ON THE KEYHOLE POSITIONS OF APOLHIS

Sokolov L.L.
Saint-Petersburg State University
e-mail: lsok@astro.spbu.ru

ABSTRACT

The structure of initial conditions, corresponding to possible collisions of asteroid Apophis with the Earth, is very complicated (Sokolov et al., Solar System Research 2013, V. 47, N 5, P. 441-447). This structure is similar to fractal structure due to resonant returns. We discuss the case of asteroid Apophis, because its possible collisions have been carefully investigated and Apophis will be hazardous as before despite orbit refinement in 2013. Other hazardous asteroids have similar complicated structure of keyholes.

The method of approaches and collisions detection includes initial conditions variation using Everhart integrator and ephemerides DE405. We change only one variable (semi-major axis). Initial conditions variation in the future (01.05.2035 for Apophis) used as well in order to overcome the loss of accuracy in the case of encounter in 2029. Important characteristics of investigated trajectories, including relative positions and sizes of keyholes, leads to collisions, are stable under small changes of motion model (Sokolov et al., Solar System Research 2012, V. 46, N 4, P. 291-300). The calculations are performed using a computer cluster of the Saint-Petersburg State University.

We derive and discuss the list of possible collisions of Apophis in XXI century. It contains very many keyholes, despite Apophis orbit refinement in 2013. Only essential keyholes are presented now in the NASA website http://neo.jpl.nasa.gov/risk. In addition to 9 possible collisions in XXI century in this website, important keyholes corresponds to collisions in 2055, 2056, 2064, 2066, 2068, 2074, 2075, 2078, 2087, 2098. Only in the region of initial semi-major axes between collisions in 2060 and in 2076 (in the NASA website) located more than 100 keyholes.

To estimate sizes of the keyholes, the range of semi-major axes $\delta a$ at 01.05.2035, corresponding to each collision, was calculated. Alternative method of estimation - using the range of minimum geocentric distances $\Delta r_{\text{min}}$ at 2051 for trajectories, corresponding to each collision.

To avoid a collision, we must move Apophis to the region of semi-major axes without keyholes (Yeomans D.K. et al., 2009 IAA Planetary Defense Conference, Granada, Spain, 2009). We investigate the time dependence of semi-major axis regions, leads to collisions, and evolution with time the regions without collisions. Sizes of regions, leads to collisions, as well as regions without collisions, increase with time. Sizes of regions of possible asteroid motion decrease with time due to the orbit refinement. After 2029 dimensions of regions without collisions, as well as regions leads to collisions, should be multiplied about 10$^5$. The next significant change of dimensions take place in 2051. The value of multiplicator depends on the minimum geocentric distance in 2051, it may be about 10$^4$, 10$^3$.

INTRODUCTION

We investigate possible collisions of asteroids with the Earth, connected with resonant returns. Each possible collision or close approach generate possible approaches or collisions in
the future. Asteroid Apophis demonstrate this property. The first list of resonant returns of Apophis was presented in (Chesley, 2006). Possible close approaches and collisions were presented in (Sokolov et al., 2008), (Yeomans et al., 2009), (Chesley, 2011), (Sokolov et al., 2012), (Farnocchia et al., 2013), (Sokolov et al., 2013) as well. Before 2011 returns after approaches in 2029 and 2036 were discussed, later — after 2029 and 2051. Approach in 2051 is located near nominal orbit of Apophis (Chesley, 2011). Resonant returns generate possible collisions not only for Apophis, but for other hazardous asteroids. It is important to search for and investigate all possible collisions.

The list of more than 100 possible collisions of asteroid Apophis in XXI century derived in the Chair of Celestial Mechanics SpbSU is discussed, including relative positions and sizes of gaps, leads to collisions. In spite of orbit refinement in 2013, Apophis will be a hazardous asteroid.

METHOD

To search for the possible collisions of Apophis with the Earth, we applied the Everhart integrator (Everhart, 1974) and Solar System model DE405 (Standish, 1998).

To separate hazardous trajectories of Apophis, we vary the initial conditions in 2006 (JD=2453800.5) and 2035 (JD=2464448.5) . To search for most if not all possible collisions, it is sufficient to change only one variable (mean motion, or semimajor axis, or some coordinate). The method of collisions separation is fully considered in (Sokolov et al. 2008, Sokolov et al. 2012, Sokolov et al. 2013).

For the calculations the high-performance computational cluster of the Saint Petersburg State University used. This cluster consists of 384 computing cores (Intel Xeon X7560). For our task we occupy no more than 64 of all the cores. The program complex utilizes algorithms of parallelization of data flows to optimize amounts of computation time. For the control and the transfer of data the program uses message passing interface technology (realized as Open MPI).

RESULTS

In the Table 1 the ordered positions $\Delta a$ of Apophis keyholes, dates of collisions, minimum geocentric distances $r_{min}$, and sizes of keyholes $\delta a$ are presented. Collisions in XXI century, placed near nominal orbit are considered. First collision take place in 2055; collisions are found each year after 2055 except 2057, 2063, 2089.

In the Table 2 the ordered positions $\Delta a$ of important Apophis keyholes, dates of collisions, sizes of keyholes $\delta a$, and probabilities $P$ of collisions from NASA website (after probabilities refinement in 2015) are presented.

For the position of any keyhole estimation we use the necessary change $\Delta a$ (m) of semi-major axis leads to the "main" collision in 2068. Such a relative positions of keyholes one can calculate sufficiently exact and independent on the nominal orbit. Presented in the Table 1 values of $\Delta a$ and corresponding "relative" values of Sigma LOV, presented in http://neo.jpl.nasa.gov/risk/a99942.html, are in good agreement; usually the accuracy of linear approximation is about

Regions of initial (JD=2453800.5) semimajor axis without collisions between groups of keyholes have sizes about several tens meters. This sizes change significantly only after close approach to the planet. After 2029 dimensions of regions without collisions, as well as regions leads to collisions, should be multiplied on about $10^5$. The next significant change of dimensions take place in 2051. The value of multiplicator depends on the minimum geocentric distance in 2051. Let consider neighbouring collisions in 2060 (N 21) and in 2055 (N 22); in 2069 (N 123) and in 2068 (N 124).

The first pair: sizes of gaps between 2029 and 2051 is about $10^3$ m. Sizes of region between gaps is about $10^5$ m. After 2051 this values should be multiplied on about $10^4$. Minimum geocentric distances in 2051 are $129 \cdot 10^3$ km (2060) and $93 \cdot 10^3$ km (2055).

The second pair: sizes of gaps between 2029 and 2051 are about $10^6$ m (2069) and $10^3$ m (2068) Sizes of region between gaps is about $10^7$ m. After 2051 this values should be multiplied on about $10^4$. Minimum geocentric distances in 2051 are $783 \cdot 10^3$ km (2069) and $763 \cdot 10^3$ km (2068).

To estimate sizes of the keyholes, the range of semimajor axes $\delta a$ at 01.05.2035, corresponding to each collision, was calculated. Alternative method of estimation – using the range of minimum geocentric distances $\Delta r_{\text{min}}$ at 2051 for trajectories, corresponding to each collision. In the Table 4 one example is presented. The accuracy of such estimations is about 0.1-0.01.

The collisions with large sizes of keyholes are (number, year): (5, 2075), (11, 2064), (20, 2056), (21*,2060), (22, 2055), (36, 2068), (38, 2068), (40, 2078), (77, 2074), (80, 2098), (85, 2066), (92*, 2065), (95*, 2078), (98*, 2091), (108*, 2077), (112, 2087), (124*, 2068), (135*, 2076), (145*, 2068), (156*, 2069).

Possible collisions characteristics (relative positions and sizes of keyholes) are stable under small change of the model of motion. The stability is considered in (Sokolov et al. 2012). One more example: characteristics of collisions in 2056 and in 2068 ("main" collision, N 124*), derived using "old" (2006) and "new" (2013, after orbit refinement) initial conditions. Collision in 2056. Minimum geocentric distances difference is equal to 24 km. Size of keyhole $\delta a$ in 2035 difference is equal to 0.4 m. Collision in 2068. Minimum geocentric distances difference is equal to 1.3 km. Size of keyhole $\delta a$ in 2035 difference is equal to 10 m. Relative positions of keyholes difference is less than 0.1 m. The real accuracy of keyholes characteristics is usually about $10^{-2}$.

In the Table 3 relative values of semi-major axes for collisions in 2075, 2056, 2060, 2055, 2069, 2068, 2076 are presented at 06.03.2006, 28.01.2015, 01.05.2035, 23.04.2052. Relative semi-major axis is the difference between the semi-major axis and the semi-major axis, corresponding to the "main" collision in 2068 at the same time.

**CONCLUSIONS**

The positions of keyholes demonstrate complicated structure similar to fractals structure due to resonant returns of asteroids. This complicated structure must be taken into account in the design of collisions eliminating.
There are many keyholes near Apophis nominal orbit. In spite of orbit refinement in 2012-2013, Apophis will continue a hazardous asteroid.

Our results are in agreement with the results, presented in (Yeomans et al., 2009), (Chesley, 2011), (Farnocchia et al., 2013).

ACKNOWLEDGMENT

This work was partly supported by the Russian Foundation of Basic Research, project no. 14-02-00804, project no 15-02-04340, and by Saint-Petersburg State University grant N 6.37.341.2015.

The author thank N.A. Petrov and A.A. Vasiliev for helpful cooperation.

REFERENCES


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Table 1: Possible collisions of asteroid Apophis with the Earth

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Table 1: Possible collisions of asteroid Apophis with the Earth

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Table 1: Possible collisions of asteroid Apophis with the Earth

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Table 3: Time dependence of relative semi-major axes of collisions (m)

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Table 4: Estimations of keyholes sizes

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<tr>
<td>2069</td>
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Figure 1: The relative position $Da$ of the gaps leading to collisions of Apophis with the Earth and dates of collisions.

Figure 2: The relative position $Da$ and size $da$ of the gaps leading to collisions of Apophis with the Earth.
Figure 3: The relative position $Da$ and size $da$ of the gaps leading to collisions of Apophis with the Earth in the vicinity of the nominal orbit.

Figure 4: The relative position $Da$ and size $da$ of the large gaps leading to collisions of Apophis with the Earth and dates of collisions.