four reaction wheels gives redundancy to the control system in case of failure.

The reaction wheels were modeled as dc motors and it was included in the overall model of the system, taking into account that the consulted references for power consumption analysis, don't include the reaction wheels model like a dc motor, so the power consumption analysis aren't made in terms of voltage an current in the motors. Usually, the control law has units of torque and is applied to the satellite model directly.[2] In this paper, the control law has voltage units, and is applied from the controller to the reaction wheel model. The voltage is continuous and not a pulsed signal.

The simulation results show how the power consumption for this kind of reaction wheels configuration is affected by the angle of the pyramid. For several pyramid angles the power consumption

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was evaluated while the orientation process occurs. This time the power is function of the voltage and current in each wheel and the cluster. The simulation was made for a particular desired attitude (Roll, Pitch and Yaw).

SPACECRAFT MODEL

In order to evaluate the satellite power consumption for a particular attitude, the satellite was modeled like a rigid body (10x10x30 cm) with a uniform mass distribution and its rotation axes parallels to its principal axes. Several actuators are used in an attitude control system, such as reaction wheels, thrusters, magnetorquers, *etc*. In this article a cluster of four reaction wheels has been chosen; this kind of actuators allows the satellite to have soft variations of its internal torque in order to achieve a precise axis orientation. The reaction wheels are arranged in a pyramidal configuration, which presents advantages over a configuration with three wheels, typically aligned with the principal axes of the body. The pyramidal configuration represents a redundancy useful in case of failure, and from the point of view of the mathematical model, its presents a more general approach for a momentum interchange device.

Kinematic

The kinematics of the satellite is presented using a quaternion $\mathbf{q}$. The quaternion representation avoid singularities and trigonometric functions.[3,4]

$$\dot{\mathbf{q}} = \frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{\Omega}(\omega)\mathbf{q} \tag{1}$$

where

$$\boldsymbol{\Omega} = \begin{bmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{bmatrix} \tag{2}$$

A quaternion review focus in spacecraft attitude determination and control is present by Yang.[5]

Dynamic

The rotational motion about a fixed point is described by a set of equations known as *Euler's equations of motion*,[6] which describe the angular momentum conservation, $\mathbf{h} = [h_1 \quad h_2 \quad h_3]^{\mathrm{T}}$, for a rigid body.

$$\dot{\mathbf{h}} + \boldsymbol{\omega} \times \mathbf{h} = \boldsymbol{\tau} \tag{3}$$

Only the torque $\boldsymbol{\tau} = [\tau_1 \quad \tau_2 \quad \tau_3]^{\mathrm{T}}$, can change the angular momentum magnitude, and it can be external or internal.

With the addition of the reaction wheels, the body's dynamics change. Now the Eq. (3) can be rewrite as:[3,4]

$$\mathbf{I}\dot{\boldsymbol{\omega}} = -\boldsymbol{\omega} \times (\mathbf{I}\boldsymbol{\omega} + \mathbf{L}\mathbf{h}_{\mathrm{w}}) - \mathbf{L}\boldsymbol{\tau}_{\mathbf{w}} + \boldsymbol{\tau}_{\mathrm{ext}} \tag{4}$$

where $\mathbf{I}$ is the moment of inertia of the satellite (3×3), $\boldsymbol{\omega}$ is the vector of angular velocities (3×1), $\mathbf{h}_{\mathrm{w}}$ is the angular momentum provide by the wheels (4×1), $\boldsymbol{\tau}_{\mathrm{w}}$ is the torque that each wheel produce and change the satellite's attitude (4×1), in other words is the control torque. The term $\boldsymbol{\tau}_{\mathrm{ext}}$ includes all the external torque that can modify the body's attitude (3×1). In the case of a satellite its could be: gravitational gradient, aerodynamics drag, magnetic torque, *etc*.[7] Finally, $\mathbf{L}$ is the orientation matrix and will be explained below.

Reaction Wheels Configuration

The wheel orientation matrix $\mathbf{L}$ has three rows because the satellite axes of rotation. Its number of columns will depends of the number of reaction wheels, for this paper four reaction wheel has been used, so $\mathbf{L} \in \mathbb{R}^{3\times 4}$. If the reaction wheels are arranged in a pyramidal configuration as show figure 1, the orientation matrix $\mathbf{L}$ is given by:

$$\mathbf{L} = \begin{bmatrix} \cos\alpha\sin\beta & -\sin\alpha\sin\beta & -\cos\alpha\sin\beta & \sin\alpha\sin\beta \\ \sin\alpha\sin\beta & \cos\alpha\sin\beta & -\sin\alpha\sin\beta & -\cos\alpha\sin\beta \\ \cos\beta & \cos\beta & \cos\beta & \cos\beta \end{bmatrix} \tag{5}$$

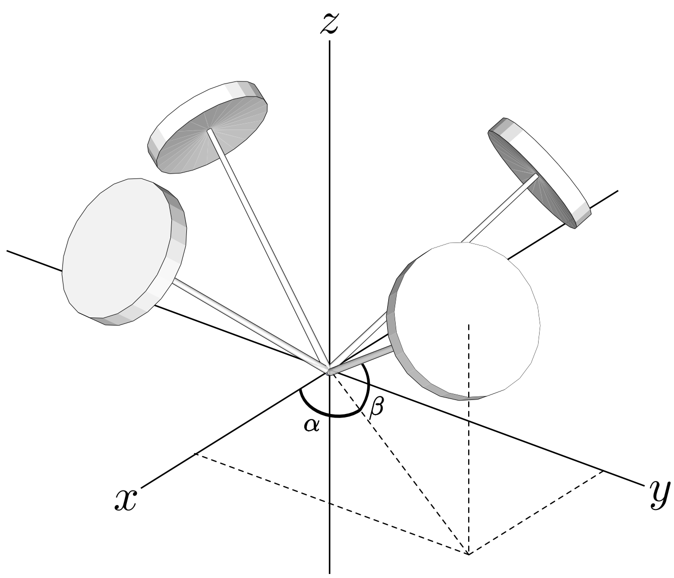

Figure 1. Reaction wheel in pyramidal configuration.

Reaction Wheels Model

In general, the reaction wheels can be classified as a momentum interchange device. The reaction wheels achieve precision in pointing through a smooth change in its momentum.[8]

The reaction wheels were modeled like a free wheel drive by a dc motor, which can spin in both directions achieving a control axis, can driven independently and its speed can be modified by the controller. The pyramidal configuration give a three axis control, because the momentum component not are in one unique axis. Each time one wheel spin, an angular momentum vector is generate on the axis where the wheel is aligned, redirecting in this way the spacecraft.[7,8]

For the electrical and mechanical representation of the wheels a dc motor model is used. The electrical representation of the motor is:

$$V - V_R - V_L - V_a = 0 \tag{6}$$

where V_R, V_L y V_a are the resistance, coil voltages and the counter–electromotive force, respectively, and can be written as:

$$\begin{aligned} V_R &= iR \\ V_L &= L\frac{di}{dt} \\ V_a &= K_e\omega_\text{w} \end{aligned} \tag{7}$$

The terms K_e and ω_w in Eq. (7) are the motor counter–electromotive force constant and angular velocity of the reaction wheel , respectively. The differential equation that depict the electrical part of the motor is:

$$\frac{di}{dt} = -\frac{K_e}{L}\omega_\text{w} - \frac{R}{L}i + \frac{V}{L} \tag{8}$$

The mechanical representation of the motor use the Newton's second law for the torques.

$$\tau_e - \tau_b = \dot{h} \tag{9}$$

where τ_e and τ_b are the electromagnetic torque and the torque generated by friction, respectively, and can be written as:

$$\begin{aligned} \tau_e &= K_m i \\ \tau_b &= b\,\omega_\text{w} \end{aligned} \tag{10}$$

The term K_m and b in Eq. (10) are the motor electromagnetic torque constant and the friction constant, respectively. The differential equation that depicts the mechanical part of the motor is:

$$K_m i = J\frac{d\omega_\text{w}}{dt} + b\omega_\text{w} \tag{11}$$

Figure 2 shown the block diagram for the Eq. (8) and (11).

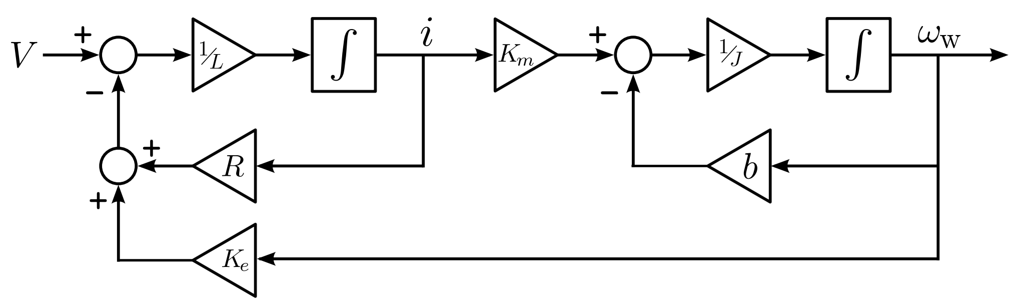

Figure 2. Block diagram for the electrical and mechanical parts of one wheel.

Is important to keep in mind that the block diagram in Figure 2 is just for one wheel and all the wheels are identical.

CONTROLLER

A PD controller algorithm was used to provide the control law for the actuators, the reaction wheels, and to achieve the desired attitude. The Equation 12 shown the control law.

$$\mathbf{u} = -\mathbf{K_P}\mathbf{q_e} - \mathbf{K_D}\boldsymbol{\omega} \tag{12}$$

where $\mathbf{K_P}$ and $\mathbf{K_D}$ are the proportional and derivative gains vectors of the controller. $\mathbf{u}$ is the control law and $\mathbf{q_e}$ is the attitude error quaternion and is defines as:

$$\begin{bmatrix} q_{e1} \\ q_{e2} \\ q_{e3} \\ q_{e4} \end{bmatrix} = \begin{bmatrix} q_{c4} & q_{c3} & -q_{c2} & -q_{c1} \\ -q_{c3} & q_{c4} & q_{c1} & -q_{c2} \\ q_{c2} & -q_{c1} & q_{c4} & -q_{c3} \\ q_{c1} & q_{c2} & q_{c3} & q_{c4} \end{bmatrix} \begin{bmatrix} q_{m1} \\ q_{m2} \\ q_{m3} \\ q_{m4} \end{bmatrix} \tag{13}$$

where $\mathbf{q_c}$ is the commanded quaternion and $\mathbf{q_m}$ is the actual quaternion.

The diagram block in Figure. 3 depict the overall system used for simulation.

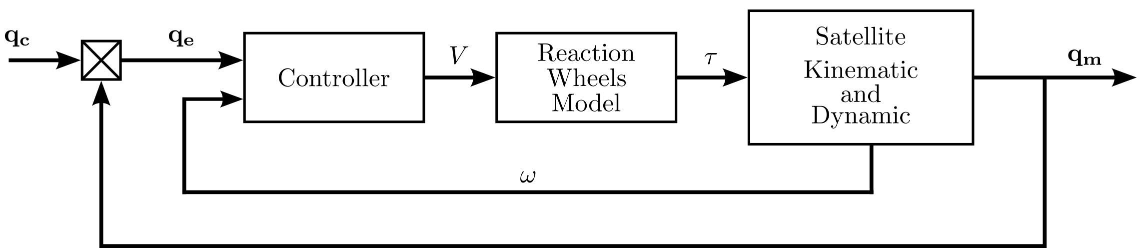

Figure 3. Scheme of the overall system considering a pyramidal reaction wheel configuration

SIMULATION PARAMETERS

Table 1 and 2 show the satellite and reaction wheels parameters and its values used for the simulation.

Table 1. Satellite parameters.

Parameter	Value	Units
Dimensions	10 x 10 x 30	cm x cm x cm
Mass	4	kg
Moments of inertia	[0.0333, 0.0333, 0.0067]	kg m^2

Table 2. Reaction wheels parameters.

Parameter	Value	Units
Maximum torque	$\pm 0.635 \times 10^{-3}$	Nm
Maximum wheel speed	± 1000	rpm
Maximum voltage	± 12	V

The angles for the pyramidal configuration in Figure 1 are: $\alpha = 0$ and $\beta = [10°, 25°, 45°, 65°, 80°]$. The values $\beta = 0°$ and $\beta = 90°$, were not considered because it not allows three axis control.

The attitude set–point for Roll, Pitch and Yaw, were: $\theta = 40°$, $\phi = 20°$ and $\psi = 60°$.

RESULTS

This section contains the simulation results for the evaluation in power consumption for each angle of the pyramid configuration β, presented in the previous section.

Figures 4, 5 and 6 show the attitude response for each pyramid angle roll, pitch and yaw, respectively. The set–point was the same for the five cases. As figures show, for all cases the system achieves a certain attitude. That means that the controller is working properly. Although, for $\beta = 10°$ the desired attitude wasn't achieved.

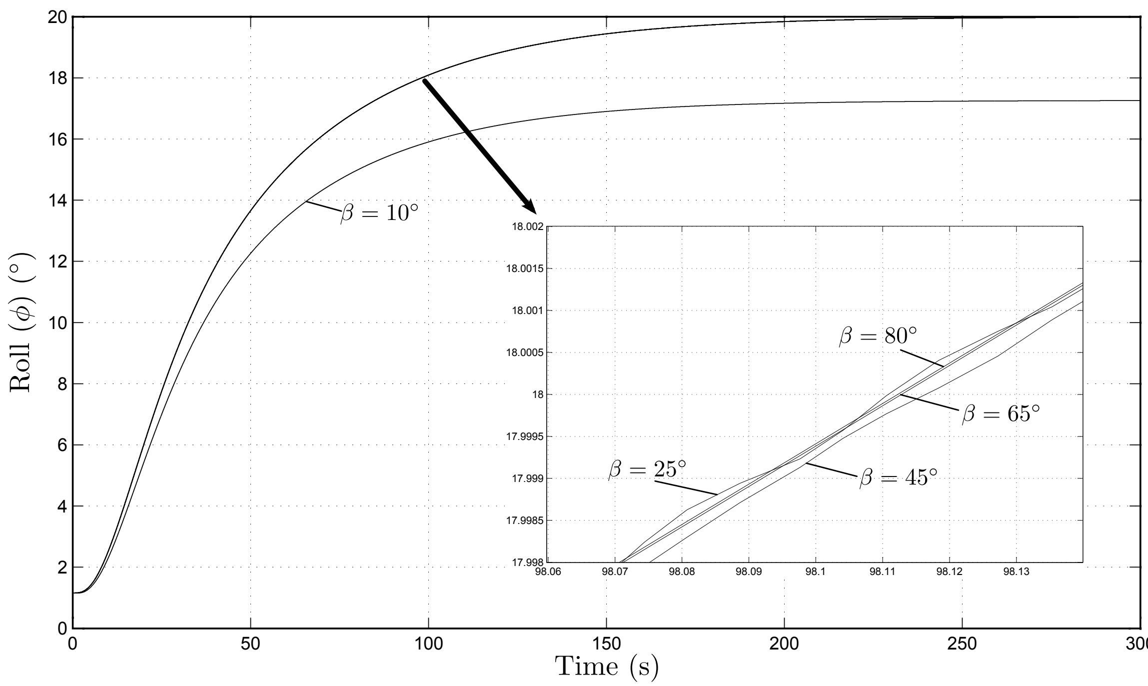

Figure 4. Attitude response for Roll (ϕ) for each pyramid angle $\beta = [10°, 25°, 45°, 65°, 80°]$

Figure 7 shows the curve of generated power by the cluster, for each angle β in the pyramid. As expected, $\beta = 45°$ have a lower pick, meanwhile $\beta = 10°$ have the pick more higher.

Figure 8 shows the energy consumption for the cluster. The results show again the deficient performance for $\beta = 10°$. For this angle, there is a major energy consumption without achieving the attitude desired. It can be infer that a region exists around $\beta = 45°$ where there is a lower energy consumption.

CONCLUSION

The power consumption for a four reaction wheels cluster in a pyramid configuration was evaluated, varying the pyramid angle. The power consumption curves (Figure 7) are function of the voltage and current in the dc motors. The results show a low power consumption for the angle in the middle of the range chosen, $\beta = 45°$. For all the angles values of the pyramid, β, the overall system achieves an attitude, but the desired attitude are not carried out for all the values of β, as can see in Figure 4, 5 and 6. The worse case is for $\beta = 10°$, where the controller is limited to get the desired attitude. The control algorithm used was a PD (Proportional-Derivative) controller. These

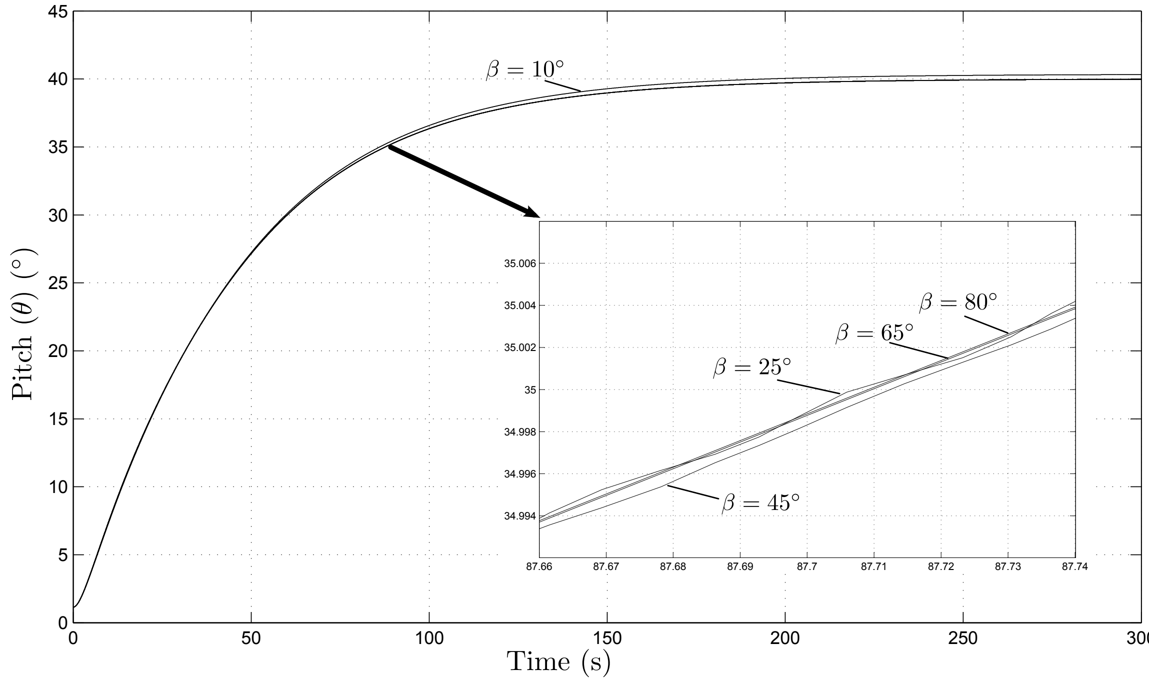

Figure 5. Attitude response for Pitch (θ) for each pyramid angle $\beta = [10°, 25°, 45°, 65°, 80°]$

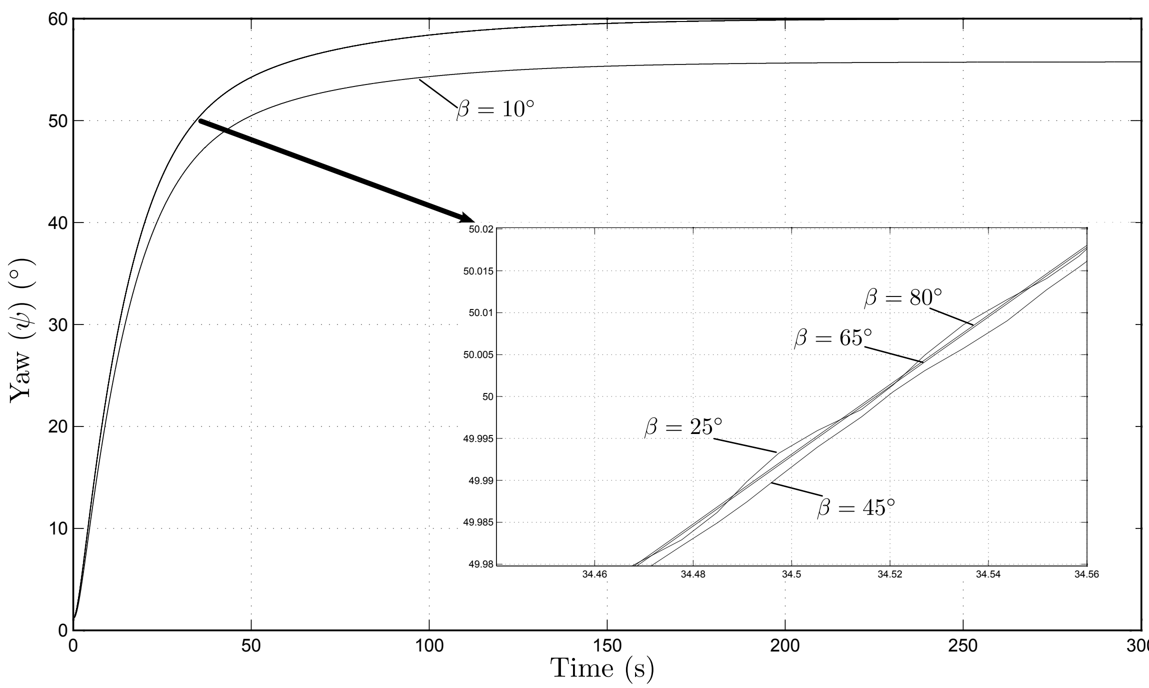

Figure 6. Attitude response for Yaw (ψ) for each pyramid angle $\beta = [10°, 25°, 45°, 65°, 80°]$

results are attributes to the satellite geometry and the arrangement of the reaction wheels cluster with respect to satellite axes. According to the Figure 7 and 8, a carefully selection of the pyramid angle implies low power consumption with attitude performance.

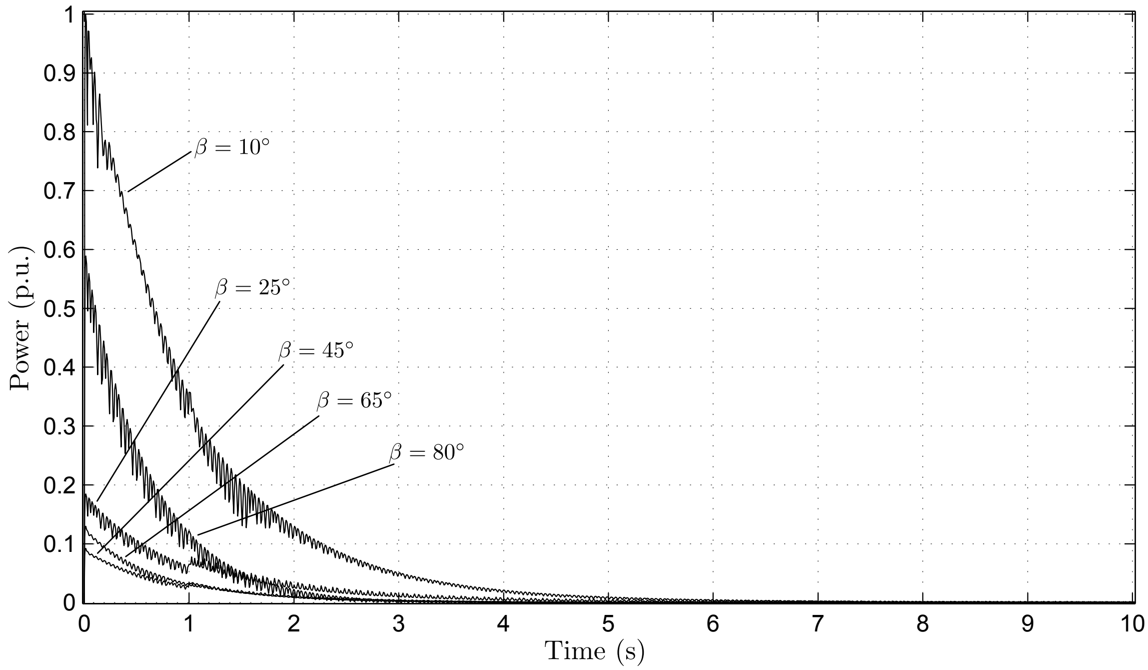

Figure 7. Power generate by the cluster for each angle.

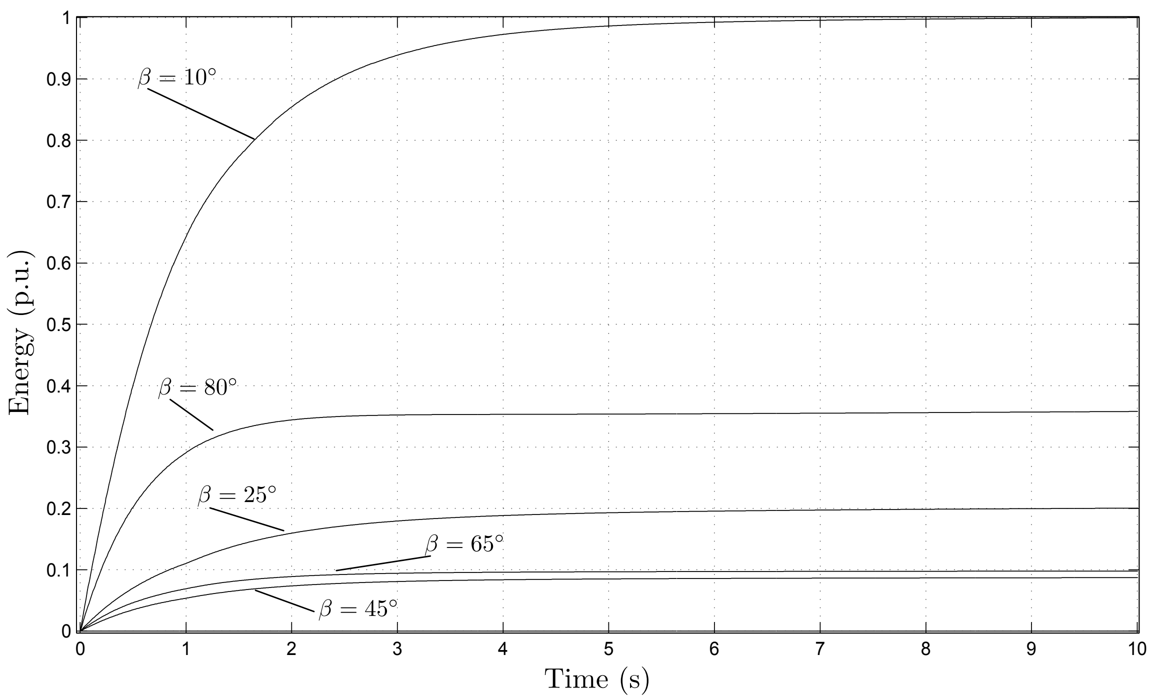

Figure 8. Energy consume by the cluster for each angle

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