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OPTIMAL SPACECRAFT TRAJECTORIES FOR FLIGHT TO ASTEROID APOPHIS WITH RETURN TO EARTH USING CHEMICAL HIGH THRUST ENGINES

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Energy optimal trajectories for the flight to asteroid Apophis, staying there during some time and following return to the Earth are determined and investigated in the paper. The Rocket "Soyuz" is proposed to be used for the spacecraft launch onto an initial LEO orbit. The upper stage "Fregat" is used for the escape. A special chemical engine is used for the following heliocentric corrections, and maneuvers, including the deceleration as well as the acceleration near the Apophis. On the first stage of the analysis, the impulse approximation is performed for the trajectories determination. Then the trajectories are varied with taking into account the thrust values of the chemical engines used. There are determined optimal trajectories and their characteristics for the expedition from Earth to Apophis and back for the flights during 2019-2022 years, with the flight duration up to two years. Comparison with the flight using the low thrust electric-jet engines is performed. The flight of the asteroid's satellite near Apophis is analyzed at the last part of the paper.

INTRODUCTION

Asteroid Apophis will have in this century some approaches to the Earth. There is even any small positive probability of the Apophis-Earth collision. Because of this, the Apophis investigation by the SC devices and an analysis of the asteroid matter in the Earth laboratories are actual. So, a space expedition to Apophis with returning to the Earth is supposed to be perspective. It is confirmed by the space experience, e.g. by Lunar missions, *Stardust* NASA Mission, *Genesis* NASA Mission, Japanese Mission *Hayabusa* (References 1-3). Energy optimal trajectories for the expedition including a flight to asteroid Apophis, staying there during some time and following return to the Earth are investigated in the paper. For the main analysis, Rocket Soyuz-FG with the upper stage "Fregat" is proposed to be used for this expedition (Reference 4). Then other rockets are considered.

The paper continues the studies (References 5 and 6). Two groups of the flights are studied there. One group is composed by the SC flights that use a high thrust chemical engine for escape and a low thrust electric-jet engine for interplanetary flight. Another group is formed by the SC flights that use only usual high thrust chemical engines. It is shown that the payload mass is more if the electric jet engines are used. Nevertheless, it is easier to realize the flight with the usual

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high thrust engines (HTE). Because of this, this case of the flight is analyzed in more details in our paper.

The flight of the asteroid's Apophis satellite near Apophis is analyzed in the paper, too. The observations and the trajectory measurements of this satellite together with a device on the Apophis surface can be used to define better the orbital and attitude motion of the asteroid. These "daughter" satellite and the surface device are supposed to be placed from the expedition main "mother" spacecraft.

SCHEME OF FLIGHT IN CONSIDERATION

We consider the flight that consists of several main stages. At the initial, geocentric stage, $t \in [t_0, t_1]$, the spacecraft (SC) is launched to a low (with an altitude H≈200 km) Earth satellite orbit (LEO) by the rocket ("Soyuz-FG" at the main analysis). After a passive flight along this orbit, in an optimal point, the SC is accelerated to a hyperbolic velocity using a high thrust engine (HTE) of the upper stage ("Fregat"). This acceleration is performed by two switch of the upper stage. After that, the upper stage is separated, and the SC moves in the Earth sphere of influence leaving it at t_1 (see References 3, 5, and 6).

Then, at the first heliocentric stage of the flight, $t \in [t_1, t_2]$, the SC moves passively from the Earth to the Apophis. The determination of the trajectory on the basis of the trajectory measurements is performed during this flight and the correction maneuvers are executed to compensate the effect of the rocket launcher operations errors and approach the Asteroid correctly.

At the near-asteroid stage of the flight, $t \in [t_2, t_3]$, after the deceleration by the second, interplanetary HTE, the SC performs the near-asteroid motion (with, may be, landing, taking the asteroid mutter samples and some investigations on the asteroid surface). The asteroid satellite and the surface device are supposed to be separated from the spacecraft and placed on their positions. At the end of this stage, the SC is accelerated by that HTE to have a necessary velocity for the flight to the Earth.

At the second heliocentric stage of the flight, $t \in [t_3, t_4]$, the SC returns to the Earth. Then, at the second geocentric stage, we have the final part of the flight, $t \in [t_4, t_5]$.The descending probe is separated here, enters to the Earth atmosphere, descents in the atmosphere and lands on the Earth surface. Here, we do not put any limitations on the SC velocity at the end of the flight.

We consider that the summary time $\Delta t_\Sigma = \Delta t = t_4 - t_1$ is taken to 730 days, and the time near the asteroid $\Delta t_A = t_3 - t_2 = 7$ days. The optimal trajectories are found varying the initial time t_1 in the range [05.01.2019; 31.12.2022].

It is necessary to define the dates t_0, t_1, t_2, t_3, t_4, t_5, value and direction of the SC velocities "at infinity" $V_\infty(t_1)$, $V_\infty(t_2)$, $V_\infty(t_3)$, and $V_\infty(t_4)$ for the departure from the Earth to Apophis, for the arrival to the asteroid from the Earth, for the departure from the Apophis to the Earth, and for the arrival to the Earth in order to maximize the final "payload" mass of the SC m_P.

AN APPROXIMATE ANALYSIS

The optimal trajectory is determined first in frame of a simplified model. In this analysis, the impulse approximation is used to perform the energy and fuel mass consumption evaluation for the active engine parts of the space flight during the acceleration near the Earth as well as during the deceleration and acceleration near the asteroid.

For an analysis of the geocentric flight of the SC near the Earth, the model of the Earth sphere of influence is used with taking into account the Earth central gravity only. When the SC is out of the Earth sphere of influence, the SC heliocentric motion is analyzed taking into account the Sun central gravity only, in model of the Earth point sphere of influence. For an analysis of the SC asteroid-centric motion near the asteroid, a model of the asteroid sphere of influence is used with taking into account the asteroid central gravity only.

After this simplified analysis, the exact numerical analysis is performed with taking into account the perturbation factors.

FIRST GEOCENTRIC PHASE OF SC FLIGHT

The SC motion in the Earth and asteroid spheres of influence is an "inner" problem.

According to the scheme of the SC flight and the approximate model of analysis taken, it is believed that the SC launch to the LEO with the altitude H_0 is carried out by a rocket (like "Soyuz", "Zenit" etc.), which ensures the delivery of mass m_0 to this initial orbit, then subsequent impulse acceleration of the spacecraft is performed by the upper first engine stage (like "Fregat") to a geocentric hyperbolic orbit with a velocity at "infinity" $\mathbf{V}_{\infty 1}$. After that, this first accelerating engine stage with its mass of $m_{1\mathrm{E}}$ is separated from the SC, and the SC moves to the boundary of the Earth sphere of influence to begin there the heliocentric stage of the flight.

The SC mass m_1 at the start of the heliocentric motion is given by the equation:

$$m(t_1) = m(t_0)\,\boldsymbol{exp}\left(-\frac{\Delta V_1}{W_{1E}}\right) - m_{1E} \tag{1}$$

where

$$\Delta V_1 = \sqrt{V_{\infty 1}^2 + \frac{2\mu_{\mathrm{E}}}{r_0}} - \sqrt{\frac{\mu_{\mathrm{E}}}{r_0}} \tag{2}$$

is the acceleration impulse value; W_{1E} is a jet exhaust velocity for the accelerating upper first engine stage; μ_E is a gravity parameter of the Earth; $r_0=r_E+H_0$ is a radius of the initial LEO; r_E is the Earth's middle radius.

HELIOCENTRIC PHASES OF SC FLIGHT

Then there is the first passive (with some corrections) phase of the SC heliocentric flight from the Earth to Apophis. After the motion near the asteroid, we have the second passive (with some corrections) phase of the SC heliocentric flight from Apophis to the Earth. This is an "outer" problem, which determines the trajectory.

To calculate the characteristics of the expedition, we set: the initial time t_1, duration Δt_1 of the flight from the Earth to Apophis, and the total duration $\Delta t = \Delta t_\Sigma$. Then: $t_2 = t_1 + \Delta t_1$ is the time of the SC arrival to Apophis, $t_3 = t_2 + 7$ days is the time of the SC departure from Apophis to the Earth, $t_4 = t_1 + \Delta t_\Sigma$ is the time of the SC arrival to the Earth orbit.

According to the simplified model taken, the SC heliocentric motion is analyzed taking into account the Sun central gravity only:

$$\dot{\mathbf{r}} = \mathbf{V}; \;\; \dot{\mathbf{V}} = \mathbf{g}(\mathbf{r}) = -\mu_S \frac{\mathbf{r}}{r^3}; \tag{3}$$

where **r** and **V** are the SC heliocentric position and velocity vectors, μ_S is the Sun gravity parameter.

For this model, using the times (t_1, t_2) and (t_3, t_4), and the Earth and asteroid orbits, first we determine by usual way the initial and final SC position vectors ($\mathbf{r}_1$; $\mathbf{r}_2$) and ($\mathbf{r}_3$; $\mathbf{r}_4$) as well as the Earth and Apophis velocities for these two phases:

$$\mathbf{r}_1=\mathbf{r}_E(t_1);\ \mathbf{V}_E(t_1);\ \mathbf{r}_2=\mathbf{r}_A(t_2);\ \mathbf{V}_A(t_2); \tag{4}$$

$$\mathbf{r}_3=\mathbf{r}_A(t_3);\ \mathbf{V}_A(t_3);\ \mathbf{r}_4=\mathbf{r}_E(t_4);\ \mathbf{V}_E(t_4). \tag{5}$$

Using these initial and final SC position vectors and corresponding times (t_1; $\mathbf{r}_1$; t_2; $\mathbf{r}_2$) and (t_3; $\mathbf{r}_3$; t_4; $\mathbf{r}_4$), we determine the elements of heliocentric orbits $\mathbf{q}_{12}$ and $\mathbf{q}_{34}$ and the SC initial and final heliocentric velocities ($\mathbf{V}_1$; $\mathbf{V}_2$) and ($\mathbf{V}_3$; $\mathbf{V}_4$) for the flight from the Earth to Apophis, $t_1 \le t \le t_2$, and back, $t_3 \le t \le t_4$, by a two-fold solution of the Euler-Lambert Problem (taking into account the possibility of a one passive revolution in the flight), see References 7-11:

$$\mathbf{V}_1=\mathbf{V}(\mathbf{q}_{12}, t_1);\ \mathbf{V}_2=\mathbf{V}(\mathbf{q}_{12}, t_2);\ \mathbf{V}_3=\mathbf{V}(\mathbf{q}_{34}, t_3);\ \mathbf{V}_4=\mathbf{V}(\mathbf{q}_{34}, t_4). \tag{6}$$

Using these velocities as well as the Earth's and asteroid's ones, we determine the SC relative velocities; they are supposed for this model to be equal to the velocities at "infinity" $\mathbf{V}_{\infty 1}$ – for the flight away from the Earth, $\mathbf{V}_{\infty 2}$ - for the approach to Apophis, $\mathbf{V}_{\infty 3}$ – for the flight away from the asteroid and $\mathbf{V}_{\infty 4}$ - at the approach to the Earth:

$$\mathbf{V}_{\infty 1} = \mathbf{V}_1 - \mathbf{V}_E(t_1);\ \mathbf{V}_{\infty 2} = \mathbf{V}_2 - \mathbf{V}_A(t_2);\ \mathbf{V}_{\infty 3} = \mathbf{V}_3 - \mathbf{V}_A(t_3);\ \mathbf{V}_{\infty 4} = \mathbf{V}_4 - \mathbf{V}_E(t_4). \tag{7}$$

The velocity $\mathbf{V}_{\infty 1}$ determines the value of the velocity impulse ΔV_1 and the fuel mass for the acceleration from the Earth by the first HTE (1, 2). Similarly, the velocities $\mathbf{V}_{\infty 2}$ and $\mathbf{V}_{\infty 3}$ determine the velocity impulses ΔV_2 – for the arrival deceleration near the asteroid by the second high thrust engine, and ΔV_3 – for the departure acceleration from Apophis by this engine, too.

This makes it possible to find the total fuel mass consumption and the SC mass after the impulses ΔV_2 and ΔV_3. The SC mass $m(t_2)$ after the second impulse:

$$m(t_2)=m(t_1)\exp(-\Delta V_2/W_{2E}); \tag{8}$$

and the SC mass $m(t_3)$ after the third impulse:

$$m(t_3)=m(t_2)\exp(-\Delta V_3/W_{2E}); \tag{9}$$

where W_{2E} is the jet exhaust velocity for the second engine block HTE.

We suppose here that this mass $m(t_3)$ is a final mass m_f. After the subtraction of the second HTE block mass m_{2E} from the final mass, we have a "payload" mass:

$$m_P=m(t_3) - m_{2E}; \tag{10}$$

where

$$m_{2E} = m_{20} + a_{F2}\, m_{F2}, \tag{11}$$

here m_{20} is a constant component of the engine mass, a_{F2} is the coefficient of the mass for the fuel tanks, m_{F2} is the mass of fuel for the SC deceleration and acceleration near the asteroid

$$m_{F2}=m(t_1) - m(t_3). \quad (12)$$

The optimum trajectories with a maximum payload mass m_P are found varying the initial time t_1, and the time for the flight from Earth to Apophis Δt_1 for various durations of the flight Δt_Σ.

NEAR-ASTEROID PHASE

The calculated maneuvers of the spacecraft near Apophis are a transfer from the SC heliocentric orbit into an orbit around the asteroid and back. Radius of the Asteroid's sphere of influence is estimated by

$$R_A \approx r_{AS}\,(m_A/m_S)^{2/5} \approx 2 \text{ km}.$$

Here r_{AS} is the Asteroid-Sun distance, m_A is the asteroid mass ($(2.7\text{-}4.3)\cdot 10^{10}$ kg), and m_S is the Sun mass. It is assumed in the paper that the radius of the orbit of an artificial satellite of the asteroid is 0.5 km or more.

The gravitational parameter of Apophis is very small, ~ 2-3 m^3/s^2. So, the gravity effect of the Apophis is low enough. The circular and escape velocities are small (about 0.1 m/s). So, the velocity impulses ΔV_2 and ΔV_3 are practically equal to the corresponding velocities at "infinity".

SOME NUMERICAL RESULTS

For the initial analysis, it is believed that the SC launch to the LEO is carried out by the rocket "Soyuz-FG", which ensures the delivery of mass $m_0 \approx 7130$ kg to an initial orbit with the altitude H_0=200 km. Subsequent acceleration of the spacecraft is performed by the upper block "Fregat". Its thrust value is of about 2000 N, a jet exhaust velocity is supposed to be W_{1E} ~ 3198 m/s with a specific thrust of 326 s; a separated mass of the stage "Fregat" m_{1E} is of 970 kg. For $V_{\infty 1} = 0$, the SC mass $m(t_1)$ is 1630 kg.

For the second engine HTE, a specific thrust is supposed to be 304 s, $W_{2E} \approx 2982$ m/s. The thrust value is of about 400 N. For this HTE, we believe that for its mass (11): constant component m_{20} is 100 kg, the coefficient of the fuel tanks mass a_{F2} is 0.15.

The optimum trajectories with a maximum payload mass m_P are found varying start time t_1 in the range [01.05.2019; 31.12. 2022], and the time for the flight from Earth to Apophis Δt_1 for various durations of the expedition Δt_Σ from 1 year to 2 years. Two groups of the expedition trajectories are analyzed. For the first group, the transfer arcs of the heliocentric phases are less than one revolution, $\Delta u<2\pi$. For the second group, at least one heliocentric part includes a whole passive revolution; its angular arc $\Delta u>2\pi$.

For the first group, Table 1 gives optimal data for initial time t_1, duration of the first heliocentric phase Δt_1, final mass m_f and the payload mass m_P.

The best variant here corresponds to the mission duration Δt = 450 days with the payload mass m_P=182 kg, initial time t_1=23.01.2021, the time for the flight from the Earth to Apophis Δt_{12} = 120 days, the back duration Δt_{34} = 323 days, the return time t_4=18.04.2022.

For departure from the Earth: the velocity at "infinity" $V_{\infty 1}$ is 3.784 km/s, the velocity impulse value ΔV_1 is 3.856 km/s. For the arrival to Apophis: $\Delta V_2 \approx V_{\infty 2}$= 2.296 km/s. For the departure from Apophis: $\Delta V_3 \approx V_{\infty 3}$= 0.912 km/s.

Figure 1 gives the final mass depending the initial time if the time for the flight from the Earth to Apophis Δt_{12} = 120 days, and the summary time Δt = 450 days. Figures 2 and 3 give this optimal trajectory. Figure 2 shows the first part of the flight, from the Earth initial point $P_1(t_1)$ to the point $P_2(t_2)$. Figure 3 shows the flight from Apophis $P_3(t_3)$ to Earth $P_4(t_4)$.

Table 1. Optimal data in case of Δu<2π

N	Δt, days	t_1 , date	Δt_1, days	m_f , kg	m_P , kg
1	390	10.04.2021	116	313	81
2	420	22.02.2021	97	382	180
3	450	23.01.2021	120	397	182
4	480	08.01.2021	214	287	68
5	510	20.05.2020	272	353	110
6	540	22.10.2019	223	336	175
7	570	22.09.2019	258	346	139
8	600	23.08.2019	284	323	77
9	630	24.07.2019	313	290	17
10	730	15.14.2020	370	345	144

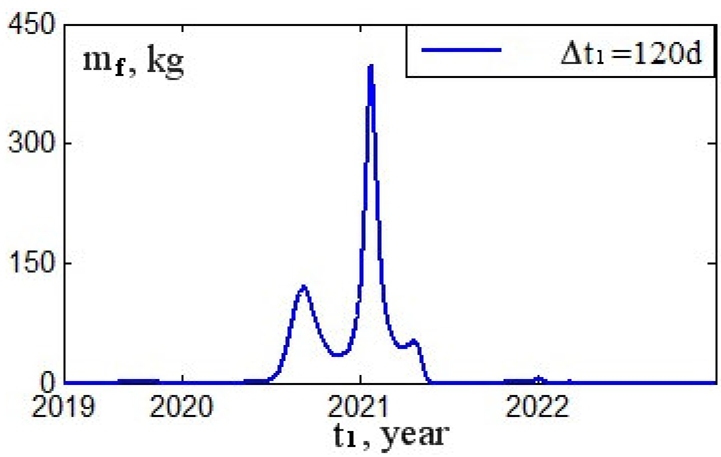

Figure 1. Final mass versus initial time for Δt_{12} =120 days, mission time Δt = 450 days.

Table 2 gives for the second group of trajectories the optimal data in initial time t_1, duration of the first heliocentric phase Δt_1, final mass m_f and the payload mass m_P.

We can see here that characteristics of the mission are improved. The best variant here corresponds to the mission duration Δt = 690 days with the payload mass m_P=265 kg, initial time t_1=24.05.2019, the time for the flight from the Earth to Apophis Δt_{12} = 335 days, the back duration Δt_{34} = 348 days, the return time t_4=13.04.2021. For departure from the Earth, the velocity at "infinity" $V_{\infty 1}$ is ~1.892 km/s, the velocity impulse value ΔV_1 is ~3.386 km/s. For the arrival to Apophis: $\Delta V_2 \approx V_{\infty 2}$= 2.834 km/s. For the departure from Apophis: $\Delta V_3 \approx V_{\infty 3}$= 0.370 km/s.

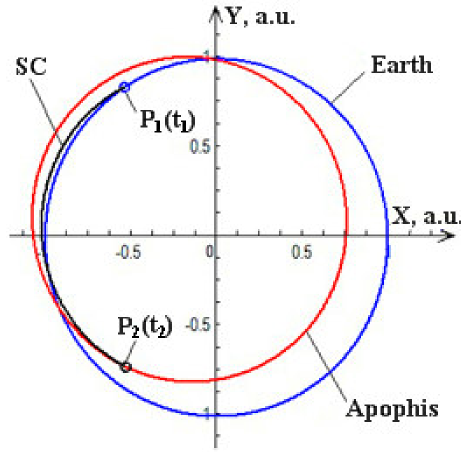

Figure 2. Flight from Earth to Apophis for optimal trajectory if Δu<2π.

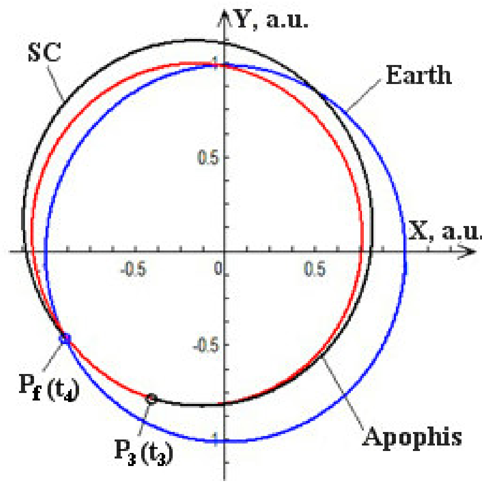

Figure 3. Flight from Apophis to Earth for optimal trajectory if Δu<2π.

Table 2. Optimal data in case of Δu>2π

N	Δt, days	t_1, date	Δt_1, days	m_f, kg	m_P, kg
11	540	22.10.2019	211	343	186
12	570	22.09.2019	245	280	61
13	600	23.08.2019	267	379	135
14	630	23.07.2019	280	438	180
15	660	23.06.2019	304	492	235
16	690	24.05.2019	335	513	265
17	730	27.04.2020	308	437	224

Figure 4 gives the SC final mass depending the initial time if the time for the flight from the Earth to Apophis Δt_{12} =335 days, and the mission time Δt=690 days.

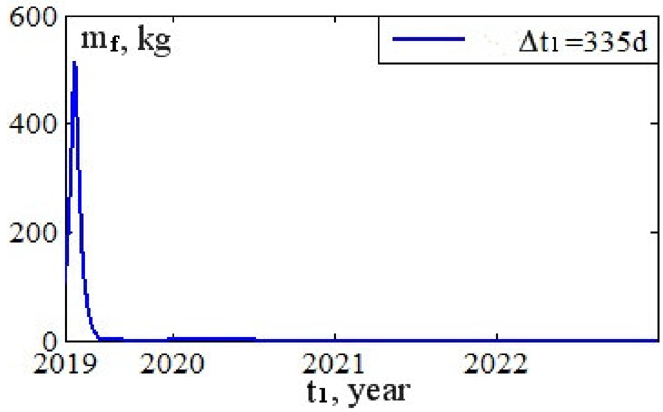

Figure 4. Final mass versus initial time for Δt_1 =335 days, and mission time Δt=690 days.

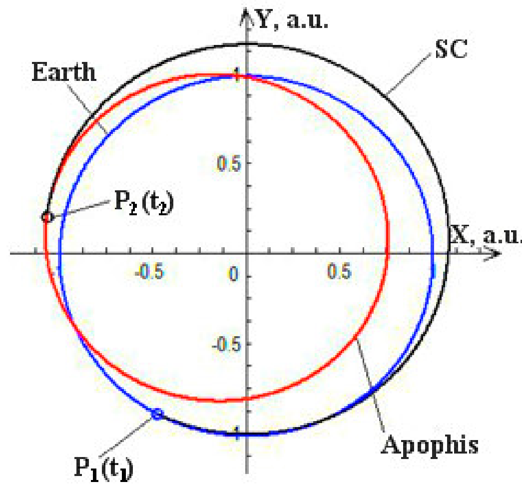

Figure 5. Flight from Earth to Apophis for optimal trajectory if $\Delta u_{34}>2\pi$.

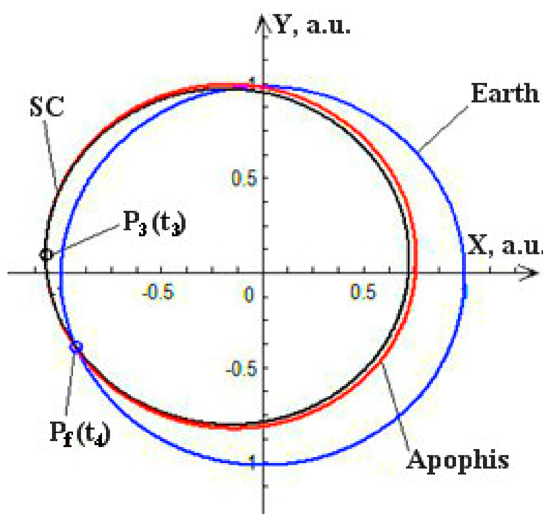

Figure 6. Flight from Apophis to Earth for optimal trajectory if $\Delta u_{34}>2\pi$.

Figures 5 and 6 give this optimal trajectory. Figure 5 shows the first part of the flight, from the Earth initial point $P_1(t_1)$ to the point $P_2(t_2)$. Figure 6 shows the second part of the flight, from the Apophis point $P_3(t_3)$ to the Earth point $P_4(t_4)$.

We note that the SC return to the Earth takes place in the ascending node of the Apophis orbit for both optimal trajectories. So, the SC second heliocentric orbit is at the Apophis orbit plane practically.

MORE PRECISE CALCULATION OF THE MISSION TRAJECTORIES

At the second phase of an analysis, we performed a precise calculation of the optimal trajectories received. It is made in several directions. There are two main groups of the factors. The first one is a correction of the trajectory to take into account the real gravity field. The second one is the correction of the mission energy characteristics.

At the first, main analysis, we wanted to verify that the perturbations, which in the Lambert analysis are not considered, are in fact very small for the trajectories above and there are not some close approaches, to the Moon, for example. To take into account the real gravity field for the Earth-Apophis-Earth flight, we calculated the SC motion first for here according to the following differential equation of the SC motion:

$$\ddot{\mathbf{r}} = -\frac{\mu_E}{|\mathbf{r}|^3}\mathbf{r} - \sum_i \mu_i \left(\frac{\mathbf{r}_i}{|\mathbf{r}_i|^3} + \frac{\mathbf{r}-\mathbf{r}_i}{|\mathbf{r}-\mathbf{r}_i|^3} \right) + \Delta; \qquad (13)$$

here: $\mathbf{r}$ $(x,\ y,\ z)$ is the radius-vector of the spacecraft in the Cartesian nonrotating geocentric geoequatorial frame XYZ, where the X axis points in the mean Vernal Equinox direction, the Z axis is perpendicular to the mean geo-equator for J2000.0; $\mathbf{r}_i$ is the radius-vector of the i-th celestial body, they are taken according to the JPL-ephemerides DE405 for the planets and Moon coordinates; μ_E and μ_i are the gravity parameters of the Earth and i-th celestial body; additional term Δ takes into account the Earth oblateness.

For the first Earth-Apophis phase we integrate (13) from the initial time for the beginning of a passive flight after near-Earth engine acceleration, $t_{in}=t_{0p}$, to the final time of the SC-Apophis approach, $t_f=t_2$, for the transfer to the asteroid satellite orbit. For the final time, we calculate a residual, the difference $\Delta\mathbf{r}_{\mathrm{f}}$ of the SC real position $\mathbf{r}(t_f)$ and the nominal one $\mathbf{r}_n(t_f)$. This vector is a measure of the perturbations. Its initial value for the trajectories obtained is 1-2 mln km.

Varying initial control parameters of the flight, we decrease the final residual $\Delta\mathbf{r}_{\mathrm{f}}$ to zero correcting the trajectory. For the first phase, we vary the start time t_0, the acceleration beginning time and the acceleration final velocity. They have small variations. So, for the trajectory 3 (Fig. 2) with $\Delta u<2\pi$, the Lambert start time t_0 is 2021,1,20,10:17:32. After correction according to (13), it decreases to 5^{m}. For the optimal trajectory 16 (Figure 4) with $\Delta u>2\pi$, the Lambert start time t_0 is 2019,5,19,15:37:24.14. After correction according to (13), it increases to $1^{\mathrm{m}}\ 21^{\mathrm{s}}$.

By similar way, the second phase is tested and corrected. For this phase we integrate (13) from the initial time for the beginning of a passive flight after near-asteroid engine acceleration, $t_{in}=t_3$, to the final time of the SC-Earth approach, $t_f=t_5$, for the entrance to the Earth atmosphere.

Some energy characteristics of the trajectories obtained are corrected, too. The impulse approximation for the LEO acceleration is changed adding the "gravity losses" δV_{gr1} to the impulse ΔV_1. This additional impulse we defined by two methods. One is by direct numerical integration of the SC motion equations for the acceleration with the limited thrust value of "Fregat" stage and optimal orientation. Another is according to its theoretical analysis (References 12, and 13):

$$\delta V_{gr} \approx 0.019(\mu / r^3) t_e^2 \Delta V . \tag{14}$$

Here t_e is the engine time for the limit thrust realization of the impulse ΔV at the distance r from a center with the gravity parameter μ. Both methods results are in good agreement. To decrease this additional velocity δV_{gr}, we usually consider a case of two sub-impulses, two engine operations which are separated by a passive revolution. An analysis gave the optimal part of the first sub-impulse:

$$\Delta V_{11} \approx 0.4\ \Delta V_1.$$

For the trajectory 3, δV_{gr1} is taken of ~ 21 m/s. For the trajectory 16, δV_{gr1} is taken of ~ 15 m/s. Gravity losses for maneuvers $\Delta \mathbf{V}_2$ and $\Delta \mathbf{V}_3$ near Apophis are very small.

The specific thrust for the near-Earth acceleration is taken by 333 s, the exhaust velocity $W_{1E} \approx$ 3267 m/s according to the Lavochkin Association recommendation. It was also recommended to take the separated mass of the Fregat stage depending on the fuel mass required (Reference 14).

To take into account the fuel mass consumptions for the trajectory corrections and for the control near the asteroid, we consider approximately that the correction velocity impulses are 50 m/s before asteroid, 25 m/s after it, and 10 m/s for the near-asteroid control.

To compensate the possible variations of the second engine parameters, the guaranty supply of its fuel in size of 0.01 m_{F2} (12) is introduced.

To study the Apophis characteristics, it is important to leave its satellite and a surface device. Their masses are taken by 10 kg for the satellite (according to the Lavochkin Association recommendation) and 20 kg for the surface device.

If we take into account these all factors, for the rocket "Soyuz-FG" the payload mass is $m_P \approx 158$ kg for the trajectory 3 with Δt=450 days; and $m_P \approx 230$ kg for the trajectory 16 with Δt=690 days. The use of the rocket "Soyuz-2" with initial mass m_0=8250 kg increases the payload mass to $m_P \approx 208$ kg for the trajectory 3 and to $m_P \approx 301$ kg for the trajectory 16. Using the rocket "Zenith" increases some more the SC possible payload mass - to $m_P \approx 545$ kg for Δt=450 days; and to $m_P \approx 685$ kg for Δt=690 days (three-impulse near-Earth acceleration is considered here).

Thus, the space expedition to the asteroid Apophis with the return to Earth, in principle, can be implemented using conventional propulsion systems with a high thrust engines and the modern rockets like "Soyuz", "Zenit". Nevertheless, it is seen that the use of more advanced electro-jet propulsion thrusters can significantly improve the energy characteristics of the Mission (References 5, and 6).

ANALYSIS OF SC MOTION NEAR ASTEROID

The spacecraft motion near the asteroid is analyzed to model the dynamics of the satellite of the asteroid. Two perturbations are taken into account: the effect of far celestial bodies (Sun, Earth, Venus, and Jupiter) and the influence of the non-spherical structure of Apophis. The equations of the SC asteroid-centric motion are used for this. The gravity parameter of Apophis μ_A is taken in a range 1.8-2.86 m^3/s^2. The calculations and analysis are performed for the short 7-days SC motion near the asteroid and for the long 5-years satellite motion after the SC departure. The inclination of the orbit is (0; 45º; 90º). Initial orbit is circular with a radius in range (0.5-5) km.

Perturbation accelerations from the far bodies in (13) are taken in a modified form (see, e.g., Reference 15) to improve the calculation accuracy.

The main result of the analysis is that these motions of the spacecraft and satellite are stable enough for the conditions considered.

Figures 7-11 give the orbit evolution for the effect of the far celestial bodies gravity, for the initial asteroid position corresponding to the SC optimal trajectory 16.

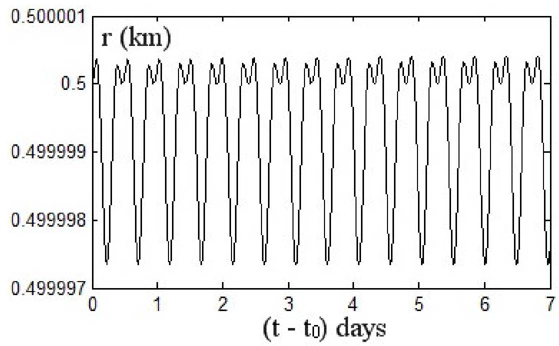

Figure 7. Distance to the asteroid center *r*, r_0=0.5 km (effect of far bodies gravity).

For the SC initial circular orbit with the radius r_0=0.5 km, Figure 7 gives the distance to the center *r* versus the time during 7 days. Perturbations are very small. So, the radius changes at about 3 mm. If initial radius increases as much as two times, the variations of the SC motion increases at about one and a half order, being small enough and increasing to ~ 25 m for r_0=5 km.

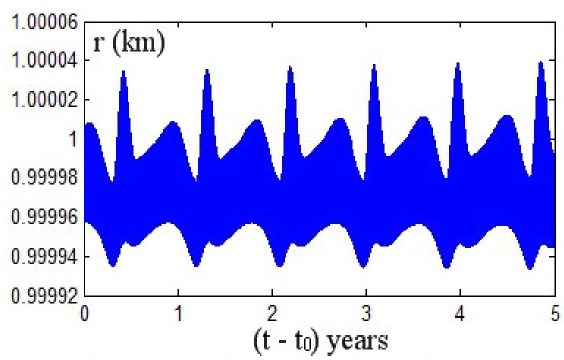

Figure 8. Distance to asteroid center during 5 years, r_0=1 km (effect of far bodies gravity).

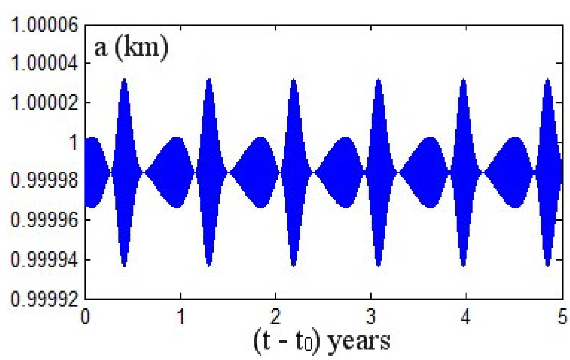

Figure 9. Semi-major axis, a_0=1 km (effect of far bodies gravity).

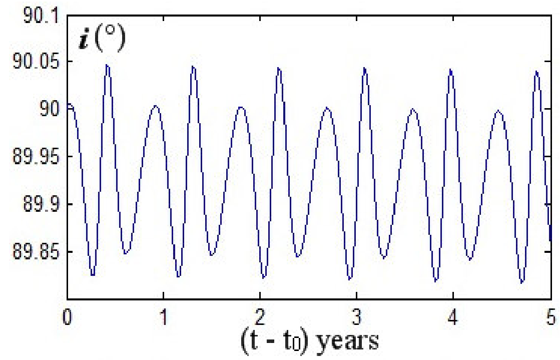

Figure 10. Inclination, a_0=1 km (effect of far bodies gravity).

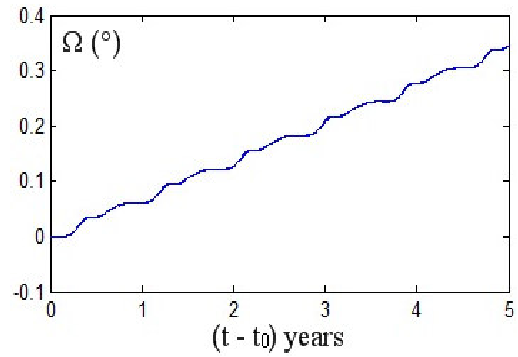

Figure 11. Ascending node Ω, a_0=1 km (effect of far bodies gravity).

Figures 8-11 give the variation of this distance as well as the semi-major axis a, inclination i, and ascending node Ω of the spacecraft orbit during 5 years for initial circular orbit with the radius 1 km, and for the same asteroid initial position. Perturbations here are more but nevertheless the satellite orbit is stable enough. The radius variation is less that 1 m. If the orbit radius increases to 5 km, the distance to the center of asteroid varies at ~0.3 km, and semi-major axis varies at ~0.04 km.

An effect of the no-spherical structure of the asteroid is approximately analyzed, too. Various methods of this analysis are used, e.g., see References 16-18. At this stage of the study, we used an approximate model of homogeneous ellipsoid of rotation around its axis of minimal moment of inertia, that is a prolate spheroid, e.g., see References 18-21.

If R_A, a (=b) and c (>a) are middle radius of asteroid, semi-minor and semi-major axes of its ellipsoid, so the lengthening of ellipsoid is $\alpha = c/a$. The motion is analyzed for α = (1.1; 1.5; and 2). Here, the results for the main variant is shown where α is equal to 2, then $a \approx 127$ m, $c \approx 254$ m for R_A =160 m. It is supposed that there is one-axis rotation of the ellipsoid around its long axes that has constant orientation in space.

Figure 12 gives the distance to asteroid center r during 7 days if initial orbit is a circular one with the radius r_0=0.5 km. Figure 13 gives this distance r during 5 years if initial orbit is a circular one with the radius r_0=1 km. Here the effect of the ellipsoid gravity is only taken into account. We can see that the perturbation is bigger in this case than in the previous one being a small enough nevertheless. It reaches ~16 m during 7 days and ~18 m for 5 years. If the initial radius r_0

increases, the perturbation decreases and the r variation is about 9 m for r_0 =1 km (Figure 13), and ~2 m for r_0 =5 km.

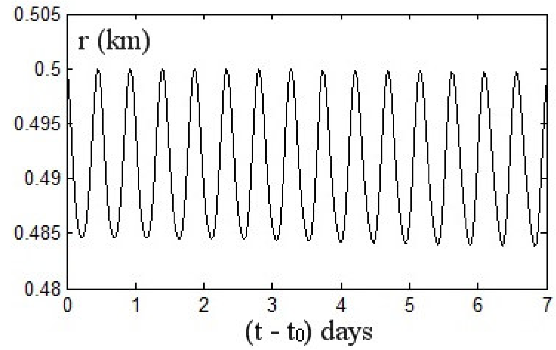

Figure 12. Distance to the asteroid center r, during 7 days, for r_0=0.5 km, under effect of ellipsoid gravity.

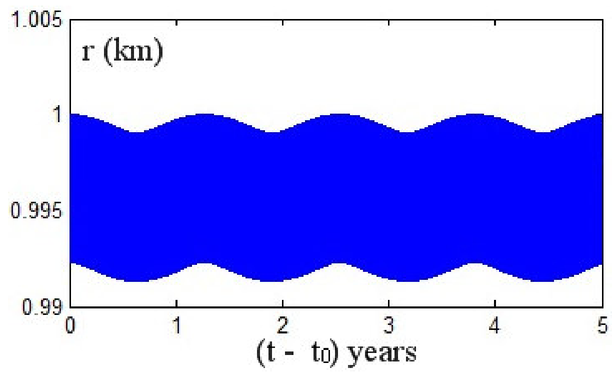

Figure 13. Distance to the asteroid center r, during 5 years, for r_0=1 km, under effect of ellipsoid gravity.

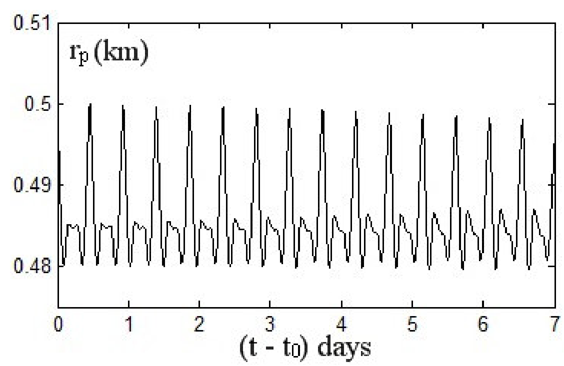

Figure 14. Pericenter radius r_p during 7 days, for r_0=0.5 km under summary effect of far bodies and ellipsoid gravity.

Summary effect of the far celestial bodies and no-spherical structure of the asteroid for the prolate spheroid model is considered, too. Figure 14 gives the pericenter radius r_p of the SC orbit for the 7 days flight duration, for the initial orbit radius r_0=0.5 km, and for α=2 (the distance r is here practically given by Figure 12). The radius r_p variation is ~20 m here. It is determined mainly by the no-spherical structure. With increasing the orbit radius r_0, this variation decreases to ~8 m for r_0=1 km, then to ~4.5 m for r_0=2 km, then it increases to ~27 m for r_0=5 km being determined mainly by the far bodies gravity. Figure 15 gives the distance to the asteroid center *r*, during 5 years, for r_0=2 km, where the minimum variations are approximately realized.

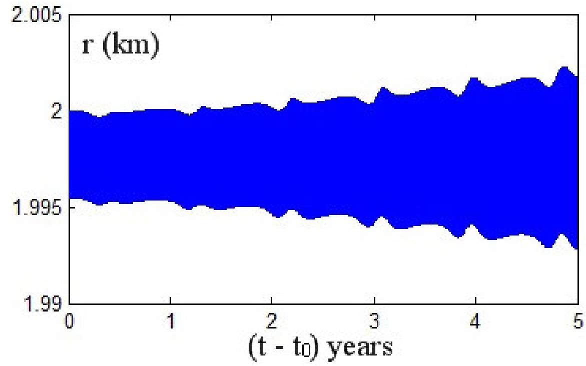

Figure 15. Distance to the asteroid center *r*, during 5 years, for r_0=2 km under summary effect of far bodies and ellipsoid gravity.

This analysis of the orbit evolution for the 5 years duration shows the orbit stability for the perturbation approximate model that is considered.

CONCLUSIONS

The energy optimal space trajectories for the space Mission Earth-Apophis-Earth are determined in the paper. The quantitative evaluation of the characteristics for this flight is performed in condition that the rocket "Soyuz" (or “Zenith”) launches the spacecraft to the LEO and the SC escape from LEO is performed by the high thrust engine of the upper stage "Fregat". Then, for following maneuvers – for the corrections and for the maneuvers of the acceleration and deceleration near the asteroid – another jet propulsion system with a chemical high thrust engine is proposed to be used.

An analysis has shown that the mission may be performed in the 2019-2021 using these rockets and high thrust engines. The trajectories are received for various durations of the mission. The satellite near the asteroid and the surface device are supposed to be left to continue the studies of Apophis after the spacecraft departure from the asteroid. The payload spacecraft mass after separation of the engine unit is about 200-680 kg for different mission duration if the rockets Soyuz-2 or Zenith are used. Of course, using the electro-jet propulsion engines can improve the energy characteristics of the Mission.

The SC and satellite motions on the orbit around the asteroid are analyzed, too. The first stage of the analysis showed that it is possible to make stable enough orbits for the spacecraft and satellite during a long enough time of about several years.

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