KINETIC DEFLECTION UNCERTAINTIES FOR REAL ASTEROID SHAPES

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ABSTRACT

The ΔV imparted on an asteroid via a kinetic impactor is a function of uncertain system inputs – the impact location of the deflector and the β parameter of the asteroid – so that the ΔV itself is of a stochastic nature. This study uses modeling techniques to generate an analytic solution for the distribution of the ΔV of an asteroid for a given incoming velocity vector of the impactor. The posterior distribution is then broken down into the relative contributions by each of the system inputs using analysis of variance (ANOVA) parameters known as the Sobol' indices.

STOCHASTIC SYSTEM

Change in velocity of an asteroid resulting from a kinetic impact is defined as:

$$\Delta \boldsymbol{V} = \gamma[\boldsymbol{V}_\infty + (\beta - 1)(\hat{\boldsymbol{n}} \cdot \boldsymbol{V}_\infty)\hat{\boldsymbol{n}}$$

Stochastic input dimensions: β parameter, Impact location → $\hat{\boldsymbol{n}}$

Probability of Hitting a Facet

1. Project facet i onto the $\hat{\boldsymbol{u}} - \hat{w}$ plane
2. Integrate the pdf of the impact location over triangular facet i
3. Identify regions of self-shadowing by other facets and divide the region into triangular components
4. Integrate the pdf of the impact location over the overlapping regions
5. Subtract the probability over the overlapping regions from the original probability of impacting facet i

Gaussian Distribution of $\boldsymbol{\beta}$

ΔV within a facet is linearly related to β, and its distribution is defined by:

$$E(\Delta \boldsymbol{V}) = \gamma[\boldsymbol{V}_\infty + (E(\beta) - 1)(\hat{\boldsymbol{n}} \cdot \boldsymbol{V}_\infty)\hat{\boldsymbol{n}}$$

$$V(\Delta \boldsymbol{V}) = \gamma[\boldsymbol{V}_\infty + (V(\beta) - 1)(\hat{\boldsymbol{n}} \cdot \boldsymbol{V}_\infty)\hat{\boldsymbol{n}}$$

Gaussian Mixtures

A weighted sum of N component distributions can be used to represent an overall pdf

$$p(\boldsymbol{x}) = \sum_{i=1}^{N} w_i g(x|\mu_i, \boldsymbol{\Sigma}_\text{i})$$

The distribution of the $\Delta \boldsymbol{V}$ imparted on an asteroid for a single $\boldsymbol{V}_\infty$ can be represented as a mixture of the Gaussian distributions in β weighted by the probability of hitting each facet.

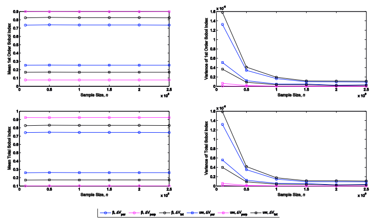
ANOVA

Method of global sensitivity analysis. Provides a measure of the extent to which uncertainties in β and the impact location are reflected in the uncertainty of the resulting $\Delta \boldsymbol{V}$.

Sobol' Indices

$$S_j^1 = \frac{D_j}{D}$$

Sobol' indices are calculated using Monte Carlo simulation. Mean and variance of the indices wrt sample size shows statistical convergence.



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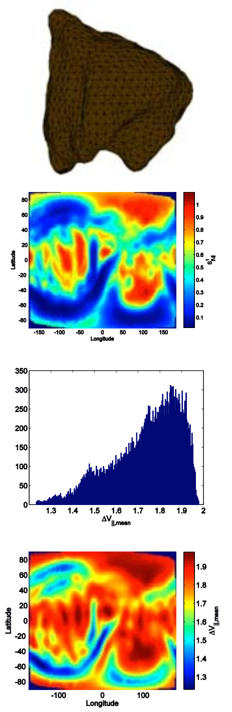

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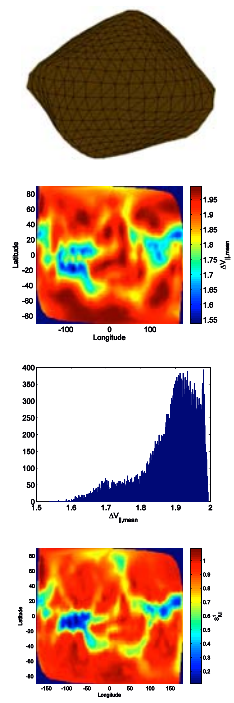
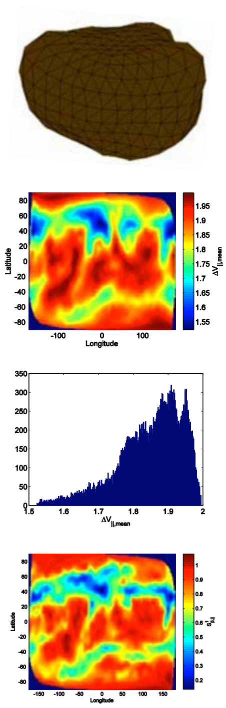


$\Delta \boldsymbol{V}_{||}$

$\Delta \boldsymbol{V}_{||}$

S_β^1

YORP



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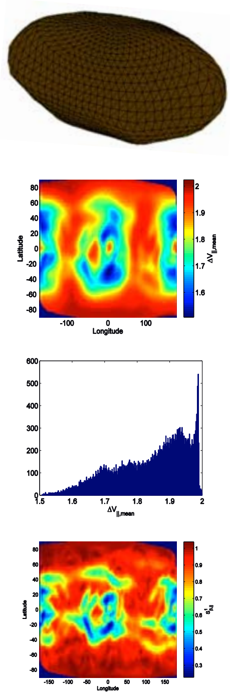

CONCLUSIONS

- Gaussian mixtures method allows for analytic computation of the distribution of the $\Delta \boldsymbol{V}$ imparted on an asteroid, without costly Monte Carlo analysis
- Overall distribution of the $\Delta\mathbf{V}$ is highly dependent on the topography of the asteroid, which can cause variations in the $\Delta\mathbf{V}$ of almost 40%
- Relative influence of uncertain input parameters is dependent on the presence of localized irregularities about the asteroid rather than the general shape of the body

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