IMPROVED DESIGN OF ON-ORBIT SEPARATION SCHEMES FOR FORMATION INITIALIZATION BASED ON $J_2$ PERTURBATION

Jiang Chao,¹ Wang Zhaokui,† and Zhang Yulin‡

Using one Multi-satellite Deployment System to provide each satellite a velocity increment, the initial formation configuration can be formed without any satellite maneuver. To enhance the stability of the formed configurations, relationships between separation parameters and the stability of initial configurations were analyzed at first. Then, new separation schemes were designed to avoid collision for satellites without propulsion systems. And based on $J_2$ invariant conditions, separation schemes were also improved to minimize the relative drifts for satellites with orbit maneuver capability. Results simulated in the high precision numerical environment, show that the improved design of separation schemes are effective.

INTRODUCTION

With the development of the micro-satellite technology, more and more research institutes and universities start to develop their own satellites, most of which are small satellites. To save costs, several small satellites are usually launched together by the same Launch Vehicle. In November 21, 2013, Russia launched a “Dnepr” rocker in Yasniy, send 32 satellites successfully into orbit, breaking a record created by an Orbital Sciences Minotaur I rocket of the United States, placed 29 satellites into orbit in November 19§,**††. The primary payloads were 300-kilogram DubaiSat-2 and 170-kilogram STSAT-3, while the remaining payloads were less than 100 kilogram, and most of them were CubeSats without propulsion systems. For those satellites without orbit maneuver capability, the on-orbit release by separation is the only way to get a better initial orbit, especially for the satellites of formation missions. For those satellites with orbit maneuver capability, the on-orbit separation could provide enough safe time to start up their control systems, as well.

The past study in Reference 1 shows that using one MSDS (Multi-satellite Deployment System) to provide each satellite a velocity increment, the initial formation configuration can be formed by particular separation schemes without any satellite maneuver. This separation schemes design method has been studied to find the design constraint conditions of separation parameters. Based on Hill-Clohessy-Wiltshire equations, the constraint conditions of each velocity increment have been brought out and described in Eq. (1). As the separation velocity increment is provided

¹ Ph.D Candidate, School of Aerospace, Tsinghua University, Beijing, China 10084.
² Ph.D, School of Aerospace, Tsinghua University, Beijing, China 10084.
³ Ph.D, School of Aerospace, Tsinghua University, Beijing, China 10084.
§ http://www.russianspaceweb.com/dnepr_019.html
†† http://www.space.com/23659-night-rocket-launch-record-29-satellites.html
by mechanical device, the constraint of separation means that on-orbit separations must along the direction of x-axis of the reference orbit frame.

\[ v_{\text{sat}} = 0 \]  

(1)

Where, \( v_{\text{sat}} = [v_{x}, v_{y}, v_{z}]^T \) is the velocity increment for payload satellite given by the \( k^{th} \) separation in the reference orbit frame.

However, as orbit perturbations were not considered in the separation schemes design, the stability of initial formation configurations could not be confirmed to keep all separation schemes safe and useful.

In this paper, separation schemes were improved for small satellites in near-circular low Earth orbits. For payload satellites without propulsion systems, new separation schemes were designed to avoid collision. And for payload satellites with orbit maneuver capability, separation schemes were improved to minimize the relative drifts, based on \( J_2 \) invariant conditions.

**STABILITY ANALYSIS OF INITIALIZATION FORMATION CONFIGURATIONS**

To improve the separation schemes, the relationships between separation parameters and the stability of formed initial configurations must be analyzed at first.

As the relative motion of the z-axis direction is independent from the x-axis and y-axis direction, the relative velocity provided by separation of the z-axis direction is not involved in this text. Taking a virtual satellite along the original orbit of MSDS before first separation as reference, then for the \( k^{th} \) payload satellite after separation, its orbit elements are depended on the orbit of MSDS before the \( k^{th} \) separation and the velocity increment provided by the \( k^{th} \) separation.

\[
\delta \sigma_{\text{sat}} = \sigma_{\text{sat}} - \sigma_{\text{MSDS}} = A_{\text{MSDS}}^{-1} (0, v_{x}, v_{y}, v_{z}, 0, 0)^T
\]

(2)

Where, \( \sigma = (a, \theta, i, q_1, q_2, \Omega)^T \) are the osculating orbital elements when \( \theta = \omega + f \), \( q_i = e \cos \omega \), \( q_2 = e \sin \omega \). \( a \) is the semi-major axis, \( e \) is the eccentricity, \( i \) is the inclination, \( \Omega \) is the longitude of the ascending node, \( \omega \) is the argument of perigee, and \( M \) is the mean anomaly. The subscript \( \text{sat} \) means the parameters of the \( k^{th} \) satellites after the separation time of \( t_k \), subscript of \( \text{MSDS} \) means the parameters of MSDS before the separation time of \( t_k \). \( A^{-1} \) is the state transition matrix for Hill states to the orbital elements differences, and is represented in Eq. (3) in.

\[
A^{-1} \begin{pmatrix} 0 \\ v_x \\ 0 \\ v_y \\ 0 \\ 0 \end{pmatrix} = v_i \begin{pmatrix} \frac{2(q_1 \sin \theta - q_2 \cos \theta)}{\eta} \\ 0 \\ 0 \\ \eta \sin \theta \sqrt{\frac{a}{\mu}} \\ -\eta \cos \theta \sqrt{\frac{a}{\mu}} \\ 0 \end{pmatrix} + v_r \begin{pmatrix} \frac{2(1 + q_1 \cos \theta + q_2 \sin \theta)}{\eta} \\ 0 \\ 0 \\ (2 \cos \theta + \frac{q_1 \sin \theta - q_2 \cos \theta}{1 + q_1 \cos \theta + q_2 \sin \theta}) \eta \sqrt{\frac{a}{\mu}} \\ (2 \sin \theta - \frac{q_1 \sin \theta - q_2 \cos \theta}{1 + q_1 \cos \theta + q_2 \sin \theta}) \eta \sqrt{\frac{a}{\mu}} \\ 0 \end{pmatrix}
\]

(3)

Where, \( \eta = \sqrt{1 - q_1^2 - q_2^2} \).
For satellites in low Earth orbits with altitude from 500km to 800km, the primary perturbation effect is due to the equatorial bulge term $J_2$. Then under the influence of $J_2$, the long-term drifts of formation satellite to reference satellite are just depended on the orbital elements of reference satellite, and their initial orbit differences. The geometric transformation between them can be represented by Eq. (4).

$$X(t) = \{ A(t) + 3J_2 R^2 B(t) \} \phi_{\delta}(t, t_o) \delta \sigma(t_o)$$  \hspace{1cm} (4)$$

Where, $X = (x, \dot{x}, y, \dot{y}, z, \dot{z})^T$ are the Hill state parameters in the reference orbit frame. The transition matrix $\{ A(t) + 3J_2 R^2 B(t) \} \phi_{\delta}(t, t_o)$ is formulated in many literatures\textsuperscript{3,4,5,6}, and just depended on orbital elements of the reference satellite.

So the long-term drifts of the $k^{th}$ payload satellite to the MSDS before the $k^{th}$ separation can be represented in Eq. (5).

$$X_{\text{sat}}(t) = \{ A_{\text{MSDS}}(t) + 3J_2 R^2 B_{\text{MSDS}}(t) \} \phi_{\delta_{\text{sat}}}(t, t_k) A_{\text{MSDS}}^{-1}(0, v_{x_k}, 0, 0, 0, 0)^T$$  \hspace{1cm} (5)$$

As the stability of formed initial configurations can be described by the long-term drifts of each payload satellite to the reference satellite, then based on Eq. (5), the relationships between MSDS after each separation and the reference satellite must be analyzed.

Considering that $\delta a$ of MSDS is $10^{-1}$m of magnitude, and $\delta q_1, \delta q_2$ are $10^{-5}$ of magnitude based on Eq. (3) for near-circular low Earth orbits, when separation velocity increment within the range from 1m/s to 3m/s and the mass ratio of payload satellite to MSDS is smaller than 0.1. Then the orbital elements differences between MSDS and the reference satellite can be ignored.

So, the stability of formed initial configurations described by the long-term drifts of each payload satellite to the reference satellite can be approximated in Eq. (6).

$$X_{\text{sat}}(t) \approx \{ A_{\text{sat}}(t) + 3J_2 R^2 B_{\text{sat}}(t) \} \phi_{\delta_{\text{sat}}}(t, t_k) A_{\text{sat}}^{-1}(0, v_{x_k}, 0, 0, 0, 0)^T$$  \hspace{1cm} (6)$$

Based on Eq. (6), the stability of formed initial configurations is just depended on the orbital elements of reference satellite and the velocity increment for payload satellite of each separation.

Then according to Eq. (3), the design constraint conditions formulated in Eq. (1) are useful when $e = 0$ or $\sin f_{\text{ref}} = 0$, because that there aren’t long-term drifts when $\delta a = 0$. It means that the separation schemes design method based on Hill-Clohessy-Wiltshire equations is applicable for exact circular orbit, or for separation schemes of which separations implemented at the right time. However, $e = 0$ is difficult to realize in engineering, then $\sin f_{\text{ref}} = 0$ were considered in the improved design of separation schemes in the following.

**IMPROVED DESIGN OF SEPARATION SCHEMES FOR SATELLITES WITHOUT PROPULSION SYSTEMS**

**Theoretical Analysis**

For payload satellites without propulsion systems, safety is more important than to keep the initialization configurations for a long time. Then the separation initialization schemes should bring suitable drift rates for each satellite to avoid collision.

As the velocity increment provided by on-orbit separation is commonly 1~2m/s, the maximum drift rate for payload satellite is about $10^5$m/day of magnitude, based on Eq. (6) when $v_{x_k} = 0$. So,
even if the separation schemes designed in Reference 1 could provide enough time for payload satellites to work together for formation missions, their formed initial formation configurations have the risk of collision. Then, the value of $v_{ik}$ should be designed to set suitable drift rates for each satellite to avoid collision.

However, the orbital differences between MSDS and the reference satellite can’t be ignored when $v_{ik} \neq 0$ and the on-orbit separations release one payload satellite once. Then, new separation schemes which release two satellites once were considered, to keep MSDS have the same orbital elements with reference satellite.

Firstly, the velocity increments of two satellites released together need to be in opposite directions of the same size, to keep the velocity increment for MSDS equals to zero.

At the same time, $t_k$, $v_{ik}$, $v_{ik}$ need to be designed to keep payload satellites far away enough from each other and to drift apart slowly. As the primary drift is along the direction of y-axis, design constraint conditions could be described in Eq. (7).

$$\begin{cases} y_{sat-k-1}(t) < y_{sat-k+1,i-1}(t) < 0; & \dot{y}_{sat-k-1}(t) < \dot{y}_{sat-k+1,i-1}(t) < 0 \\ y_{sat-k-2}(t) > y_{sat-k+1,i-2}(t) > 0; & \dot{y}_{sat-k-2}(t) > \dot{y}_{sat-k+1,i-2}(t) > 0 \end{cases} \quad (7)$$

Where, the subscript $sat-k-1$ means the parameters of first satellite of $k^{th}$ separation, and $sat-k-2$ means the parameters of second satellite.

Considering that the separation velocity increment is provided by mechanical device, then $v_{ik}$, $v_{ik}$ can be designed through the direction changes of on-orbit separation, when the value of velocity increment is fixed.

**Improved Design and Numerical Evaluation**

Based on Eq. (6) and Eq. (7), the separation schemes which release two satellites once were designed for four payload satellites with different time intervals and separation velocities and listed in Table 1.

<table>
<thead>
<tr>
<th>Schemes Number</th>
<th>$t_k$</th>
<th>$v_{ik}/v_{ik}$</th>
<th>$V_k = \sqrt{v_{ik}^2 + v_{ik}^2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$2\pi/n$</td>
<td>$\alpha_1 = 0.01, 0.9\alpha_1$</td>
<td>$V, V$</td>
</tr>
<tr>
<td>2</td>
<td>$\pi/n$</td>
<td>$\alpha_2 = 0.8, 0.9\alpha_2$</td>
<td>$V, V$</td>
</tr>
<tr>
<td>3</td>
<td>$\pi/n$</td>
<td>$\alpha_3 = 0.15, \alpha_3$</td>
<td>$V, 0.5V$</td>
</tr>
<tr>
<td>4</td>
<td>$\pi/2n$</td>
<td>$\alpha_4 = 0.12, \alpha_4$</td>
<td>$V, 0.5V$</td>
</tr>
<tr>
<td>5</td>
<td>$\pi/4n$</td>
<td>$\alpha_5 = 0.06, \alpha_5$</td>
<td>$V, 0.5V$</td>
</tr>
</tbody>
</table>

Separation schemes designed in Table 1 were simulated in high precision orbital dynamics environment. While,

- The separation velocity error is $10^{-2}$ m/s of magnitude.
- The attitude control error of MSDS is 0.01° of magnitude.
- The gravity mode of 20 × 20 is considered in simulation.
- The simulation time step is 1 sec.

For Scheme 1, $\alpha_1 = 0.01$ was selected to keep the differences between $v_{i,k}$ and $v_{i(k+1)}$ clear enough, and not too much, considering that the attitude control error of satellite is generally 0.01° of magnitude. For Scheme 2, $\alpha_2 = 0.6$ was selected to keep the value of velocity increments the same. $\alpha_3$, $\alpha_4$ and $\alpha_5$ were selected under Eq. (7), when $V_2 = 0.5V_1$.

Then the evolutions of the formed initial configurations are shown from Fig.1 to Fig.5, when $\sigma_{ref} = (6978 km, 0.977, 0.0016, 0, 0)^T$ and $V = 2 m/s$.

Figure 1. Evolutions of the Initial Configuration Formed by Scheme 1 in 2 Days.
Figure 2. Evolutions of the Initial Configuration Formed by Scheme 2 in 2 Days.
Figure 3. Evolutions of the Initial Configuration Formed by Scheme 3 in 2 Days.

Figure 4. Evolutions of the Initial Configuration Formed by Scheme 4 in 2 Days.
Results show that the separation schemes designed under the constraint conditions described in Eq. (7) could provide suitable drift rates for payload satellites to drift apart from each other and to avoid collision. Especially, the maximal relative drifts of Scheme 1 is 11 km/day, it means that the on-orbit separation method could provide at least one or two days for payload satellites without propulsion systems to work together safely.

IMPROVED DESIGN OF SEPARATION SCHEMES FOR SATELLITES WITH ORBIT MANEUVER CAPABILITY

Theoretical Analysis

For payload satellites with orbit maneuver capability, the primary object of improved design is to enhance the stability of the formed initial configurations, as payload satellites have the maneuver capability to avoid collision actively.

According to Eq. (3), the long-term drifts of payload satellites could be quite small for the separation schemes designed based on Eq. (1), when separations implemented at the right time. However, on-orbit separations can’t always implemented at the right time, then the initial conditions of formed configurations to minimize the drift resulting from the $J_2$ perturbation should be analyzed.
The constraints of $J_2$ invariant have been studied in many literatures\textsuperscript{7,8,9}, and can be formulated in forms of Eq. (8).

$$\frac{\delta e}{\delta a} = -\frac{2\eta^i a}{e J_2 R_i^2 (4+3\eta)(1+5\cos^2 i)}$$

(8)

Based on the Lagrange equations of motion, there are:

$$\begin{align*}
\delta a &= \frac{2}{n\sqrt{1-\epsilon^2}}\left[\sin f \delta V_a + (1+e \cos f)\Delta V_i\right] \\
\delta e &= \frac{\sqrt{1-\epsilon^2}}{na}\left[\sin f \delta V_a + (\cos f + \cos E)\Delta V_i\right]
\end{align*}$$

(9)

Combine Eq. (8) and Eq. (9), $\frac{\Delta V_a}{\Delta V_i}$ can be calculated. And for each separation, $\frac{\Delta V_a}{\Delta V_i} = \frac{v_{st}}{v_{st}}$. Then,

$$v_{st} = \left(\frac{e \sin f \left[4a^2\eta^4 - J_2 R_i^2 (4+3\eta)(1+5\cos^2 i)\right]}{e J_2 R_i^2 (4+3\eta)(1+5\cos^2 i)(\cos f + \cos E) - 4a^2\eta^4 (1+e \cos f)}\right)_{ref}$$

(10)

The constraints of velocity increment formulated in Eq. (10) can be implemented by an attitude offset of the MSDS. As the velocity increment provided by on-orbit separation is commonly 1~2m/s, $v_{st}/v_{st}$ is $10^{-3}$ of magnitude for near-circular low Earth orbits, and for the attitude offset of the MSDS is about 0.1 ° of magnitude.

**Improved Design and Numerical Evaluation**

In order to validate the improved design of constraints of velocity increment formulated in Eq. (10), typical separation schemes once release one payload satellite were simulated in high precision orbital dynamics environment described above, with different orbital elements of the reference satellite.

Parameters of improved separation schemes with four payload satellites were given in Table 2.

**Table 2. Parameters for Improved Design of Different Separation Schemes.**

<table>
<thead>
<tr>
<th>$\sigma_{ref\theta}$</th>
<th>Schemes Number</th>
<th>$t_a$</th>
<th>Directions of $v_{st}$</th>
<th>$v_{st}/v_{st}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(6978\text{km}, 90°, 97.7°, 0.0016, 0, 0)^T$</td>
<td>6</td>
<td>$\pi/n$</td>
<td>+</td>
<td>$-1.597\times 10^{-3}, 1.556\times 10^{-3}, -1.531\times 10^{-3}$</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>$\pi/2n$</td>
<td>+</td>
<td>$-1.597\times 10^{-3}, -1.748\times 10^{-3}, 1.557\times 10^{-3}, 1.729\times 10^{-3}$</td>
</tr>
<tr>
<td>$(6978\text{km}, 90°, 63.4°, 0.0016, 0, 0)^T$</td>
<td>6</td>
<td>$\pi/n$</td>
<td>+</td>
<td>$-1.595\times 10^{-3}, 1.583\times 10^{-3}, -1.583\times 10^{-3}, 1.582\times 10^{-3}$</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>$\pi/2n$</td>
<td>+</td>
<td>$-1.595\times 10^{-3}, -1.193\times 10^{-3}, 1.583\times 10^{-3}, 1.177\times 10^{-3}$</td>
</tr>
</tbody>
</table>
When $\mathbf{r}_{0} = (6978\text{km}, 90.00017, 0.0016, 0, 0)^{T}$, evolutions of the formed initial configurations were compared between the improved schemes and unimproved ones. Results are shown below from Fig.6 to Fig.7.

Figure 6. Evolutions of the Initial Configuration Formed by Scheme 6 in 2 Days

When $\mathbf{r}_{0} = (6978\text{km}, 90.00017, 0.0016, 0, 0)^{T}$. 
Figure 7. Evolutions of the Initial Configuration Formed by Scheme 7 in 2 Days
When \( \sigma_{ref} = (6978\text{km}, 90, 63.4, 0.0016, 0, 0)^T \).

As the orbit of \( i = 63.4 \) is usually exceptive, \( \sigma_{ref} = (6978\text{km}, 90, 63.4, 0.0016, 0, 0)^T \) were also simulated, and their evolutions of the formed initial configurations compared between the improved schemes and unimproved ones, are shown below from Fig.8 to Fig.9.
Figure 8. Evolutions of the Initial Configuration Formed by Scheme 6 in 2 Days
When $\sigma_{ref} = (6978\text{km}, 90^\circ, 63.4^\circ, 0.0016, 0, 0)^T$. 
Simulation results of other separation schemes designed in Reference 1 were similar. Results compared from Fig.6 to Fig.9 indicate that the improved constraints of velocity increment for separation schemes are effective to enhance the stability of the formed configurations.

**CONCLUSION**

Using one Multi-satellite Deployment System to provide each payload satellite a velocity increment, the initial formation configuration can be formed by particular separation schemes without any satellite maneuver. To enhance the stability of the formed configurations, relationships between separation parameters and the stability of initial configurations were analyzed at first. Then, new separation schemes were designed to avoid collision for satellites without propulsion systems. And based on $J_2$ invariant conditions, separation schemes were also improved to minimize the relative drifts for satellites with orbit maneuver capability.

The on-orbit separation formation initialization method improved in this paper, could not only provide formation application chances for satellites without orbit maneuver capability, but also save fuel consumption of initialization for satellites with orbit maneuver capability, and provide them enough safe time to start up on-board control systems.

**ACKNOWLEDGMENTS**

This paper is supported by the National High Technology Research and Development Program of China (863 Program), Number: 2012AA120603.

**REFERENCES**


