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THE POPULATION OF NEAR-EARTH ASTEROIDS

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ABSTRACT In this paper we briefly describe the methodology used to estimate the population (size-frequency distribution) of NEAs using the “re-detection ratio” of surveys over a recent interval, in this case two years from 2012 to 2014. We find this a more robust parameter than attempting to estimate population with a controlled sample from a single survey instrument, which suffers from small numbers. We report our latest population estimate, employing some improvements in modeling over previous (2010, 2012) estimates. Our most recent estimate is in very good agreement with the recent WISE population estimate at the largest size (D > 1 km), and recent estimates at the smallest size range from bolide frequencies.

INTRODUCTION For more than a decade I have been estimating the population of NEAs from discovery statistics of the various surveys in progress, using the method of “re-detection ratio” first elaborated by D’Abramo et al. (2001). This method involves tabulating, in discrete size bins, in our case 0.5 magnitude bins of absolute magnitude H, the number of NEAs seen by a given survey over the most recent time interval, we normally use 2 years. In each size bin, we track the fraction of detected objects that are re-detections of already known asteroids, and how many are new discoveries, seen for the first time. The re-detection ratio is just the ratio of number of re-detections to the total number detected:

$$R(H) = n_r(H)/[n_r(H)+n_d(H)] \qquad (1)$$

In our first population estimates, as reported in D’Abramo et al. (2001), we equated the re-detection ratio with the completion, $C(H)$, in that size range. In that formulation, the total population in a size bin, $n_{tot}(H)$, is simply:

$$n_{tot}(H) = n_d(H)/C(H) = n_d(H)/R(H). \qquad (2)$$

If all asteroids were equally easy to discover, this would be true. But all asteroids are not equally easy to discover, or to re-observe, some are in orbits that make them intrinsically more observable than others, and we expect any survey will tend to find more of the easier ones first, and re-observe them more often, thus the true completion is bound to be less than the re-detection ratio, which translates to an underestimate of the full population. In order to make a more accurate population estimate, we need to evaluate the relation between the re-detection ratio and the actual completion. We do this with a computer simulation, using a set of synthetic orbits that matches as closely as possible the orbit distribution of the actual NEAs, and attempt to match as closely as possible the observing situation of the real surveys, in terms of cadence, sky coverage, limiting magnitude, “trailing loss” due to target motion, and a host of other parameters. In the computer simulation, we track the re-detection ratio, but since we also know the number of objects used, we can compute the completion for the simulated survey and thereby estimate the relation between $R(H)$ and $C(H)$. With the relation between $R(H)$ and $C(H)$ in hand, we then fit the computed $R(H)$ to the actual survey values of $R(H)$ in the most recent two years to effectively "calibrate" the current survey, so that we can then infer the actual completion $C(H)$ of the present survey, and from that estimate the actual full population of NEAs versus size. Our analysis methods and results are presented in greater detail in Harris and D'Abramo (2015).

SYNTHETIC NEA ORBITS In order to accurately simulate a survey, we need to have a set of synthetic orbital elements that closely matches the distribution of orbits of the real NEA population. To estimate the distribution of orbits, we took the thousand largest (lowest H magnitude) discovered NEAs as a reference population. this amounted to NEAs with $H < 18.0$, for which we estimate, after the fact, that current completion is around 90%. Thus, the bias from easier or harder to discover orbits should be fairly minimal. In order to generate a larger sample of orbits (100,000 used in the current simulations), we used random number generation "molded" to fit the distribution of the 1,000 real elements. However, the real elements (a,e,i) are not mutually independent, they are correlated in various ways, in addition to observational biases. We found a somewhat less correlated set of variables to be perihelion distance, q, and aphelion distance, Q, rather than semi-major axis, a, and eccentricity, e, so we actually selected sets matching (q,Q,i) of real NEAs. With these variables, we also augmented the distribution using a nearly constant function of Q going down to 0.95 AU, whereas the observed distribution is very deficient below 1.0 AU for the obvious reason that such objects never come to opposition, or even reach greater than 90° elongation from the sun. We also augmented the number with large Q, although we truncated that function at about Jupiter-crossing, where most such NEAs are quickly removed from the population by Jupiter perturbations. We made these modifications somewhat iteratively, such that when the adjusted populations were run through the computer model to a level of 90% completion, the distribution of "discovered" objects closely matched the real discovered population used to define the distribution. We further limited the synthetic orbits to exclude any that stayed far from any planet orbit, that is with low eccentricity and semi-major axis far from any planet, because such orbits are dynamically difficult to get into by planetary encounters. A last restriction, to which we will return later, was to eliminate any orbit with an Earth encounter velocity, v_∞, less than 2.5 km/sec. This is because such a low encounter velocity implies that the object cannot reach a neighboring planet, Venus or Mars, no matter in which direction it might be gravitationally scattered by the Earth, or conversely, cannot have come from an orbit that could have been scattered by Venus or Mars. Since such inter-planet encounters are the normal way objects get into Earth-crossing orbits, such low-v_∞ orbits are essentially (but not quite) non-existent. More on that later.

After selecting a set of randomly generated distributions as above, we converted to regular elements, a, e, i, and then assigned for each orbit random values of the angle variables mean anomaly, M, argument of node, Ω, and argument of perihelion, ω, uniformly distributed from zero to 360°. We did not assign specific values of absolute magnitude, H, for each object, as will be explained presently.

SURVEY SIMULATION The next step was to compute positions and essential ancillary data for dates following assumed cadences over the survey simulation interval, initially ten years, but most recently expanded to twenty years. We used lunar months rather than calendar months, since actual surveys are modulated to some degree by the moon. Our nominal cadence was three visits of a given area per month, spaced by five days each. Thus the ephemeris set for each object consists of 244 “months”, for 20 years, each with 3 “observations”, or a total of 732 ephemeris points for each object, for a total of 73.2 million ephemeris points for the 100,000 objects. In addition to RA and Dec for each time, we tabulate longitude and latitude, rate of motion, solar elongation, solar phase angle, heliocentric and geocentric distances, galactic latitude, and the “reduction in magnitude, *dm*:

$$dm = 5\log(r\Delta) + \Phi(\alpha), \tag{3}$$

where r and Δ are the heliocentric and geocentric distances and $\Phi(\alpha)$ is the phase relation at solar phase angle α. Thus, for a given absolute magnitude H, the predicted V magnitude in the sky would be $V = H + dm$.

The second step in the survey simulation is to filter the ephemeris file with another program to determine if an object is “seen” or not. The same ephemeris file can be multiply filtered assuming different conditions for successful detection, including cadence, sky coverage, sky condition (clouds, air mass extinction, moonlight interference, galactic plane limitations, or anything else one might impose). And most importantly, the file can be filtered according to object brightness. The survey limiting magnitude is essentially linearly correlated (anti-correlated) with object absolute magnitude, thus

$$dm_{\mathrm{lim}} = V_{\mathrm{lim}} - H \tag{4}$$

We can then filter the ephemeris file for stepped values of *dm* to obtain a record of “detections” as a function of *dm*. In our most recent simulations, we have used a variable dm_{lim}, linearly improving by 2 magnitudes over the 20 year time span of the survey simulation, which closely matches the performance of the actual surveys over the last 20 years. This file of detections vs. *dm* can then be further analyzed to determine the re-detection ratio, $R(dm)$, for the last two years of the simulated survey, as well as the actual completion, $C(dm)$, since, unlike the real sky survey, we know the total

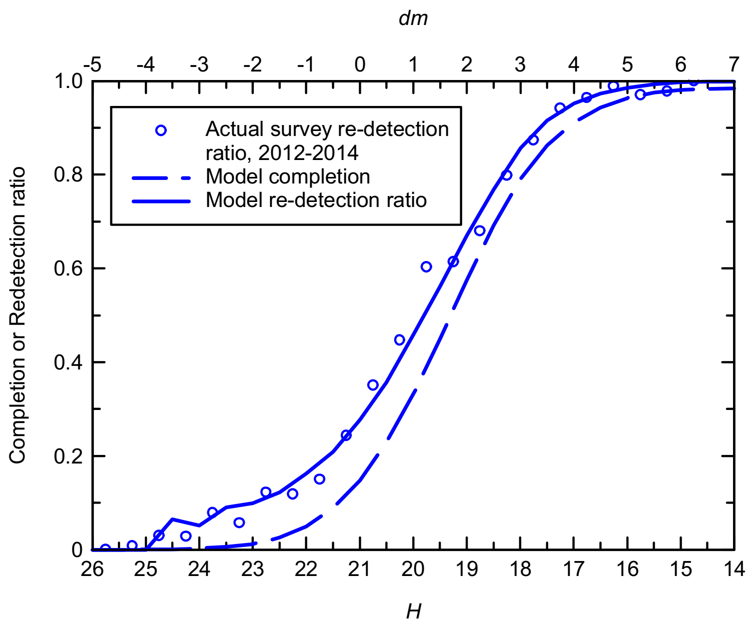

Figure 1. Re-detection ratio and completion versus magnitude for computer simulation and real survey.

number of objects, 100,000, in our simulations. Figure 1 is the plot of R and C versus dm.

The solid line in Figure 1 is the re-detection ratio versus dm, $R(dm)$, and the dashed line is the corresponding completion, $C(dm)$, from the survey simulation. Both are plotted versus dm, the top scale in the figure. The open circle plot points are the actual re-detection ratios of the current surveys (Catalina-703, Mt. Lemmon-G96, Spacewatch 1-691, PanSTARRS 1-F51), versus absolute magnitude H of the objects detected, the bottom scale in the figure. We have adjusted the two scales horizontally to best fit the actual redetection ratios (plot points), with the computed re-detection ratio (curve), thereby essentially "calibrating" the model survey to fit the actual surveys in the range over which $R(H)$ is well defined. Note that the "calibration" has $dm = 0$ equivalent to $H = 21$. From Eq. (4), this implies that V_{lim} of the current surveys is about 21, which is consistent with current performance. The size range over which $R(H)$ is well defined is from about $H = 18$, above which almost no new discoveries were recorded in the last two years, and $H = 23$, below which almost no previously known objects were re-detected. However, once calibrated, the completion function, which we can now call $C(H)$, is well defined by the simulated survey to the very largest sizes, since we have 100,000 objects and not just the very few large NEAs that actually exist. And in the smaller size range, we can extend well below the range where re-detections occur down to the level where only a hundred or so of the 100,000 simulated objects are "detected", that is, to completion of around 0.001. We can extend the completion function even further using an analytical function assuming constant signal-to-noise detection threshold and "particle in a box" motion rather than orbits, since very small objects are detected so close to the Earth that rectilinear motion suffices to model them, and obtain a proportionality $C(H) \propto 10^{-0.8H}$ (see Harris & D'Abramo 2015 for a more complete discussion). Thus we can define the completion function over the entire range of discovered NEAs, from multi-km diameter down to a few meters diameter.

NEA POPULATION

Having now estimated the current completion over the entire range of H magnitude, we can estimate the total population in each size bin according to the first equality in Eq. (2), $n_{\mathrm{tot}}(H) = n_{\mathrm{d}}(H)/C(H)$. We have tabulated the total numbers of discovered objects versus H as of the end of July, 2014. With the completion function described above, we can estimate the total population in each size range, and forming a running sum

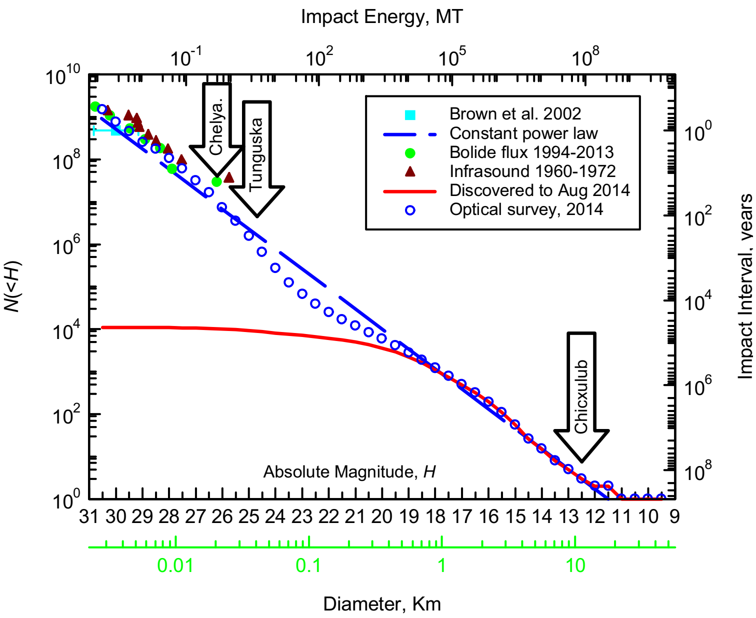

Figure 2. Estimated cumulative NEA population.

from the largest, obtain the estimated cumulative population, $N(<H)$, which is plotted in Figure 2. We also plot some population estimates from bolide statistics (Brown et al. 2013), and the number of actually discovered NEAs versus size. As a word of caution, note that in the smallest size range we are estimating a population of ~10^9 objects based on a sample of only just over 10^4 objects; thus we are depending on our survey model to correctly "de-bias" the discovered numbers by five orders of magnitude. Nevertheless, we are encouraged to note agreement with the bolide frequency numbers, which are of course more accurate in the smaller size range and become tenuous above about 10 m diameter, within a factor of two or less, a remarkable agreement considering all the uncertainties in both estimates.

THE POPULATION OF SMALL, LOW ENCOUNTER VELOCITY OBJECTS There has recently been interest in so-called "ARM target" asteroids, that is, NEAs in the ~10 m diameter size range in very Earth-similar orbits such that they have very low Earth encounter velocities, v_∞ < 2.6 km/sec, and orbits that are within 0.03 AU of intersection with the Earth's orbit (*MOID* < 0.03 AU). These are the limits set on the Asteroid Redirect Mission, ARM (Chodas 2013). Well, the limit on v_∞ is almost identical to the threshold we imposed in the nominal orbit distribution for our simulation, so there are essentially no such objects in the simulation presented above. This does not affect the completion estimates much, since such objects are extremely rare, but also doesn't allow us to answer the question, how many of them are there. But the re-detection method is extremely general and can be used to investigate even rare sub-populations such as this one. To do so, we generated a uniform distribution (in three dimensions) of v_∞ orbits from zero out to 2.5 km/sec. At larger v_∞, the distribution of encounter velocity vectors is not isotropic, but at such low values, multiple close encounters are expected to randomize velocity directions before objects are removed by collisions, and at such low v_∞ they can't reach any other planets to be disturbed. The "uniform" distribution is somewhat more questionable, since unlike molecules in thermal equilibrium, planets selectively eat the slower moving ones, so one expects the actual distribution to be even more deficient in very low v_∞ objects. Nevertheless, the uniform, isotropic distribution seems adequate obtain a rough estimate of the population of these low-v_∞ objects.

We ran out ephemerides of the 100,000 synthetic low-v_∞ orbits for 20 years for the same cadence as assumed for the regular survey, and then ran the same filters as with the previous survey to estimate the fraction discovered after 20 years. We applied the same "calibration" as found for the regular population, $dm = 21 - H$. The result (Figure 3) was remarkable: it appears that the low-v_∞ population is about 1,000 times more efficiently detected than the regular population. In the ~10 m size range, we estimate the

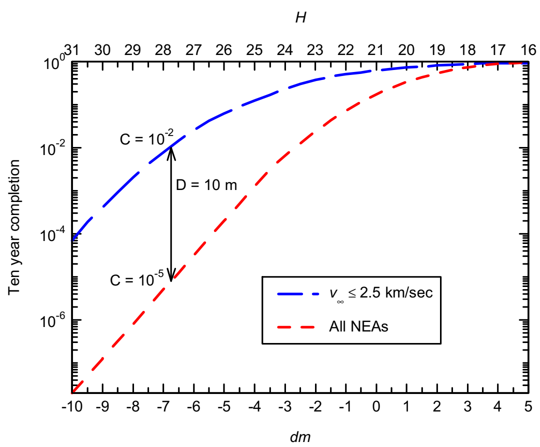

Figure 3. Completion of low v_∞ NEAs compared with completion of average v_∞ NEAs.

completion of current surveys to be only ~10^{-5}, but for the low-v_∞ fraction, we find a completion of ~10^{-2}. Currently there are about 30 such objects discovered, thus we infer that there may be only a few thousand of them total.

An interesting verification of this completion estimate comes from the fact that of the 30 or so discovered objects, two of them are known definitely to be old rocket hardware, and a couple others are suspected of being space junk. We know how many of those are out there, since we put them there. Chodas (2013) estimates about 80 objects in that size range in such heliocentric orbits, thus having serendipitously discovered a couple of them is consistent with ~1% survey completion, indeed even suggests a somewhat higher completion, and hence lower total population of natural objects.

Another source of these low-v_∞ objects is lunar impact ejecta. Gladman et al. (1995) studied the orbital evolution of lunar ejecta and found that of the fraction that escapes immediately from the lunar gravity field, most is subsequently perturbed into heliocentric orbits of very low v_∞, which then evolve over ~10^5 – 10^6 years into ever increasing v_∞ until they either impact the Earth or moon, or become other planet crossing (Mars or Venus) and then diffuse off into the regular NEA cloud. The time scale of ~10^5 years is consistent with cosmic ray exposure ages of lunar meteorites (Gladman et al. 1995). Artemieva and Shuvalov (2008) estimate that the mass of lunar impact ejecta that escapes from the moon is of the order of the mass of the impactors coming in. Thus it seems likely that the mass of lunar ejecta in low-v_∞ heliocentric orbits should be comparable to the mass of impactors striking the moon over 10^5 years or so, which, scaling for the moon from Fig. 2 should be equivalent to a ~150 m diameter ball of rock, but in the form of a distribution of collisional debris of smaller size. It would seem plausible that many, perhaps most, of the ~10 m sized objects we see in these low-v_∞ orbits are actually lunar ejecta.

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REFERENCES

Artemieva, N. A., Shuvalov, V. V., 2008. Numerical simulation of high-velocity impact ejecta following falls of comets and asteroids onto the Moon. *Solar System Research* **42**, 329-334.

Brown, P. G., *et al*., 2013. A 500-kiloton airburst over Chelyabinsk and an enhanced hazard from small impactors. *Nature* **503**, 238-241.

Chodas, P., 2013. Estimated ARM candidate target population and projected discovery rate of ARM candidates. Presentation at Target NEO 2 Workshop, Washington DC, July 9, 2013, available on line at http://targetneo.jhuapl.edu/pdfs/2013_Presentations/session2/Session2_Chodas_Presentation.pptx, and summarized in the workshop report (B. Barbee, ed.) at http://targetneo.jhuapl.edu/pdfs/2013_Presentations/TargetNEO2WorkshopReport.pdf.

D'Abramo, G., Harris, A. W., Boattini, A., Werner, S. C., Harris, A. W., Valsecchi, G. B., 2001. A simple probabilistic model to estimate the population of near-Earth asteroids. *Icarus* **153**, 214-217.

Gladman, B. J., Burns, J. A., Duncan, M. J., Levison, H. F., 1995. The dynamical evolution of lunar impact ejecta. *Icarus* **118**, 302-321.

Harris, A. W., D'Abramo, G., 2015. The population of Near-Earth Asteroids. *Icarus*, submitted.